CHAPTER 2

THEORY

2.1 Earthquakes and Radon

Earthquakes result from long term deformation of materials of the crust and upper mantle under the action of tectonic forces. This deformation leads to block instability. Fracturing of blocks occurs as a result of high stress concentration on individual blocks. A block can accumulate a large amount of elastic strain energy that produces fracture instability. A large amount of strain energy instantly released during fracturing is the prime cause of an earthquake.

Fleischer (1981) suggested that changes in a strain field are the most obvious long-term effect expected during a stress buildup prior to an earthquake or during a stress release by the event and its aftershocks. Radon could be affected by opening or closing of cracks that either influences a release of radon or causes its flow with interstitial fluid in the Earth's crust.

A mechanism by which radon anomalies are associated with recent crustal movement suggests that during a slow movement (fault creep) or a rapid movement (earthquake) in the crust, the strain is accumulated and released. As a constituent part of a rock body, fractures in the rocks under stress cause changes in the pore pressure and increase the number of microcracks, causing the migration of radon from a greater depth through the soil surface.

2.2 Movement of Radon in Overburden

Uranium in soils and rocks is the source of most radon. The decay series, beginning with ^{238}U, is a major source of natural radiation exposure. Local high levels of uranium are due mainly to the underlying rock type and its component minerals. A significant uranium daughter is radium, ^{226}Ra, which has a half-life of 1,600 years. Radium's daughter, radon, ^{222}Rn, is the only inert radioactive gas and has a half-life of 3.8 days.

Because the law of radioactive disintegration is simple and unvarying, it is rather easy to calculate the amount of radon generated by a given amount of ^{238}U, provided the radioactive series is in equilibrium at least to the level of radon. However, analysis for the escape of gas from its source and its movement through the overburden is a more complex matter, depending upon a large number of variables, such as the emanation coefficient of the source, the diffusion coefficient of the overburden, and the geometrical configuration of the whole system.

In the case of a radioactive series in equilibrium, it can be shown that

$$\lambda_1 N_1 = \lambda_2 N_2 = \cdots = \lambda_n N_n \qquad (1)$$

where λ_n is the decay constant of isotope n of series, and N_n, is the number of atoms of isotope n. Using this relationship, it can be shown that

1 g of natural uranium is in equilibrium with $3.713\text{x}10^{-7}$ curies or $3.713\text{x}10^{5}$ picocuries (pc) of radon. However, only a fraction of the radon so generated can escape from the generating medium and enter the overburden.

Radon can migrate through rocks and overburden by diffusion, a flow of moving fluid, or by a combination of both.

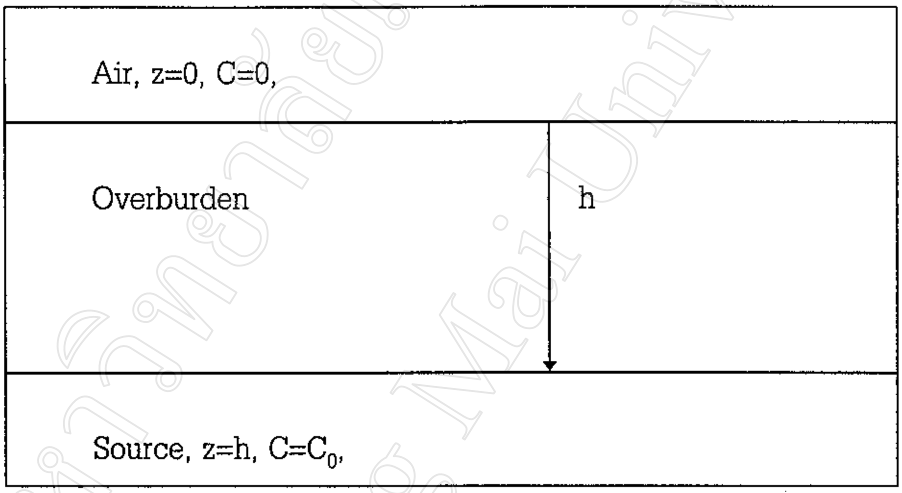

Figure 2.2a An infinite source with overburden.

A two-layer case of infinite source is shown in Figure 2.2a. In this figure, an infinite source is overlain by an overburden of thickness h. This overburden contains no radon source. If radon is transported only by diffusion under steady state conditions (dC/dt =0), radon concentration (C) within the overburden can be represented by the equation:

$$\frac{d^2C}{dz^2} - \frac{\lambda}{D}C = 0$$

where D is the diffusion coefficient and λ is the decay constant. With boundary conditions $C=C_0$ at z=h and C=0 at the surface (z=0), it can be shown that

$$C = C_0 \frac{\sinh\left(z\sqrt{\lambda / D}\right)}{\sinh\left(h\sqrt{\lambda / D}\right)} \tag{2}$$

where C_0 is radon concentration at the source point. Equation (2) was derived assuming that soil air was stationary and the transport of radon was due to diffusion only.

In the case where the overburden contains radon source with production rate P_0, the diffusion transport of radon produced within a homogeneous ground is represented by the equation:

$$\frac{d^2C}{dz^2} - \frac{\lambda}{D} C + P_0 = 0$$

The solution is

$$C = \frac{P_0}{\lambda}\left[1 - \exp\left(-z\sqrt{\lambda / D}\right)\right] \tag{3}$$

In the case of an infinite source with convection in the overburden, the movement of radon is the result of two processes, diffusion and radon flow with a velocity v. The equation for this case where the overburden contains no radon source is:

$$D\frac{d^2C}{dz^2}+v\frac{dC}{dz}-\lambda C=0$$

The solution is

$$C=C_0\exp\left[(h-z)v/2D\right]\frac{\sinh\left(z\sqrt{v^2/4D^2+\lambda/D}\right)}{\sinh\left(h\sqrt{v^2/4D^2+\lambda/D}\right)} \quad (4)$$

where v is the flow velocity in soil (v is positive upward).

The case of diffusion transport plus flow movement of radon in homogeneous ground with production rate (P_0) is represented by the equation:

$$D\frac{d^2C}{dz^2}+v\frac{dC}{dz}-\lambda C+P_0=0$$

The solution is

$$C=\frac{P_0}{\lambda}\left[1-\exp\left(-v/2D-\sqrt{v^2/4D^2+\lambda/D}\right)z\right] \quad (5)$$

A three-layer Earth, which is assumed to represent the study area, where the overburden contains no radon source is shown in Figure 2.2b. Diffusion transport and movement of radon in three layers above the source are as follows:

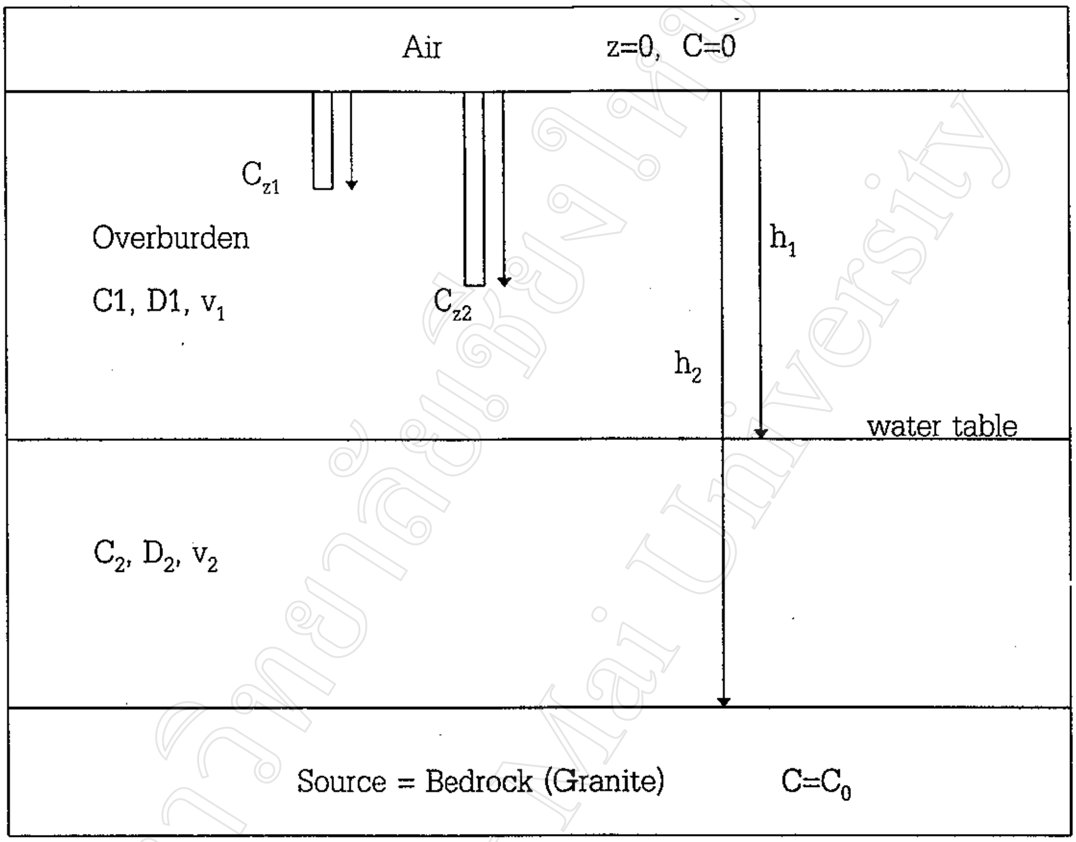

Figure 2.2b. An infinite source with three layers.

For the first layer, overburden:

$$D_1 \frac{d^2C_1}{dz^2} + v_1 \frac{dC_1}{dz} - \lambda C_1 = 0$$

For the second layer:

$$D_2 \frac{d^2C_2}{dz^2} + v_2 \frac{dC_2}{dz} - \lambda C_2 = 0$$

With the following boundary conditions:

a) $z=0$, $C_1=0$

b) $z=h_1$, $C_1=C_2$, $-D_1 \dfrac{dC_1}{dt} = -D_2 \dfrac{dC_2}{dt}$

c) $z=h_2$, $C_2=C_0$

The solution for the first layer and the second layer are as follow:

$$C_1(z) = 2C_0\left[\frac{(D_2 b_2 - D_2 a_2)\exp(h_1 - h_2) b_2}{M\exp(b_2 - a_2)(h_1 - h_2) - N}\right]\exp\left[-v_1 z/2D_1\right]\sinh\left(z\sqrt{v_1^2/4D_1^2 + \lambda/D_1}\right) \quad (6)$$

$$C_2(z) = C_0 \exp(-b_2 h_2)\left[\frac{N\exp(b_2 z) - M\exp(b_2 h_1 - a_2 h_1 + a_2 z)}{N - M\exp(b_2 h_1 - a_2 h_1 - b_2 h_2 + a_2 h_2)}\right] \quad (7)$$

where $M = (D_2 b_2 - D_1 a_1)\exp(a_1 h_1) + (D_1 b_1 - D_2 b_2)\exp(b_1 h_1)$

$N = (D_2 a_2 - D_1 a_1)\exp(a_1 h_1) + (D_1 b_1 - D_2 a_2)\exp(b_1 h_1)$

$a_1 = -v_1/2D_1 + \sqrt{v_1^2/4D_1^2 + \lambda/D_1}$

$a_2 = -v_2/2D_2 + \sqrt{v_2^2/4D_2^2 + \lambda/D_2}$

$b_1 = -v_1/2D_1 - \sqrt{v_1^2/4D_1^2 + \lambda/D_1}$

$b_2 = -v_2/2D_2 - \sqrt{v_2^2/4D_2^2 + \lambda/D_2}$

There fore, the ratio of radon concentration in the first layer is:

$$\frac{C_{z1}}{C_{z2}} = \exp\left[\left(z_2 - z_1\right)v_1/2D_1\right]\frac{\sinh\left(z_1\sqrt{v_1^2/4D_1^2 + \lambda/D_1}\right)}{\sinh\left(z_2\sqrt{v_1^2/4D_1^2 + \lambda/D_1}\right)} \quad (8)$$

where C_{z1} is radon concentration at $z=z_1$, C_{z2} is radon concentration at $z=z_2$.

2.3 The mathematical model used to distinguish the variation of radon concentration due to meteorological factors from the total radon anomaly

The mathematical model used to distinguish the variation of radon concentration due to meteorological factors from the total radon anomaly , is more exact than the observation base. There now follows a discussion of the mathematical model: Let Y(n), n=1,2,...N be a time series representing the radon concentration. For a given discrete time n, Y(n) can be expressed as a sum of two terms. The first of these terms is linearly correlated with environmental parameters, such as soil temperature, precipitation, and atmospheric pressure. The second term, denoted Ψ, is not correlated to any environmental parameter and is the component that is interesting for monitoring subsurface phenomena. Therefore, if $x_1, x_2, ..., x_m$ are time series representing the environmental parameters, the general formulation of this assertion is expressed as follows:

$$Y = [\, \Gamma_1 * X_1 + \Gamma_2 * X_2 + ... + \Gamma_m * X_m \,] + \Psi \qquad (9)$$

The row vectors: $\Gamma_1 = [\, \gamma_1(0) ... \gamma_1(p) \,], ..., \Gamma_m = [\, \gamma_m(0) ... \gamma_m(p) \,]$ are the so-called impulse responses of order p. The asterisk stands for the discrete convolution product and equation (9) can be written as:

$$Y(n) = \sum_{i=1}^{m} \sum_{k=-\infty}^{\infty} \gamma_i(n-k) X_i(k) + \Psi(n)$$

$$Y(n) = \sum_{i=1}^{m} \sum_{k=-\infty}^{n} \gamma_i(n-k) X_i(k) + \Psi(n)$$

$$Y(n) = \sum_{i=1}^{m} \sum_{k=0}^{\infty} \gamma_i(k) X_i(n-k) + \Psi(n) \qquad (10)$$

If p is the order of the multichannel linear system, equation (10) can be written as:

$$Y(n) = \sum_{i=1}^{m} \sum_{k=0}^{p} \gamma_i(k) X_i(n-k) + \Psi(n)$$

$$Y(n) = \sum_{k=0}^{p} \Gamma(k) X(n-k) + \Psi(n) \qquad (11)$$

where, $\Gamma(k) = [\gamma_1(k) \ldots \gamma_m(k)]$ $$X(n-k) = \begin{bmatrix} X_1(n-k) \\ \vdots \\ X_m(n-k) \end{bmatrix}$$

Equation (10) can be written as the block vector inner product:

$$Y(n) = \underline{\Gamma}_p . \underline{X}_p(n) + \Psi(n) \qquad (12)$$

where, $\underline{\Gamma}_p = [\Gamma(0) \ldots \Gamma(p)]$ and $$\underline{X}_p(n) = \begin{bmatrix} X(n) \\ \vdots \\ X(n-p) \end{bmatrix}$$

Multiplying the two left-hand members of equation (11) by $\underline{X}_p^T(n)$, the transposed matrix of $\underline{X}_p(n)$, and taking the expectation:

$$E\left\{Y(n).\underline{X}_p^T(n)\right\} = E\left\{\underline{\Gamma}_p.\underline{X}_p(n).\underline{X}_p^T(n)\right\} + E\left\{\Psi(n).\underline{X}_p^T(n)\right\}$$

$$= \underline{\Gamma}_p\ E\{\ \underline{X}_p(n).\ \underline{X}_p^T(n)\ \} \quad (13)$$

Since $E\left\{\Psi(n).\underline{X}_p^T(n)\right\} = 0$, because the time series Ψ is not correlated with the time series $X_1, X_2, \ldots, X_m$ by definition, which does not depend on the index n, due to the stationary nature of time series

$$\Rightarrow \underline{R}_p^{yx} = \underline{\Gamma}_p\,\underline{R}_p^{xx} \quad (14)$$

Thus, the row block vector solution $\underline{\Gamma}_p$ is given by the relationship:

$$\underline{\Gamma}_p = \underline{R}_p^{yx}\,\underline{R}_p^{xx^{-1}} \quad (15)$$

where $\underline{R}_p^{yx}$ is the row correlation vector of block dimension *p+1*

$$\underline{R}_p^{yx} = \left[R^{yx}(0) \ldots R^{yx}(p)\right]$$

$$R^{yx}(k) = \left[E\left\{Y(n).X_1(n-k)\right\} \ldots E\left\{Y(n).X_m(n-k)\right\}\right]$$

$$R_j^{yx}(k) = \frac{1}{N-k+1}\sum_{n=k}^{N} Y(n)X_j(n-k)$$

and $\underline{R}_p^{xx}$ is the symmetrical block-Toeplitz correlation matrix of block dimension (p+1)x(p+1):

$$\underline{R}_p^{xx} = \begin{bmatrix} R^{xx}(0) & R^{xx}(1) & \ldots & R^{xx}(p) \\ R^{xx}(-1) & R^{xx}(0) & \ldots & R^{xx}(p-1) \\ \vdots & \vdots & \ldots & \vdots \\ R^{xx}(-p) & R^{xx}(-p+1) & \ldots & R^{xx}(0) \end{bmatrix}$$

$$R^{xx}(k) = E\left\{X(n).X^T(n-k)\right\}$$

$$R_{i,j}^{xx}(k) = \frac{1}{N-k+1} \sum_{n=k}^{N} X_i(n) X_j(n-k)$$

In order to make the system $\underline{R}_p^{yx} = \underline{\Gamma}_p \underline{R}_p^{xx}$ well posed, a regularization method must be applied. The method proposed here is the so-called Tikhonov regularization method, adapted to the special case of an Hermitian block-Toeplitz correlation matrix in the frame of this work.

The estimator $\underline{\tilde{\Gamma}}_p^{\mu} = \underline{\tilde{R}}_p^{yx}\left(\underline{\tilde{R}}_p^{xx} + \mu I\right)^{-1}$ is regularized, I being the m (p+1)xm(p+1) size unity matrix and μ being a positive number, known as a regularization parameter.

Substitute equation (15) in equation (12), the result is:

$$\Psi(n) = Y(n) - \left[\frac{Y(n) \cdot \underline{X}_p(n-k)}{\underline{X}_p(n) \cdot \underline{X}_p{}^T(n-k)}\right] X(n) \qquad (16)$$

Equation (16) can be simply solved for $\Psi(n)$ because the parameters Y(n) and X(n) are known from the observation. Y(n) is a time series of radon concentration and X(n) is a time series of meteorological data, such as soil temperature, precipitation, and barometric pressure.

The sequence of mathematical model used for processing soil gas radon in this study is shown in Figure 2.3

where Y(n) is the time series of radon concentration,

Y'(n) is the time series of radon concentration relate to meteorological factors,

$X_1(n)$ is the time series of soil temperature,

$X_2(n)$ is the time series of barometric pressure,

$X_3(n)$ is the time series of rainfall

and n is a discrete time (week), n=1,2,...,40.

$$Y(n)=Y'(n)+\Psi(n)$$

$$Y(n) = \underline{\Gamma}_1(n).\underline{X}_1(n)+\underline{\Gamma}_2(n).\underline{X}_2(n)+\underline{\Gamma}_3(n).\underline{X}_3(n)+\Psi(n)$$

$$Y(n)=\begin{bmatrix}\gamma_1(0) & \cdots & \gamma_1(p)\end{bmatrix}\begin{bmatrix}X_1(n)\\ \vdots \\ X_1(n-p)\end{bmatrix}+\begin{bmatrix}\gamma_2(0) & \cdots & \gamma_2(p)\end{bmatrix}\begin{bmatrix}X_2(n)\\ \vdots \\ X_2(n-p)\end{bmatrix}+\begin{bmatrix}\gamma_3(0) & \cdots & \gamma_3(p)\end{bmatrix}\begin{bmatrix}X_3(n)\\ \vdots \\ X_3(n-p)\end{bmatrix}+\Psi(n)$$

↓

$$Y(n) = \underline{\Gamma}_p(n).\underline{X}_p(n)+\Psi(n)$$

$$Y(n)=\begin{bmatrix}\gamma_1(0) & \cdots & \gamma_1(p)\\ \gamma_2(0) & \cdots & \gamma_2(p)\\ \gamma_3(0) & \cdots & \gamma_3(p)\end{bmatrix}\begin{bmatrix}X_1(n) & X_2(n) & X_3(n)\\ \vdots & \vdots & \vdots \\ X_1(n-p) & X_2(n-p) & X_3(n-p)\end{bmatrix}+\Psi(n)$$

↓

$$E\left\{Y(n).\underline{X}_p^T(n)\right\}=E\left\{\underline{\Gamma}_p.\underline{X}_p(n).\underline{X}_p^T(n)\right\}+E\left\{\Psi(n).\underline{X}_p^T(n)\right\} \quad (\to 0)$$

$$\underline{\Gamma}_p = E\{Y(n).\underline{X}_p^T(n)\}E\{\underline{X}_p(n).\underline{X}_p^T(n)\}^{-1}$$

$$\begin{bmatrix}\gamma_1(0) & \cdots & \gamma_1(p)\\ \gamma_2(0) & \cdots & \gamma_2(p)\\ \gamma_3(0) & \cdots & \gamma_3(p)\end{bmatrix}=Y(n)\begin{bmatrix}X_1(n) & X_2(n) & X_3(n)\\ \vdots & \vdots & \vdots \\ X_1(n-p) & X_2(n-p) & X_3(n-p)\end{bmatrix}^T\begin{bmatrix}X_1(n) & X_2(n) & X_3(n)\\ \vdots & \vdots & \vdots \\ X_1(n-p) & X_2(n-p) & X_3(n-p)\end{bmatrix}^{-1}\begin{bmatrix}X_1(n) & X_2(n) & X_3(n)\\ \vdots & \vdots & \vdots \\ X_1(n-p) & X_2(n-p) & X_3(n-p)\end{bmatrix}^{T^{-1}}$$

↓

$$Y'(n)=\begin{bmatrix}\gamma_1(0) & \cdots & \gamma_1(p)\end{bmatrix}\begin{bmatrix}X_1(n)\\ \vdots \\ X_1(n-p)\end{bmatrix}+\begin{bmatrix}\gamma_2(0) & \cdots & \gamma_2(p)\end{bmatrix}\begin{bmatrix}X_2(n)\\ \vdots \\ X_2(n-p)\end{bmatrix}+\begin{bmatrix}\gamma_3(0) & \cdots & \gamma_3(p)\end{bmatrix}\begin{bmatrix}X_3(n)\\ \vdots \\ X_3(n-p)\end{bmatrix}$$

Figure 2.3 The sequence of mathematical model used for processing soil gas radon.