CHAPTER V
CONCLUSION

In this research, we first studied about preclosed sets and generalized preclosed sets in a general topological space, and we also studied about preclosed, gp-closed and pre gp-closed functions. Here are the main results which we obtained.

1. Let $\{G_\alpha : \alpha \in \mathcal{A}\}$ be a family of preopen subsets of a topological space X. Then $\underset{\alpha\in\mathcal{A}}{\cup} G_\alpha$ is preopen in X.

2. Let A and B be subsets of a topological space X and $x \in X$. Then

(a) $x \in pCl(A)$ if and only if $A \cap U \neq \emptyset$ for every preopen set U containing x

(b) $pCl(A)$ is preclosed

(c) A is preclosed if and only if $A = pCl(A)$

(d) $X - pCl(X - A) = pInt(A)$

(e) $X - pInt(A) = pCl(X - A)$

(f) $pInt(A)$ is preopen

(g) If $A \subseteq B$, then $pCl(A) \subseteq pCl(B)$.

3. A subset A of a topological space X is gp-open in X if and only if $F \subseteq pInt(A)$ whenever $F \subseteq A$ and F is closed in X.

4. A surjective function $f : X \longrightarrow Y$ is gp-closed (resp. $pre\ gp-closed$) if and only if for each subset B of Y and each open (resp. *preopen*) subset U of X containing $f^{-1}(B)$, there exists a gp-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$.

5. If $f : X \longrightarrow Y$ is continuous, pre gp-closed and A is gp-closed in X, then $f(A)$ is gp-closed in Y.

6. If $f : X \longrightarrow Y$ is continuous, pre gp-closed and bijective and U is gp-open in X, then $f(U)$ is gp-open in Y.

7. If $f : X \to Y$ is open, pre-irresolute and bijective and B is gp-closed in Y, then $f^{-1}(B)$ is gp-closed in X.

8. If $f : X \to Y$ is open, pre-irresolute and bijective and A is gp-open in Y, then $f^{-1}(A)$ is gp-open in X.

9. If a function $f : X \to Y$ is gc-irresolute and Y is a T_g-space, then f is gp-irresolute.

The second main purpose of this research is to study about pre-normal space and we obtain the following results.

10. If a topological space X is normal, then X is pre-normal, but not conversely.

11. The following are equivalent for a topological space X :

(a) X is pre-normal.

(b) For any pair of disjoint closed sets A and B of X, there exist disjoint gp-open sets U and V of X such that $A \subseteq U$ and $B \subseteq V$.

(c) For any closed set A of X and any open set V of X containing A, there exists a gp-open set U of X such that $A \subseteq U \subseteq pCl(U) \subseteq V$.

12. If $f : X \to Y$ is a continuous, gp-closed and surjective mapping and X is normal, then Y is pre-normal.

13. If $f : X \to Y$ is a continuous, pre gp-closed and surjective mapping and X is pre-normal, then Y is pre-normal.

In the last main purpose of our study, we defined a topological space which is a generalization of pre-normal space and we call it, a gp-normal space, and we give characterizations of being gp-normal space. Furthermore, we study about preservation theorems concerning gp-normal space. The followings are our results:

14. If a topological space X is pre-normal, then X is gp-normal, but not conversely.

15. If a topological space X is gp-normal and regular $T_{\frac{1}{2}}$-space, then X is pre-normal, but not conversely.

16. If a topological space X is gp-normal which is a T_g-space, then X is g-normal, but not conversely.

17. The following are equivalent for a topological space X :

(a) X is gp-normal.

(b) For each closed set A in X and each regular open set U in X containing A, there exists a gp-open set V in X such that $A \subseteq V \subseteq pCl(V) \subseteq U$.

(c) For each closed set A in X and each regular closed set B in X such that $A \cap B = \emptyset$, there exists a gp-open set V in X such that $A \subseteq V$ and $pCl(V) \cap B = \emptyset$.

(d) For each closed set A in X and each regular closed set B in X such that $A \cap B = \emptyset$, there exist gp-open sets U and V in X such that $A \subseteq U, B \subseteq V$ and $pCl(U) \cap V = \emptyset$.

18. If $f : X \to Y$ is regular continuous, pre gp-closed and bijective and X is gp-normal, then Y is gp-normal.

19. If $f : X \to Y$ is regular closed, open, pre-irresolute and bijective and Y is gp-normal, then X is gp-normal.

20. If $f : X \to Y$ is regular closed, gp-irresolute and injective and Y is gp-normal, then X is gp-normal.