CHAPTER 1

INTRODUCTION

The Taylor-matrix method and the Chebyshev-matrix method for solving linear differential equations

$$P(x)y'' + Q(x)y' + R(x)y \;=\; f(x) \tag{1.1}$$

have been studied by C. Keşan [2] and M.Sezer and M. Kaynak [5], respectively.

The Taylor-matrix method and the Chebyshev-matrix method for solving Freadholm integral equations

$$g(x) \;=\; f(x) + \int_a^b K(x,y)g(y)dy \tag{1.2}$$

have been presented by R.P Kanwal and K.C Liu [1] and M. Sezer and S. Doğan [4], respectively. The objectives of this thesis are to study a Taylor-matrix method to find an approximate solution of integro-differential equations

$$P(x)y'' + Q(x)y' + R(x)y \;=\; f(x) + \int_a^b K(x,\xi)d\xi \tag{1.3}$$

$$P(x)y'' + Q(x)y' + R(x)y \;=\; f(x) + \int_a^b K(x,\xi)y(\xi)d\xi \tag{1.4}$$

where $P(x),\ Q(x),\ R(x),\ f(x)$ and $K(x,\xi)$ are functions having Taylor expansions about c in $[a,b]$, under the given conditions, which are

$$\sum_{i=0}^{1}[a_i y^{(i)}(a) + b_i y^{(i)}(b) + c_i y^{(i)}(c)] \;=\; \lambda \tag{1.5}$$

$$\sum_{i=0}^{1}[\alpha_i y^{(i)}(a) + \beta_i y^{(i)}(b) + \gamma_i y^{(i)}(c)] \;=\; \mu \tag{1.6}$$

where $a \le c \le b$, provided that the real coefficients $a_i,\ b_i,\ c_i,\ \alpha_i,\ \beta_i,\ \gamma_i,\ \lambda,\ \mu$ are appropriate constants; and the solution of equations is expressed in the form

$$y(x) \;=\; \sum_{n=0}^{N} \frac{1}{n!} y^{(n)}(c)(x-c)^n, \quad a \le c \le b \tag{1.7}$$

which is a Taylor polynomial of degree N at $x = c$, where $y^{(n)}(c),\ n = 0, 1, \ldots, N$ are the coefficients to be determined. Then, we study the Chebyshev-matrix method to find the approximate solution of integro-differential equation

$$P(x)y'' + Q(x)y' + R(x)y = f(x) + \int_{-1}^{1} K(x,\xi)d\xi \quad (1.8)$$

$$P(x)y'' + Q(x)y' + R(x)y = f(x) + \int_{-1}^{1} K(x,\xi)y(\xi)d\xi \quad (1.9)$$

where we assume that the range of the variables is $-1 \leq x, \xi \leq 1$ under the prescribed conditions; and the solution of equations is expanded in the form

$$y(x) = {\sum_{r=0}^{N}}' a_r T_r(x), \quad -1 \leq x \leq 1. \quad (1.10)$$

Here, $\sum'$ denotes a sum whose first term is halved, T_r denotes the Chebyshev polynomial of the first kind of degree r and $a_r,\ r = 0, 1, \ldots, N$ are the Chebyshev coefficients to be determined.