CHAPTER 4

CONCLUSION

In this work, we have presented four methods for controlling chaos of the nonlinear dynamical system (1.1). The mathematical controllability conditions are derived from Routh-Hurwitz theorem. Both the methods of linear feedback control and bounded feedback control suppress the chaotic behavior of the system to one of the two unstable equilibrium points E_1 or E_2. Nonfeedback control and an approximated to the delay feedback control method suppress chaotic behavior of system (1.1) to a limit cycle.

The methods for controlling chaos that we will use are feedback control, bounded feedback control, nonfeedback control and an approximated to the delay feedback control.

1. Feedback control method

   Theorem 1. The equilibrium point $E_1 = (\beta_1, \beta_2, \gamma)$ of the controlled system (2.4) is asymptotically stable provide that $k_{11} = 0, k_{22} = 0$ and $k_{33} > 0$.

   Theorem 2. The equilibrium point $E_2 = (-\beta_1, -\beta_2, \gamma)$ of the controlled system(2.5) is asymptotically stable provide that $k_{11} = 0, k_{22} = 0$ and $k_{33} > 0$.

2. Bounded feedback control method

   Theorem 3. The equilibrium solution $E_1 = (\beta_1, \beta_2, \gamma)$ of the controlled system (2.6) is asymptotically stable if the constant $k > 0$.

   Theorem 4. The equilibrium solution $E_2 = (-\beta_1, -\beta_2, \gamma)$ of the controlled system (2.8) is asymptotically stable for $k > 0$.

3. Nonfeedback control method

Case 1. In this case, the controller (3.1) is added to the second equation of(1.1). The controlled system has the following equations:

$$\begin{aligned} \dot{x} &= -\mu x + y(z+\alpha) - bxz, \\ \dot{y} &= -\mu y + x(z-\alpha) - byz + f_1 + f_2\cos(\omega t), \\ \dot{z} &= 1 - xy. \end{aligned} \tag{3.2}$$

Chaos is suppressed to a periodic solution within the chaotic attractor for certain values of f_1, f_2 and ω.

Case 2. In this case, the controller(3.1) is added to the third equation of (1.1). The resulting controlled system has the form

$$\begin{aligned} \dot{x} &= -\mu x + y(z+\alpha) - bxz, \\ \dot{y} &= -\mu y + x(z-\alpha) - byz, \\ \dot{z} &= 1 - xy + f_1 + f_2\cos(\omega t). \end{aligned} \tag{3.3}$$

Also chaos is suppressed to another periodic solution within the chaotic attractor for certain values of f_1, f_2 and ω.

4. Approximated delay feedback control method

System (4.1) can be approximated to the following controlled system:

$$\begin{aligned} \dot{x} &= (1-k_{11}\tau)\left[y(z+\alpha) - \mu x - bxz\right], \\ \dot{y} &= (1-k_{22}\tau)\left[x(z-\alpha) - \mu y - byz\right], \\ \dot{z} &= (1-k_{33}\tau)\left(1 - xy\right), \end{aligned} \tag{4.2}$$

where some point of $0 < k_{11}\tau < 1$, $0 < k_{22}\tau < 1$ and $0 < k_{33}\tau < 1$.