CHAPTER 1

INTRODUCTION

The Newton method for solving a system of equations

$$\mathbf{f}(\mathbf{x}) = 0 \quad \text{or} \quad f_i(x_1, x_2, \ldots, x_n) = 0, i = 1, 2, \ldots, m \tag{1.1}$$

uses the iterations

$$\mathbf{x}^{k+1} = \mathbf{x}^k - (J_{\mathbf{f}}(\mathbf{x}^k))^{-1}\mathbf{f}(\mathbf{x}^k),\ k = 0, 1, \ldots \tag{1.2}$$

where the Jacobian matrix $J_{\mathbf{f}}(\mathbf{x}^k)$ is assumed nonsingular at each iteration. If this cannot be assumed, or in general if $m \neq n$, the *Moore-Penrose inverse* of the Jacobian matrix can be used in the iterations

$$\mathbf{x}^{k+1} = \mathbf{x}^k - (J_{\mathbf{f}}(\mathbf{x}^k))^{\dagger}\mathbf{f}(\mathbf{x}^k),\ k = 0, 1, \ldots \tag{1.3}$$

with stationary points of the sum of squares $\sum_i f_i^2(\mathbf{x})$ as limits. Any *{2}-inverse* of the Jacobian can be used instead of Moore-Penrose inverse. Let $T_{\mathbf{x}^k}$ denote a {2}-inverse of $J_{\mathbf{f}}(\mathbf{x}^k)$. Nashed and Chen [7] studied the method

$$\mathbf{x}^{k+1} = \mathbf{x}^k - T_{\mathbf{x}^k}\mathbf{f}(\mathbf{x}^k),\ k = 0, 1, \ldots \tag{1.4}$$

and established quadratic convergence to a solution of $T_{\mathbf{x}^0}\mathbf{f}(\mathbf{x}) = 0$ under suitable conditions on $\mathbf{f}$ and the initial point $\mathbf{x}^0$.

In 2002, Adi-Ben Israel [2] gave conditions for local convergence of this method, proved quadratic convergence, implemented an adaptive version this iterative method, allowing a controlled increase of the ranks of the {2}-inverse used in the iterations, and found that under certain conditions on $\mathbf{f}$ and $\mathbf{x}^0$, the iterates

$$\mathbf{x}^{k+1} = \mathbf{x}^k - T_{\mathbf{x}^k}\mathbf{f}(\mathbf{x}^k),\ k = 0, 1, \ldots \tag{1.5}$$

converge to a point $\mathbf{x}^*$ satisfying

$$T_{\mathbf{x}^*}\mathbf{f}(\mathbf{x}^*) = 0. \tag{1.6}$$

The main purpose of this research is to give sufficient conditions on $\mathbf{f}$ and the initial point $\mathbf{x}^0$ for which the iterates

$$\mathbf{x}^{k+1} = \mathbf{x}^k - T_{\mathbf{x}^k}\mathbf{f}(\mathbf{x}^k),\ k = 0, 1, \ldots$$

converge to a point $\mathbf{x}^*$ satisfying $T_{\mathbf{x}^*}\mathbf{f}(\mathbf{x}^*) = 0$. Many examples of using this method are given and we also construct a software program for computing iterated sequence $(\mathbf{x}^k)$ in (1.5).

This thesis contains 4 chapters. The introduction of the thesis is in Chapter 1. Some useful definitions, notations and some known results which will be used in the successive chapters are given in Chapter 2. The main result of this thesis is in Chapter 3. The conclusion of thesis is in Chapter 4.