# Chapter 2

# PRELIMINARIES

In this chapter, we introduce the keywords, Taylor polynomial, Taylor series, Volterra and Fredholm integral and Leibniz's rule which will be seen in the next chapter.

## 2.1 Taylor polynomial

Taylor polynomial is used to approximate a function on many occasions for various purpose.

If a function $y = f(x)$ has $n$ derivatives at $x = c$, then the $n$th-degree Taylor polynomial approximating $f(x)$ near $x = c$ is

$$\begin{aligned} P_n(x) &= f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n \\ &= \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k \end{aligned} \tag{2.1}$$

where $f^{(n)}(c)$ is the $n$th derivative of $y = f(x)$ evaluate at $x = c$.

## 2.2 Taylor series

In previous section we mentioned that a Taylor polynomial can be used to approximate the value of a function near $x = c$. If we extend the Taylor polynomial, we will obtain a power series that represents the function in some interval centered at $x = c$.

If $y = f(x)$ is a function which all derivatives of $y = f(x)$ exist at $x = c$, then the Taylor series representation for $y = f(x)$ centered at $x = c$ is given by

$$\begin{aligned} f(x) &= f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \frac{f^{(4)}(c)}{4!}(x-c)^4 + \ldots \\ &= \sum_{k=0}^{\infty} \frac{f^{(k)}(c)}{k!}(x-c)^k. \end{aligned} \tag{2.2}$$

## 2.3 Integral equations

Integral equations are one of the most useful mathematical tools in both pure and applied analysis. This is particularly true of problems in mechanical vibrations and the related fields of engineering and mathematical physics, where they are not only useful but often indispensable even for numerical computational.

### 2.3.1 Volterra integral equations

Let $K(x,y)$ be a complex valued continuous function defined on a domain $a \leq x, y \leq b$; $f(x)$ be a complex valued continuous function defined on the interval $a \leq x \leq b$ and $\lambda$ be an arbitrary complex number; then the following integral equations

$$\phi(x) - \lambda \int_a^x K(x,y)\phi(y)dy = f(x) \qquad (a \leq x, y \leq b) \tag{2.3}$$

and

$$\int_a^x K(x,y)\phi(y)dy = f(x) \qquad (a \leq x, y \leq b). \tag{2.4}$$

are called *Volterra integral equation* of the second and first kind, respectively,for function $\phi(x)$.

### 2.3.2 Fredholm integral equations

Let $K(x,y)$ be a complex valued continuous function defined on a domain $a \leq x, y \leq b$; $f(x)$ be a complex valued continuous function defined on the interval $a \leq x \leq b$ and $\lambda$ be an arbitrary complex number; then the general linear *Fredholm integral equation of the second kind* for function $o(x)$ is an equation of the type

$$\phi(x) - \lambda \int_a^b K(x,y)\phi(y)dy = f(x) \qquad (a \le x, y \le b) \tag{2.5}$$

while the linear *Fredholm integral equation of the first kind* is given by

$$\int_a^b K(x,y)\phi(y)dy = f(x) \qquad (a \le x, y \le b). \tag{2.6}$$

## 2.4 Leibniz's rule

The Leibniz's rule *for differentiation of an integral*

$$\frac{d}{dc}\int_{a(c)}^{b(c)} f(x,c)dx = \int_{a(c)}^{b(c)} \frac{\partial}{\partial c} f(x,c)dx + f(b,c)\frac{db}{dc} - f(a,c)\frac{da}{dc}$$

The Leibniz's rule *for differentiation of a product*

$$\frac{d^n}{dx^n}(uv) = \frac{d^n u}{dx^n}v + \binom{n}{1}\frac{d^{n-1}u}{dx^{n-1}}\frac{dv}{dx} + \binom{n}{2}\frac{d^{n-2}u}{dx^{n-2}}\frac{d^2 v}{dx^2} + \ldots + \binom{n}{r}\frac{d^{n-r}u}{dx^{n-r}}\frac{d^r v}{dx^r} + u\frac{d^n v}{dx^n}$$