Chapter 4

CONCLUSIONS

Because of the high-order integro-differential equations are difficult to find the exact solutions. The approximation method is necessary. In this purpose the Taylor method is presented to find the solution, that is, the solution is expressed as Taylor polynomial at $x = c$, as

$$y(x) = \sum_{n=0}^{N} \frac{1}{n!} y^{(n)}(c)(x-c)^n.$$

In our study, we extend (1.1) by adding more terms of high order derivatives under integral sign. The mixed conditions, in form Eq.(1.2) , are apply with our problems

The pertinent features of the method are illustrated in the Examples 1, 2, 3 and 4.

In Example 1, we observed that the Taylor solution converges rapidly and we found that for $N \geq 12$ the approximation is very close to the exact solution at any points in the interval $0 \leq x \leq 1$.

In Example 2, the errors at any points in the interval $0 \leq x \leq 1$ are less than 1.64698×10^{-6} for $N = 18$ as can be seen from Table 2.

The Example 3 showed that the Taylor solution at the points near $x = 0$ give high accurracy.

In Example 4, the errors of approximate solution of Fredholm integro-differential equation are not quite good.

The obtained approximate solutions are computed by using computer program MATLAB.

For this study, the method gives good approximation near $x = c$. The approximate solutions are better as we take more terms for Taylor polynomial, that is, the truncation limit N must be large enough.

We can conclude from the examples that for p or q less than or equal to $m+1$, the solutions of the high-order linear Volterra or Fredholm integro-differential equations always exist. If rank $\tilde{\mathrm{M}}^* = \text{rank } \mathrm{M}^* = N+1$, then the systems have unique solution. For p or q greater than $m+1$, the systems may have infinite solutions or no solution.