## CHAPTER 4

## CONCLUSION

The goal of this thesis is to construct an analytic approximation to the solution of Volterra-Fredholm's model for population growth of a species in a closed system that characterized by the nonlinear integro-differential equation

$$\kappa \frac{du}{dt} = u(t) - u^2(t) - u(t) \int_0^t u(x) dx - Cu(t) \int_{t_1}^{t_2} u(x) dx, \qquad u(0) = u_0$$

The goal has been achieved by implementing the direct solution method and three independent methods, that are the series solution method, the decomposition method, and converting the model to a nonlinear ODE. From these results the three methods give the same solution and we can not predict the behavior of population. So the solution u(t) will be estimated by Padé approximations for predicting the behavior of population. In computing the solution we used the direct solution method combined with the series solution method and using Maple 7 software.

We can guarantee that the method used is corresponding to the behavior of the population of identical individuals as shown in Figure 3.2. Figure 3.2 shows the behavior of the population before and after the perturbation occurred, the behavior of the population increases rapidly as the logistic curve and followed by the slow exponential decay after the maximum point appears. When the perturbation occurred, the population u(t) decreases immediately and decreases exponentially to extinction in the long run.

The reliability and the efficiency of the method demonstrated by the error from Table 1 and Figure 3.4.

From Figure 3.3 we can conclude that, the increasing value of C effect to the behavior of the population u(t). That is the population u(t) decrease immediately after the perturbation occurred, followed with the exponentially decay.



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