APPENDICES

APPENDIX A

Mechanical Properties of Materials

The elastic coefficients in the cylindrical coordinates for all materials in this study which are shown as the stiffness matrices, $[C]$, are given in the following.

Isotropic Material

$$
\begin{bmatrix}
3.5 & 1.5 & 1.5 & 0 & 0 & 0 \\
1.5 & 3.5 & 1.5 & 0 & 0 & 0 \\
1.5 & 1.5 & 3.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.0
\end{bmatrix}
$$

and ρ = 1.0, where all constants are dimensionless.

Aluminum

$$
\begin{bmatrix}
92.807 & 39.715 & 39.715 & 0 & 0 & 0 \\
39.715 & 92.807 & 39.715 & 0 & 0 & 0 \\
39.715 & 39.715 & 92.807 & 0 & 0 & 0 \\
0 & 0 & 0 & 26.546 & 0 & 0 \\
0 & 0 & 0 & 0 & 26.546 & 0 \\
0 & 0 & 0 & 0 & 0 & 26.546
\end{bmatrix} \text{GPa}
$$

and ρ = 2.768 g/cm^3

Boron/epoxy

$$\begin{bmatrix} 22.616 & 8.205 & 6.860 & 0 & 0 & 0 \\ 8.205 & 22.616 & 6.860 & 0 & 0 & 0 \\ 6.860 & 6.860 & 209.608 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6.895 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.895 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7.205 \end{bmatrix} \text{GPa}$$

and $\rho = 2.076 \text{ g/cm}^3$

Graphite/epoxy

$$\begin{bmatrix} 12.814 & 7.035 & 5.955 & 0 & 0 & 0 \\ 7.035 & 12.814 & 5.955 & 0 & 0 & 0 \\ 5.955 & 5.955 & 141.573 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.170 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.170 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.890 \end{bmatrix} \text{GPa}$$

and $\rho = 1.6 \text{ g/cm}^3$

Glass/epoxy

$$\begin{bmatrix} 35.285 & 22.910 & 15.713 & 0 & 0 & 0 \\ 22.910 & 35.285 & 15.713 & 0 & 0 & 0 \\ 15.713 & 15.713 & 56.785 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8.960 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8.960 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6.190 \end{bmatrix} \text{GPa}$$

and $\rho = 1.8 \ g/cm^3$

APPENDIX B

Recurrence Relations

Recurrence Relations for Regular Bessel Functions

$$\frac{dC_\nu(kr)}{dr} = \frac{\nu}{r}C_\nu(kr) - kC_{\nu+1}(kr) \tag{B-1}$$

$$\frac{dC_{\nu+1}(kr)}{dr} = kC_\nu(kr) - \frac{(\nu+1)}{r}C_{\nu+1}(kr) \tag{B-2}$$

$$\frac{d^2C_\nu(kr)}{dr^2} = \left[\frac{\nu(\nu-1)}{r^2} - k^2\right]C_\nu(kr) + \frac{k}{r}C_{\nu+1}(kr) \tag{B-3}$$

C denotes J or Y.

Recurrence Relations for Modified Bessel Functions

$$\frac{dZ_\nu(kr)}{dr} = \frac{\nu}{r}Z_\nu(kr) + kZ_{\nu+1}(kr) \tag{B-4}$$

$$\frac{dZ_{\nu+1}(kr)}{dr} = kZ_\nu(kr) - \frac{(\nu+1)}{r}Z_{\nu+1}(kr) \tag{B-5}$$

$$\frac{d^2Z_\nu(kr)}{dr^2} = \left[k^2 + \frac{\nu(\nu-1)}{r^2}\right]Z_\nu(kr) - \frac{k}{r}Z_{\nu+1}(kr) \tag{B-6}$$

Z denotes I or $e^{i\nu\pi}K$.

APPENDIX C

The Direct Method

The direct method is the method generally used to construct a frequency equation of a laminated structure. The frequency equation is formulated directly by using the exact solution of each layer containing a certain number of unknown constants to satisfy both the traction free conditions at the surfaces of the structure and the continuity conditions of displacement and stress components at interfaces between layers. Therefore, the complete solution being a transcendental relation between the frequency and the wavenumber contains all of the unknown constants of the solution of each layer in the structure.

In order to visualize the mathematical scheme of the method, the axisymmetric free vibration of the three-layered transversely isotropic cylinder is considered. The cross section of the cylinder is shown in Figure C.1.

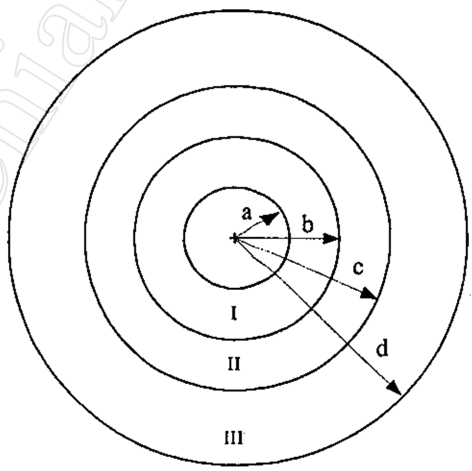

Figure C.1 Cross section of the three-layered cylinder.

In each layer, its solution given in Eqs. (18) contains four unknown constants. Therefore, the complete solution for the three-layered cylinder will contain 12 constants. These constants can be evaluated by the following boundary and interface conditions of the cylinder:

$$
\begin{array}{llll}
\text{at} & r = a \; ; & \sigma_{rr}^{(I)} = 0 , & \\
 & & \tau_{rz}^{(I)} = 0 , & \\
\text{at} & r = b \; ; & u^{(I)} = u^{(II)} , & \\
 & & w^{(I)} = w^{(II)} , & \\
 & & \sigma_{rr}^{(I)} = \sigma_{rr}^{(II)} , & \\
 & & \tau_{rz}^{(I)} = \tau_{rz}^{(II)} , & \\
\text{at} & r = c \; ; & u^{(II)} = u^{(III)} , & \\
 & & w^{(II)} = w^{(III)} , & \\
 & & \sigma_{rr}^{(II)} = \sigma_{rr}^{(III)} , & \\
 & & \tau_{rz}^{(II)} = \tau_{rz}^{(III)} , & \\
\text{at} & r = d \; ; & \sigma_{rr}^{(III)} = 0 , & \\
 & & \tau_{rz}^{(III)} = 0 . &
\end{array}
\tag{C-1}
$$

Substitution of the displacement and stress components into Eq. (C-1), a set of 12 homogeneous equations can be represented in the following form:

$$
\begin{bmatrix}
x & x & x & x & & & & & & & & \\
x & x & x & x & & & & & & & & \\
x & x & x & x & x & x & x & x & & & & \\
x & x & x & x & x & x & x & x & & & & \\
x & x & x & x & x & x & x & x & & & & \\
x & x & x & x & x & x & x & x & & & & \\
 & & & & x & x & x & x & x & x & x & x \\
 & & & & x & x & x & x & x & x & x & x \\
 & & & & x & x & x & x & x & x & x & x \\
 & & & & x & x & x & x & x & x & x & x \\
 & & & & & & & & x & x & x & x \\
 & & & & & & & & x & x & x & x
\end{bmatrix}
\begin{Bmatrix}
C_1^{(I)} \\ C_2^{(I)} \\ C_3^{(I)} \\ C_4^{(I)} \\ C_1^{(II)} \\ C_2^{(II)} \\ C_3^{(II)} \\ C_4^{(II)} \\ C_1^{(III)} \\ C_2^{(III)} \\ C_3^{(III)} \\ C_4^{(III)}
\end{Bmatrix}
= 0 \quad , \qquad \text{(C-2)}
$$

where x marks nonzero elements of the coefficient matrix. For nontrivial solution, the natural frequencies can be obtained by setting the determinant of the coefficient matrix to zero.

APPENDIX D

The Globally Averaged Stiffness

The Globally Averaged Stiffness model suggests that the stiffness matrix $[C_{ij}]$ is independent of the z-coordinate where z is the through-thickness coordinate. In this case, the actual laminated plate is replaced by "equivalent" single-layer plate. In order to calculate "effective" stiffness of the equivalent structure, some specific assumptions have to be adopted. It was assumed that the stress-strain state is such that the in-plane strains are identical for all of the layers (the iso-strain model regarding the in-plane strains) and the transverse stresses are identical for all of the layers (the iso-stress model regarding the transverse stresses). Consequently, only variation of the in-plane stresses and transverse strains is allowed from layer to layer. This combination of the iso-strain and iso-stress assumptions provides a rather simple procedure for calculating effective stiffnesses. According to this model, the generalized Hooke's law for an equivalent monoclinic body is written as follows:

$$\begin{aligned}
\langle\sigma_x\rangle &= c_{11}^*\langle\varepsilon_x\rangle + c_{12}^*\langle\varepsilon_y\rangle + c_{13}^*\langle\varepsilon_z\rangle + c_{16}^*\langle\gamma_{xy}\rangle, \\
\langle\sigma_y\rangle &= c_{12}^*\langle\varepsilon_x\rangle + c_{22}^*\langle\varepsilon_y\rangle + c_{23}^*\langle\varepsilon_z\rangle + c_{26}^*\langle\gamma_{xy}\rangle, \\
\langle\sigma_z\rangle &= c_{13}^*\langle\varepsilon_x\rangle + c_{23}^*\langle\varepsilon_y\rangle + c_{33}^*\langle\varepsilon_z\rangle + c_{36}^*\langle\gamma_{xy}\rangle, \\
\langle\tau_{yz}\rangle &= c_{44}^*\langle\gamma_{yz}\rangle + c_{45}^*\langle\gamma_{xz}\rangle, \\
\langle\tau_{xz}\rangle &= c_{45}^*\langle\gamma_{yz}\rangle + c_{55}^*\langle\gamma_{xz}\rangle,
\end{aligned} \qquad \text{(D–1)}$$

$$\langle \tau_{xy} \rangle = c_{16}^{*} \langle \varepsilon_x \rangle + c_{26}^{*} \langle \varepsilon_y \rangle + c_{36}^{*} \langle \varepsilon_z \rangle + c_{66}^{*} \langle \gamma_{xy} \rangle ,$$

where $\langle ... \rangle$ denotes the averaging. c_{ij}^{*} (i,j = 1, 2, 3,...,6) are effective stiffnesses given as follows:

$$c_{ij}^{*} = \langle c_{ij} \rangle - \left\langle \frac{c_{i3} c_{j3}}{c_{33}} \right\rangle + \left\langle \frac{c_{i3}}{c_{33}} \right\rangle \frac{\left\langle \frac{c_{j3}}{c_{33}} \right\rangle}{\left\langle \frac{1}{c_{33}} \right\rangle} \qquad \text{for i,j = 1, 2, 3, 6} \qquad \text{(D–2)}$$

$$c_{ij}^{*} = \frac{\left\langle \frac{c_{ij}}{\Delta} \right\rangle}{\left\langle \frac{c_{44}}{\Delta} \right\rangle \left\langle \frac{c_{55}}{\Delta} \right\rangle - \left(\left\langle \frac{c_{45}}{\Delta} \right\rangle \right)^{2}} \qquad \text{for i,j = 4, 5} \qquad \text{(D–3)}$$

where $\Delta = c_{44} c_{55} - c_{45}^{2}$. Expressions for the average stiffnesses in (D–2) and (D–3) in terms of the stiffnesses of individual layers $c_{ij}^{(k)}$ and their thicknesses h_k follow form the general averaging rule:

$$\langle c_{ij} \rangle = \frac{h_1 c_{ij}^{(1)} + h_2 c_{ij}^{(2)} + \ldots + h_K c_{ij}^{(K)}}{h_1 + h_2 + \ldots + h_K} \qquad \text{(D–4)}$$

where K is the number of layers in the package.

It should be noted here that the aforementioned model is for laminated plate. In order to apply for the laminated cylinder, proper transformation needs to attend.

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