

CHAPTER I

INTRODUCTION

The properties of injectivity and projectivity in the category of modules over a ring play an important role in characterizing or describing the structure of rings.

In 1960 Utumi saw that the class of continuous regular rings is a generalization of the class of self-injective regular rings. He extended the notion of continuity to any ring (The terminology of continuous here is not related to topology and analysis. It comes from the continuous geometry of Von Neumann); Jeremy [6], Mohamed and Bouhy [9], Goel and Jain [4] carried this notion to modules. Later, Chatters and Hajarnavis (1977) introduced the notion of CS-modules that are modules such that every submodule is essential in a direct summand.

Many generalizations of CS-module were introduced in recent years, for example in some papers of P. F. Smith. We now concentrated to a weaker form of CS-modules. A module is called *weak CS* if every semisimple submodule is essential in a direct summand. M is called a CS-module if every complement of a submodule of M is a direct summand of M .

In this thesis, we consider a class of module M in which every semisimple submodule of M is essential in a direct summand of M . We call them weak CS-modules. The results in the thesis are not strong enough to describe the structure

of weak CS-modules, however, we try to develop many details related to this kind of modules. It is well-known that every direct summand of a CS-module is again CS, but for weak CS-modules we do not know in general. It is true when we take a weak CS-module which is UC-module, i. e., every direct summand of a weak CS-module which is UC is a weak CS-module. Also we prove that a direct summand of non-singular weak CS is again weak CS. It is well-known that any finite direct sum of relatively injective CS-modules is a CS-module. For a weak CS-module, we obtain the same result. We hope that these results will play as a tool in proving some statements related to weak CS-modules as in case of CS-modules. The results show that it is interesting to continue studying this kind of modules.

Throughout the thesis, we write the index before the terms "Theorem", "Definition", "Proposition" and "Lemma". The first number stands for the chapter, the second for the section and the third for the order. When we refer "Proposition 2.1.1" for example, this means that "2.1.1 Proposition".

The thesis comprises of four chapters. Chapter I is an introduction. In chapter II we list some well-known results, definitions and notations that we shall use later. Chapter III is a study of direct sums of weak CS, direct summands of weak CS modules and a decomposition theorem for a non-singular weak CS-module. Chapter IV is a summary of the result.