CHAPTER IV

CONCLUSION

In this study, we found that :

1. General properties of principally N- injective modules.

1.1 If M_R is P-N-injective and $A_R \cong M_R$, then A_R is P-N-injective.

1.2 If M_R is P-N-injective and $A_R \cong N_R$, then M_R is P-A-injective.

1.3 M_R is P-N-injective if and only if M_R is P-A-injective for all submodules A of N.

1.4 If M_R is P-N-injective and every cyclic submodule of N_R is projective, then $M/_K$ is P-N-injective for all submodules K of M.

1.5 If M_R is P-N-injective and A_R is a direct summand of M_R, then A_R is P-N-injective.

1.6 A product $\prod\limits_{i \in I} M_i$ is P-N-injective if and only if each M_i is P-N-injective.

1.7 A sum $\bigoplus\limits_{i \in I} M_i$ is P-N-injective if and only if each M_i is P-N-injective.

2. Characterizations of principally N- injective modules.

The following conditions are equivalent for a right R-module M, a cyclic right R-module $N = tR$ and $_N M = \{m \in M \,/\, r_R(t) \subseteq r_R(m)\}$:

(1) M is P-N-injective.

(2) For each $n = ta \in N$ and each $f \in Hom\,(nR, M)$, $f(n) \in {_N}Ma$.

(3) For each $n = ta \in N$, $l_M\, r_R(n) = {_N}Ma$.

(4) For each $n = ta \in N$ and each $m \in M$, $r_R(n) \subseteq r_R(m)$ implies $Sm \subseteq {_N}Ma$, where $S = End(M)$.

(5) For each $n = ta \in N$ and each $b \in R$, $l_M[bR \cap r_R(n)] = l_M(b) + {}_NMa$.

3. Characterizations of C-rings.

The following conditions are equivalent :

(1) R is a C-ring.

(2) For each right ideal I of R, each principal right ideal P of R and each maximal subideal K of $P+I$ containing I, there exists a maximal right ideal M of R containing I such that $K = M \cap (P+I)$.

4. Relations between a V-ring and a C-ring.

The following conditions are equivalent for a ring R :

(1) Each simple right R-module is injective.

(2) Each simple right R-module is P-M- injective for all right R- modules M.

(3) Each simple right R-module is P-N-injective for all cyclic right R-modules N.

(4) The radical of N, $\text{Rad } N = 0$ for all cyclic right R-modules N.

(5) Each right ideal is an intersection of maximal right ideals.

We can conclude that V-ring and C-ring are the same ring.

5. Relations among a regular ring , a V-ring and a P-V-ring.

The following conditions are equivalent for a right duo ring R :

(1) R is regular.

(2) R is a V-ring.

(3) R is a P-V-ring.