CHAPTER 1

INTRODUCTION

In 1999, Abdul-Majid Wazwaz [1] has been presented the reliable numerical strategies for solving the Volterra model for population growth of a species within a closed system. The model is characterized by the nonlinear Volterra integro-differential equation

$$\kappa \frac{du}{dt} = u - u^2 - u \int_0^t u(x)dx, \qquad u(0) = 0.1, \tag{1.1}$$

where $u \equiv u(t)$ is the scaled population of identical individuals [2] at a time t, and κ is a prescribed parameter. The nondimensional parameter $\kappa = c/(ab)$, where $a > 0$ is the birth rate coefficient, $b > 0$ is the crowding coefficient, and $c > 0$ is the toxicity coefficient [3]. The coefficient c indicates the essential behavior of the population evolution before its level falls to zero in the long run. Volterra introduced this model for a population $u(t)$ of identical individuals which exhibits crowding and sensitivity to the amount of toxins produced. The nonlinear model (1.1) includes the well-known terms of a logistic equation [4], and in addition includes an integral term that characterizes the accumulated toxicity produced since time zero.

In [1], three alternative numerical strategies are proved to be effective and reliable. One of the algorithms was based on the series solution method. The other algorithm was based on the Adomian decomposition method [5, 6, 7]. In the third algorithm, the series solution method or the Adomian decomposition method has been implemented independently to the related nonlinear differential equation that results from converting Eq.(1.1)

Furthermore, the behavior of the model in that increases rapidly in the logistic curve and it decreases exponentially to extinction in the long run can be formally determined by using the Padé approximants [8, 9] of the series obtained. In addition, the essential behavior of the population $u(t)$ for small κ and for large κ has been addressed by using the Padé approximants.

In our study, the Volterra-Fredholm model for population growth of a species within a closed system

$$\kappa\frac{du}{dt} = u - u^2 - u\int_0^t u(x)dx - Cu\int_{t_1}^{t_2} u(x)dx, \qquad u(0) = u_0, \tag{1.2}$$

where u_0 is the initial population, C is a nondimensional parameter and t_1, t_2 are constants represented the interval time which perturbation had occurred in the system will be considered.

The idea of solving the model (1.2) follows from A.M. Wazwaz and S.A. Kuri [10]. The series solution method and the direct solution method were used in solving Volterra integral equations of the second kind and Fredholm integral equations of the second kind respectively. However,in (1.2) both Volterra and Fredholm integral terms are presented so that the combined method will be used. The results of combining series solution method with the direct solution method show accuracy and effectively of technique. Moreover, the combining of direct solution method with the Adomian decomposition method, and the method of converting model to nonlinear ODE give the same accuracy.

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