Chapter 1

Introduction

1.1 Statement and Significance of the Problem

Laminated composite materials have numerous applications in Engineering. They are used in high pressure vessels, spacecrafts, etc. The applications of these materials do not limit only in Engineering fields, they extend to various other fields as well. Therefore, the study of such materials is of interest in Engineering at the present.

Usually, the structures made of laminated composite materials are built in the forms of plate or shell, which are basic forms in the field of engineering. These structures are composed of perfectly bonded elastic layers. Each layer may have different mechanical properties, as well as thickness. Many researches have been carried out in order to understand both static and dynamic behaviors of these structures. In this study, the dynamic behavior of a laminated composite circular cylinder is investigated. Due to the complexity in mechanical properties, most of their dynamic behaviors were studied via experiments. Many researchers have employed several theories to study a free vibration of such cylinder. Based on the shell theories, the natural frequencies can be obtained for thin laminated shells. However, it is well known that when the thickness-to-mean radius ratio of the shell becomes larger, the assumptions of thin shell are no longer valid. For the case of thick laminated cylinders, numerical methods are widely used. For low modes of vibrations, these numerical methods provide good results in comparison to the experiments. However, the frequencies corresponding to higher modes are less accurate. On the other hand, an exact three-dimensional analysis has been always considered as a benchmark for these studies.

1.2 Literature Review

1.2.1 Free Vibration of Circular Cylinder

Pochhammer (1876) investigated the axially symmetric propagation of harmonic waves in an infinitely long homogeneous isotropic elastic cylindrical rod. He derived the exact frequency equation which implicitly related the frequency to the wavenumber.

Herrmann and Mirsky (1956) presented the three-dimensional and a shell theory analysis of axially symmetric motion of infinitely long isotropic circular cylinders. On the basis of three-dimensional linear theory of elasticity, the exact solution of the equations of motion was formed analytically in terms of Bessel functions. The dispersion equation was obtained by imposing the traction free boundary conditions on the inner and outer surfaces of shell. The analytical formulation yielded the transcendental equation relating between the frequency and the wavenumber. On the other hand, a shell theory was also applied to investigate the natural frequencies, which were compared with those obtained by the analytical method.

Gazis (1959) and Armenakas et al. (1969) presented wave propagation in infinitely long homogeneous isotropic elastic hollow cylinder. The exact solution of the general three-dimensional equations of linear elasticity theory was used to find the exact natural frequencies including both axisymmetric and nonaxisymmetric vibration cases. The frequency equation of the cylinder was formulated by applying the stress-displacement relations to satisfy the traction-free conditions on inner and outer surfaces. The set of six homogeneous equations which related to the wavenumber was solved to find the natural frequencies of the cylinder.

Mirsky (1965) studied the axisymmetric vibrations of infinitely long orthotropic cylindrical shells. Based on the three-dimensional elasticity theory, the exact solution was obtained by using the Frobenius power-series method. The closed-form solution which was expressed in terms of four unknown constants was applied to the problem of free vibration. The frequency equation was formulated by applying the solution to satisfy the traction-free boundary conditions on inner and outer surfaces of cylinder. The formulation provided a set of four homogeneous equations. For a nontrivial solution, the determinant of the coefficients was set equal to zero. Then, the natural frequencies were obtained numerically. It showed that the power-series expansions for the displacement components converged rapidly for low modes.

Keck and Armenakas (1971) studied wave propagation in infinitely long three-layered transversely isotropic circular cylinders. The natural frequencies of the axisymmetric vibration were investigated analytically. Based on the three-dimensional elasticity theory, the solution to the equations of motion for each layer was obtained in terms of Bessel functions. The solution of each layer which contained four unknown constants was applied directly to formulate the frequency equation of the cylinder. They imposed the traction-free conditions at the surfaces of the cylinder and the continuity conditions of displacement and stress components at interfaces between layers. The complete solution being a transcendental relation between the frequency and the wavenumber contained all of the unknown constants of the solution of each layer in the cylinder. For a nontrivial solution, the natural frequencies were evaluated by setting the determinant of the coefficients of the constants in the frequency equation to zero.

Chou and Achenbach (1972) studied the three-dimensional vibrations of an infinitely long orthotropic hollow circular cylinder. Based on the Frobenius method, the closed-form solution containing six unknown constants of the three-dimensional equations of linear elasticity theory was obtained in terms of power series. The solution was applied for the case of an infinitely long orthotropic solid rod by reducing the number of unknown constants of solution to three. The frequency equation was formulated by setting the solution to satisfy the traction free boundary conditions at inner and outer surfaces in the case of hollow cylinder while only the traction free boundary condition at outer surface was required in the case of a solid rod. The formulation yielded a set of six homogeneous equations for hollow cylinder case and a set of three homogeneous equations for solid rod case. The natural frequencies were obtain by setting the determinant of the coefficients of the constants to zero. The convergence of natural frequencies depended on the number of terms in power series.

Huang and Dong (1984) and Datta, et al. (1990) studied the characteristics of dispersive wave propagation in an infinitely long laminated composite circular cylinder. Since the analytical formulation for laminated composite cylinders is intractable, a displacement-based stiffness approximation was employed. The displacements at discrete nodal generalized coordinates were approximated by quadratic interpolation polynomials in this Rayleigh-Ritz type approximation. The frequency equation of the cylinders in form of an eigenvalue problem was formulated by Hamilton's principle. The method generally provided good results in comparison to the available analytical solutions.

Rattanawangcharoen, et al. (1992) and Rattanawangcharoen (1993) used another Rayleigh-Ritz type approximation to study the dynamic behavior of a composite cylinder. The method approximated the displacement distribution through the thickness of each sublayer by cubic polynomial interpolation functions. These functions were chosen to satisfy not only the displacement continuity at the interfaces between the adjoining sublayers but also the stress continuity at the interfaces. The numerical results obtained by the method were more accurate at high frequencies than those obtained by the displacement-based stiffness method.

Heyliger and Jilani (1992, 1993) presented the investigation in free vibations of laminated anisotropic cylindrical cylinders by a numerical method. The weak form of the equations of motion for laminated anisotropic composite shells were solved by using the Ritz mothod for the problems of free vibrations with various end conditions. Using a combination of power and Fourier series as the approximating functions for the three displacement components, the natural frequencies were evaluated for a number of geometric and material combinations. Because the full equations of elasticity are used in the formulation, no assumption was required regarding the type of motion. The transverse shear strains, deformation of the normals, and all inertial terms were included in the formulation. The approximating functions assumed that the displacement components and their derivatives are continuous through the thickness of the shell. Several problem geometries were considered, including a single-layer isotropic shell, a three-layer shell composed of isotropic materials, a single-layer of orthotropic shell, a single-layer anisotropic shell, and laminated shells composed of anisotropic materials. Comparisons were made with results of isotropic and anisotropic shell theories, with very good agreement being obtained.

Yuan and Hsieh (1998) presented an exact solution of free harmonic wave propagation in an infinitely long laminated composite cylindrical shell. The formulation for three-dimensional wave propagation based on the linear cylindrically anisotropic elasticity was derived. The Frobenius method was used to find the solution of the three-dimensional equations of motion in terms of power series. The solution satisfied the equations of motion of each layer contained six unknown constants. The frequency equation was obtained by requiring the traction free boundary conditions at inner and outer surfaces of the shell. For an N-layer laminated composite cylindrical shell and by assuming the interfaces between the layers were perfectly bonded, the continuity of displacements and tractions along the interfaces and traction-free boundary conditions provided a homogeneous equation of order 6Nx6N. For a nontrivial solution, the determinant of the coefficients was set equal to zero. The dispersion relation was a transcendental function relating the frequencies, axial wavelength and circumferential wavenumbers.

1.2.2 Propagator Matrix (Transfer Matrix)

Karunasena, et al. (1991) used the propagator matrix approach to obtain the exact natural frequencies of a multilayered transversely isotropic plate. The exact solution to the equations of motion of each transversely isotropic layer was applied to formulate the propagator matrix of the layer. By applying the continuity of both displacements and stresses at the interfaces of layers, the complete 6 x 6 coefficient matrix relating the displacement and the stress components at one outer surface to those at the other outer surface of multilayered plate was formed. By invoking the zero-traction conditions at these surfaces, the exact dispersion relation of the plate was obtained by setting determinant of the 3 x 3 coefficient matrix being a submatrix in the complete 6 x 6 matrix to zero. The exact frequencies obtained analytically by the propagator matrix approach were used to compare to those which were obtained by the stiffness method.

Rattanawangcharoen and Shah (1992), Rattanawangcharoen (1993), and Rattanawangcharoen, et al. (1994) applied the analytical method to study wave propagation in an infinitely long two-layered isotropic circular cylinder. Gazis's solution which was expressed in terms of Bessel functions was applied in the propagator matrix method. By applying the exact solution, displacement and stress components, which contained six unknown constants of each sublayer, the natural frequencies of multilayered cylinder were obtained by setting the determinant of the 3x3 coefficient matrix to zero. This investigation showed that the method was practically used for cylinders consisted of a large number of layers because the size of matrix of coefficient which was solved did not depend on the number of layers. Since being the analytical method, numerical results by using propagator matrix were considered as a benchmark for the results by numerical method, the Rayliegh-Ritz type approximations, applied in the same study.

Taylor and Nayfeh (1996) applied the matrix transfer technique to study the free vibration of thick rectangular multilayered plates. By using the existing exact solution of free vibration of simply supported thick orthotropic rectangular plate, the 6 x 6 coefficient matrix relating the displacement and stress components of each layer was formulated. Consequently, the complete 6 x 6 coefficient matrix relating the displacement and stress components of the first face to those of the last face of multilayered plate was found by applying the continuity of displacements and stresses at interfaces between layers. It was shown that the size of the complete matrix was independent of the number of lamina. Finally, by applying the zero stress conditions at the first and the last faces, the natural frequencies were obtained by setting the determinant of the 3 x 3 coefficient matrix being a submatrix in the complete 6 x 6 matrix to zero.

1.3 Purpose of the study

The main purpose of this study is to obtain the exact frequency equation of the vibration in an infinitely long, laminated transversely isotropic circular cylinder by the propagator matrix method.

1.4 Scope of study

a) The laminated cylinder considered in this study has an infinite length.

b) Only hollow cylinder is considered.

c) The laminae of the cylinder are perfectly bonded at the interfaces.

d) All the motions considered are independent of θ.