Chapter 1

Introduction

In this dissertation, we shall give an explicit formula of the solution of the Diamond operator. Moreover, we give a solution of an equation related to the wave equation and a solution of an equation related to the elastic wave.

In chapter 2, we give some useful definitions, properties and auxiliary results of gamma functions, distributions and partial differential equations.

In chapter 3, we evaluate the Diamond kernel $< K_{2k,2k}(x), \varphi(x) >$ where $x \in \mathbb{R}^n$, k is a positive integer, $\varphi \in \mathcal{D}$ the space of all continuous and infinitely differentiable function with compact support, and $K_{2k,2k}(x)$ is given by(2.27) with $\alpha = \beta = 2k$. In addition, we know from [7] that the convolution $(-1)^k K_{2k,2k}(x) * f(x)$ is a solution of the equation $\diamondsuit^k u(x) = f(x)$ where the operator $\diamondsuit^k$ was first introduced by A. Kananthai [8] and is named the Diamond operator iterated k- times and is defined by

$$\diamondsuit^k = \left[\left(\sum_{i=1}^{p} \frac{\partial^2}{\partial x_i^2} \right)^2 - \left(\sum_{j=p+1}^{p+q} \frac{\partial^2}{\partial x_j^2} \right)^2 \right]^k , \tag{1.1}$$

where $p + q = n$ is the n-dimensional of Euclidean space $\mathbb{R}^n$, k is a positive integer, $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$, f is a distribution and $u(x)$ is an unknown function. In this work, we consider $u(x) = (-1)^k K_{2k,2k}(x) * \varphi(x)$ as a solution of the equation $\diamondsuit^k u(x) = \varphi(x)$ where $\varphi(x) \in \mathcal{D}$ and such solution is given in the explicit form under some conditions of p and n.

In chapter 4, under some conditions of a given function f, we can find a solution of the equation of the form

$$\diamondsuit^k u(x) = f(x, \triangle^{k-1} \Box^k u(x)), \tag{1.2}$$

where $x \in \mathbb{R}^n, \diamondsuit^k$ is given by (1.1), $\triangle, \Box$ are Laplacian and ultra-hyperbolic operator defined by

$$\triangle = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \ldots + \frac{\partial^2}{\partial x_n^2} \tag{1.3}$$

and

$$\square = \frac{\partial^2}{\partial x_1^2} + \ldots + \frac{\partial^2}{\partial x_p^2} - \frac{\partial^2}{\partial x_{p+1}^2} - \ldots - \frac{\partial^2}{\partial x_{p+q}^2}, \tag{1.4}$$

where $p+q = n$. Moreover, such solution relates to the wave equation depending on the conditions of p, q and k.

In chapter 5, under assumptions of a given function f, we can give a solution of the equation of the form

$$\Diamond_{c_1}^k \Diamond_{c_2}^k u(x) = f(x, \triangle_{c_1}^{k-1} \square_{c_1}^k \Diamond_{c_2}^k u(x)), \tag{1.5}$$

where c_1, c_2 are positive and $\triangle_{c_1}, \square_{c_1}, \Diamond_{c_1}$, and $\Diamond_{c_2}$ are given by (2.42), (2.40), (5.1), and (5.2) respectively. Moreover, such solution relates to the elastic wave depending on the conditions of p, q, and k.