CHAPTER 4

CONCLUSION

This research give sufficient of Newton method for solving the system of nonlinear equations with m equations and n unknowns, and several examples of using this method are given. The following are the main results obtained in this study:

(1) Let C be a convex subset of $\mathbb{R}^n$, let $\mathbf{f} : C \to \mathbb{R}^m$ be differentiable and suppose $F : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is monotonically decreasing such that

$$\|J_{\mathbf{f}}(\mathbf{x}) - J_{\mathbf{f}}(\mathbf{y})\| \leq F(\|\mathbf{x} - \mathbf{y}\|)\|\mathbf{x} - \mathbf{y}\|, \ \ \forall \mathbf{x}, \mathbf{y} \in C$$

Then

$$\|\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{y}) - J_{\mathbf{f}}(\mathbf{y})(\mathbf{x} - \mathbf{y})\| \leq M\|\mathbf{x} - \mathbf{y}\|^2$$

where $M = F(0)/2$.

(2) Let $\mathbf{x}^0 \in \mathbb{R}^n$, $r > 0$ and let $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$ be differentiable in the open ball $B(\mathbf{x}^0, r)$. Suppose $F : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is monotonically decreasing such that

$$\|J_{\mathbf{u}} - J_{\mathbf{v}}\| \leq F(\|\mathbf{u} - \mathbf{v}\|)\|\mathbf{u} - \mathbf{v}\|$$

for all $\mathbf{u}, \mathbf{v} \in B(\mathbf{x}^0, r)$, where $J_{\mathbf{u}}$ is the Jacobian of $\mathbf{f}$ at $\mathbf{u}$. Further, assume that for all $\mathbf{x} \in \overline{B(\mathbf{x}^0, r)}$, the Jacobian $J_{\mathbf{x}}$ has a $\{2\} - inverse$ $T_{\mathbf{x}} \in \mathbb{R}^{n \times m}$

$$T_{\mathbf{x}} J_{\mathbf{x}} T_{\mathbf{x}} = T_{\mathbf{x}},$$

$$such\ that\ \ \|T_{\mathbf{x}^0}\|\|\mathbf{f}(\mathbf{x}^0)\| < \alpha a_0,$$

$$and\ for\ all\ \mathbf{u},\ \mathbf{v} \in \overline{B(\mathbf{x}^0, r)}, \|(T_{\mathbf{u}} - T_{\mathbf{v}})\mathbf{f}(\mathbf{v})\| \leq N\|\mathbf{u}\text{-}\mathbf{v}\|^2,$$

$$and\ \ M\|T_{\mathbf{u}}\| + N \leq K < 1,$$

for some positive scalars N, K and α with

$$\alpha K < \frac{a_{i+1}}{a_i^2}, \ r > \alpha S \quad \text{for all } i \in \mathbb{N}.$$

where $M = F(0)/2$ and $S = \Sigma_{i=0}^{\infty} a_i$, $a_i > 0$ for all $i \in \mathbb{N}$.

Then:

(a) Starting at $\mathbf{x}^0$, all iterates

$$\mathbf{x}^{k+1} = \mathbf{x}^k - T_{\mathbf{x}^k}\mathbf{f}(\mathbf{x}^k),\ \ k = 0, 1, 2, \ldots$$

lie in $B(\mathbf{x}^0, r)$.

(b) The sequence $(\mathbf{x}^k)$ converges, as $k \to \infty$, to a point $\mathbf{x}^* \in \overline{B(\mathbf{x}^0, r)}$, that is a solution of

$$T_{\mathbf{x}^*}\mathbf{f}(\mathbf{x}) = 0.$$

(c) For all $k \geq 0$

$$\|\mathbf{x}^k - \mathbf{x}^*\| \leq \alpha(S - S_{k-1})$$

where $S_k = \Sigma_{i=0}^{k} a_i$.