APPENDIX

The Program "Direct-Series" (written in Maple7) is used to find the approximation solution of Volterra-Fredholm's population model

$$\kappa\frac{du}{dt} = u(t) - u^2(t) - u(t)\int_0^t u(x)dx - Cu(t)\int_{t_1}^{t_2} u(x)dx, \qquad u(0) = 0.1$$

Input : κ, C, t_1, t_2, and $u(0)$.

Output : All alphas, the smallest positive real alpha, an approximation solution of $u(t)$, Padé approximants [4/4] of $u(t)$, and graph for expressing the essential behavior of $u(t)$.

```
> restart;
> Direct-Series := proc(kappa, C, t1, t2, u0)
> local     a, b, c, d, A, B, i, u, Pade-of-u-pure, Pade-of-u, ans, ans-list, ans-array,
s. min-ans, ans-array-sorted, g1, g2 ;
> a := array(0..10):
> b := array(0..10):
> c := array(0..10):
> d := array(0..10):
> A := sum('n*a[n]* t^(n-1)', 'n'=1..10):
> B := (1/kappa)*(sum('a[n]*t^n', 'n'=0..10) -(sum('a[n]*t^n', 'n'=0..10))^2 -
(sum('a[n]*t^n', 'n'=0..10) *int(sum('a[n]*x^n', 'n'=0..10), x=0..t)):
> for i from 1 to 8 do
>     a[0] := u0 :
>     b[i] := coeff(A, t, i-1) :
>     c[i] := coeff(B, t, i-1) :
```

```
        >       a[i] := c[i]/i :
>       end do :
> u(t) := simplify(expand(sum('a[i]*t^i', 'i'=0..8))):
> with(numapprox) :
> Pade-of-u-pure := pade(u(t), t, [4,4]) :
> with(plots) :
> g1 := plot(Pade-of-u-pure , t=0..t1, u=0..1, discont=true, color=red):
> a := array(0..10) :
> b := array(0..10) :
> c := array(0..10) :
> d := array(0..10) :
> A := sum('n*a[n]* t^(n-1)', 'n'=1..10):
> B := (1/kappa)*((1-C*alpha)*sum('a[n]*t^n', 'n'=0..10) -(sum('a[n]*t^n', 'n'=0..10))^2
-(sum('a[n]*t^n', 'n'=0..10) *int(sum('a[n]*x^n', 'n'=0..10), x=0..t)):
> for i from 1 to 8 do :
>       a[0] := u_0 :
>       b[i] := coeff(A, t, i-1) :
>       c[i] := coeff(A, t, i-1) :
>       a[i] := c[i]/i :
> end do :
> u := proc(t)
>       simplify(expand(sum('a[i]*t^i', 'i'=0..8)))
> end proc :
> ans := solve(alpha=int(u(t), t=t1..t2), alpha) :
> ans-list := [ans] :
> ans-array := convert(ans-list, array) :
> ans-array-sorted := sort(ans-array) :
```

```
        > printf(' Solving for all alpha, we obtained n') :
> for i from 1 to 8 do
>       if type(ans-array[i], float) and type(ans-array[i], nonnegative) then
>       min-ans := ans-array[i] :
>       break :
>       end if :
> end do :
> s:= (subs(alpha=min-ans, u(t))) :
> printf('The smallest positive real alpha is n') :
> print('alpha'=min-ans) :
> print('u(t)'=s) :
> with(numapprox) :
> Pade-of-u := pade(s, t, [4,4]) :
> print('Pade'=Pade-of-u) :
> with(plots) :
> g2:= plot(Pade-of-u, t=t1..5, u=0..1, discont=true, color=blue) :
> display(g1, g2) :
> end proc :

> Direct-Series(0.1, 0.1, 0.9, 1.0, 0.1);
```

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