CHAPTER VI

CONCLUSION

In this research, we obtain the main objectives as the followings :

1. We obtain relations among continuous, g-continuous, r-g-continuous, gc-irresolute and r-g-irresolute functions and here are our results.

   1.1 Let X be a $T_{\frac{1}{2}}-space$ and Y be a topological space. Then $f:X \to Y$ is continuous if and only if f is g-continuous.

   1.2 Let X and Y be topological spaces. If $f:X \to Y$ is closed and g-continuous, then f is gc-irresolute.

   1.3 Let X be a $T_{\frac{1}{2}}^{*}-space$ and Y be a topological space. Then $f:X \to Y$ is continuous if and only if f is g-continuous.

   1.4 Let X be a $T_{rg}-space$ and Y be a topological space. Then $f:X \to Y$ is g-continuous if and only if f is r-g-continuous.

   1.5 Let X and Y be topological spaces. If $f:X \to Y$ is regular closed and r-g-continuous, then f is r-g-irresolute.

   1.6 Let X be a $T_{rg}-space$ and Y be topological space. If $f:X \to Y$ is is onto, open and r-g-continuous, then f is r-g-irresolute.

2. We obtain some results concerning of g-open, g-closed, r-g-open and r-g-closed in a product space. The results are as the followings :

   2.1 Let $\{ X_\alpha \mid \alpha \in I \}$ be a family of topological spaces and $G_\alpha \subseteq X_\alpha$ for each $\alpha \in I$. If G_α is g-open in X_α for all $\alpha \in I$ and $G_\alpha = X_\alpha$ for all but finitely many $\alpha \in I$, then $\prod_{\alpha \in I} G_\alpha$ is g-open in $\prod_{\alpha \in I} X_\alpha$.

2.2 Let $\{ X_\alpha \mid \alpha \in I \}$ be a family of $T_{\frac{1}{2}}-spaces$, G_α a nonempty subset of X_α for each $\alpha \in I$ and $Int(\prod_{\alpha \in I} G_\alpha) \neq \varnothing$. Then $\prod_{\alpha \in I} G_\alpha$ is g-open in $\prod_{\alpha \in I} X_\alpha$ if and only if G_α is g-open in X_α for all $\alpha \in I$ and $G_\alpha = X_\alpha$ for all but finitely many $\alpha \in I$.

2.3 Let $\{ X_\alpha \mid \alpha \in I \}$ be a family of topological spaces and $F_\alpha \subseteq X_\alpha$ for each $\alpha \in I$. Then $\prod_{\alpha \in I} F_\alpha$ is g-closed in $\prod_{\alpha \in I} X_\alpha$ if and only if F_α is g-closed in X_α for each $\alpha \in I$.

2.4 Let $\{ X_\alpha \mid \alpha \in I \}$ be a family of topological spaces, G_α a nonempty subset of X_α such that G_α contains a nonempty regular closed subset F_α of X_α for each $\alpha \in I$. If $\prod_{\alpha \in I} G_\alpha$ is r-g-open in $\prod_{\alpha \in I} X_\alpha$ and $Int(\prod_{\alpha \in I} G_\alpha) \neq \varnothing$, then G_α is r-g-open in X_α for each $\alpha \in I$.

2.5 Let $\{ X_\alpha \mid \alpha \in I \}$ be a family of topological spaces and $F_\alpha \subseteq X_\alpha$ for each $\alpha \in I$. If $\prod_{\alpha \in I} F_\alpha$ is r-g-closed in $\prod_{\alpha \in I} X_\alpha$, then F_α is r-g-closed in X_α for each $\alpha \in I$.

3. We have studied g-continuous, r-g-continuous, gc-irresolute and r-g-irresolute functions from any topological spaces into a product space. The results are as the followings :

3.1 Let Y be a $T_{\frac{1}{2}}-space$ and let $\{ X_\alpha \mid \alpha \in I \}$ be a family of topological spaces. Let $f : Y \to \prod_{\alpha \in I} X_\alpha$ be a function. Then f is g-continuous if and only if the composite function $\pi_\alpha \circ f : Y \to X_\alpha$ is g-continuous for each $\alpha \in I$.

3.2 Let Y be a topological space and let $\{X_\alpha | \alpha \in I\}$ be a famjly of topological spaces. Let $f : Y \to \prod_{\alpha \in I} X_\alpha$ be a function. If f is gc-irresolute then the composite function $\pi_\alpha \circ f : Y \to X_\alpha$ is gc-irresolute for each $\alpha \in I$.

3.3 Let Y be a $T_{\frac{1}{2}}^{*}-space$ and let $\{X_\alpha | \alpha \in I\}$ be a family of topological spaces. Let $f : Y \to \prod_{\alpha \in I} X_\alpha$ be a function. Then f is r-g-continuous if and only if the composite function $\pi_\alpha \circ f : Y \to X_\alpha$ is r-g-continuous for each $\alpha \in I$.

3.4 Let Y be a topological space and let $\{X_\alpha | \alpha \in I\}$ be a family of topological spaces. Let $f : Y \to \prod_{\alpha \in I} X_\alpha$ be a function. If f is r-g-irresolute, then the composite function $\pi_\alpha \circ f : Y \to X_\alpha$ is r-g-irresolute for each $\alpha \in I$.

3.5 Let $\{X_\alpha | \alpha \in I\}$ be a family of $T_{\frac{1}{2}}-spaces$ and $\{Y_\alpha | \alpha \in I\}$ be a family of topological spaces. For each $\alpha \in I$, let $f_\alpha : X_\alpha \to Y_\alpha$ be a function and $f : \prod_{\alpha \in I} X_\alpha \to \prod_{\alpha \in I} Y_\alpha$ be defined by $f((x_\alpha)_{\alpha \in I}) = (f_\alpha(x_\alpha))_{\alpha \in I}$. Then f is g-continuous if and only if f_α is g-continuous for each $\alpha \in I$.

3.6 Let $\{X_\alpha | \alpha \in I\}$ be a family of $T_{\frac{1}{2}}^{*}-spaces$ and $\{Y_\alpha | \alpha \in I\}$ be a family of topological spaces. For each $\alpha \in I$, let $f_\alpha : X_\alpha \to Y_\alpha$ be a function and $f : \prod_{\alpha \in I} X_\alpha \to \prod_{\alpha \in I} Y_\alpha$ be defined by $f((x_\alpha)_{\alpha \in I}) = (f_\alpha(x_\alpha))_{\alpha \in I}$. Then f is r-g-continuous if and only if f_α is r-g-continuous for each $\alpha \in I$.

4. We obtain preservation theorems on some topological spaces under g-continuous, r-g-continuous, gc-irresolute and r-g-irresolute functions. The results are as the followings :

   4.1 If $f : X \to Y$ is g-continuous and onto and X is connected*, then Y is connected.

   4.2 If $f : X \to Y$ is gc-irresolute and onto and X is connected*, then Y is connected*.

   4.3 If $f : X \to Y$ is r-g-continuous and onto and X is connected**, then Y is connected.

   4.4 If $f : X \to Y$ is r-g-irresolute and onto and X is connected**, then Y is connected**.

   4.5 Let X be a topological space and Y be Hausdorff. If $f : X \to Y$ is injective and g-continuous, then X is g-Hausdorff.

   4.6 Let X be a topological space and Y be g-Hausdorff. If $f : X \to Y$ is injective and gc-irresolute, then X is g-Hausdorff.

   4.7 Let X be a topological space and Y be Hausdorff. If $f : X \to Y$ is injective and r-g-continuous, then X is rg-Hausdorff.

   4.8 Let X be a topological space and Y be rg-Hausdorff. If $f : X \to Y$ is injective and r-g-irresolute, then X is rg-Hausdorff.