Chapter 1

INTRODUCTION

A Taylor method for solving Fredholm integral equations has been presented by Kanwal and Liu [2]. Then this method has been extended by Sezer to Volterra integral equations [4], to second-order linear differential equations [3] and to high-order linear Volterra-Fredholm integro-differential equations in the form

$$\sum_{j=0}^{m} P_j(x)\, y^{(j)}(x) = f(x) + \lambda_1 \int_a^x K_1(x,t)\, y(t)\, dt + \lambda_2 \int_a^b K_2(x,t)\, y(t)\, dt. \tag{1.1}$$

with the mixed conditions

$$\begin{gathered}\sum_{j=0}^{m-1} \left[a_{ij} y^{(j)}(a) + b_{ij} y^{(j)}(b) + c_{ij} y^{(j)}(c)\right] = \mu_i \\ i = 0,1,2,\ldots,m-1, \qquad a \le c \le b,\end{gathered} \tag{1.2}$$

where $P_k(x)$ $(k = 0,1,2,\ldots,m)$, $f(x)$, $K_1(x,t)$ and $K_2(x,t)$ are functions having nth $(n \ge m)$ derivatives on an interval $a \le x,t \le b$ and a, b, c, a_{ij}, b_{ij}, c_{ij}, λ_1, λ_2 and μ_i are appropriate constants [6]. The solution, which is a Taylor polynomial of degree N, is expressed in the form

$$y(x) = \sum_{n=0}^{N} \frac{1}{n!} y^{(n)}(c)\,(x-c)^n, \qquad a \le x, c \le b,\ N \ge m. \tag{1.3}$$

The algorithm of finding Taylor solution is following. First, we differentiate both sides of high-order linear Volterra-Fredholm integro-differential equation n times and then substituting the Taylor series for the unknown function in the resulting equation. Then,we wrote the resulting equation in the matrix form and apply the given conditions. Finally, the obtained linear algebraic system has been solved approximately by a suitable truncation scheme.

In our study, the basic ideas of method in [2] , [3] , [4] and [6] are developed and applied to the Volterra-Fredholm integro-differential equations, with more terms of high order derivative under integral sign, in the form

$$\sum_{k=0}^{m} P_k(x)\, y^{(k)}(x) = f(x) + \lambda_1 \int_a^x \sum_{i=0}^{p} A_i(x,t)\, y^{(i)}(t)\, dt + \lambda_2 \int_a^b \sum_{j=0}^{q} B_j(x,t)\, y^{(j)}(t)\, dt \qquad (1.4)$$

for some p and q where $P_k(x)$ $(k = 0,1,2,...,m)$, $f(x)$, $A_i(x,t)$ $(i = 0,1,2,...,p;\ p \leq m+1)$, $B_j(x,t)$ $(j = 0,1,2,...,q;\ q \leq m+1)$, are functions having nth $(n \geq m)$ derivatives on an interval $a \leq x,t \leq b$. The conditions and the approximate solution of this equation are of the same as in (1.2) and (1.3). Thus, $y^{(n)}(c)$ $\ n = 0,1,2,...,N$ are coefficients to be determined.