APPENDICES

APPENDIX A

Parameter for the IEEE 14-Bus Test System

Table A.1 Line parameter at fundamental frequency comparing between data from IEEE 14-bus and data modeling from TL program

		IEEE			TL			|Error|		
Bus #	Bus #	R (pu)	X (pu)	B (pu)	R (pu)	X (pu)	B (pu)	R (pu)	X (pu)	B (pu)
1	2	0.01938	0.05917	0.05280	0.01913	0.05904	0.05359	2.51E-04	1.25E-04	7.85E-04
1	5	0.05403	0.22304	0.04920	0.05408	0.22228	0.04962	5.15E-05	7.61E-04	4.18E-04
2	3	0.04699	0.19797	0.04380	0.04746	0.19873	0.04340	4.65E-04	7.57E-04	4.03E-04
2	4	0.05811	0.17632	0.03740	0.05814	0.17648	0.03690	3.07E-05	1.62E-04	5.04E-04
2	5	0.05695	0.17388	0.03400	0.05604	0.17773	0.03422	9.08E-04	3.85E-03	2.19E-04
3	4	0.06701	0.17103	0.03460	0.06700	0.17006	0.03474	1.16E-05	9.72E-04	1.36E-04
4	5	0.01335	0.04211	0.01280	0.01328	0.04210	0.01294	6.73E-05	6.47E-06	1.37E-04
4	7	0.00000	0.20912	0.00000						
4	9	0.00000	0.55618	0.00000						
5	6	0.00000	0.25202	0.00000						
6	11	0.09498	0.19890	0.00000	0.09484	0.19896	0.01849	1.37E-04	5.72E-05	1.85E-02
6	12	0.12291	0.25581	0.00000	0.12274	0.25559	0.02414	1.69E-04	2.22E-04	2.41E-02
6	13	0.06615	0.13027	0.00000	0.06608	0.13071	0.01364	7.09E-05	4.44E-04	1.36E-02
7	8	0.00000	0.17615	0.00000						
7	9	0.00000	0.11001	0.00000						
9	10	0.03181	0.08450	0.00000	0.03183	0.08407	0.00729	2.17E-05	4.28E-04	7.29E-03
9	14	0.12711	0.27038	0.00000	0.12698	0.27034	0.02446	1.28E-04	4.49E-05	2.45E-02
10	11	0.08205	0.19207	0.00000	0.08205	0.19235	0.02115	2.44E-06	2.82E-04	2.11E-02
12	13	0.22092	0.19988	0.00000	0.22047	0.19976	0.02276	4.48E-04	1.21E-04	2.28E-02
13	14	0.17093	0.34802	0.00000	0.17085	0.34755	0.03450	8.14E-05	4.74E-04	3.45E-02

Table A.2 Line parameter at each harmonics frequency from TL program

H-Order		5			7			11		
Bus #	Bus #	R (pu)	X (pu)	B (pu)	R (pu)	X (pu)	B (pu)	R (pu)	X (pu)	B (pu)
1	2	0.01956	0.29143	0.26958	0.01983	0.40292	0.37977	0.02016	0.60971	0.60823
1	5	0.05136	1.06127	0.25369	0.04779	1.41934	0.36343	0.03621	1.93529	0.61486
2	3	0.04493	0.95956	0.22078	0.04228	1.29700	0.31464	0.03434	1.82868	0.52298
2	4	0.05768	0.85907	0.18691	0.05666	1.17119	0.26518	0.05202	1.69706	0.43436
2	5	0.05536	0.86715	0.17320	0.05438	1.18452	0.24550	0.05028	1.72648	0.40096
3	4	0.06464	0.82982	0.17575	0.06202	1.13413	0.24904	0.05400	1.65621	0.40627
4	5	0.01406	0.20997	0.06475	0.01473	0.29328	0.09075	0.01635	0.45782	0.14308
4	7	0.00000	1.04500	0.00000	0.00000	1.46300	0.00000	0.00000	2.29900	0.00000
4	9	0.00000	2.78090	0.00000	0.00000	3.89326	0.00000	0.00000	6.11798	0.00000
5	6	0.00000	1.25100	0.00000	0.00000	1.75140	0.00000	0.00000	2.75220	0.00000
6	11	0.09564	0.98028	0.09314	0.09606	1.35226	0.13137	0.09543	2.03169	0.21120
6	12	0.12129	1.24678	0.12220	0.11926	1.70249	0.17325	0.11042	2.47853	0.28315
6	13	0.06767	0.64895	0.06843	0.06904	0.90209	0.09615	0.07207	1.38744	0.15274
7	8	0.00000	0.88075	0.00000	0.00000	1.23305	0.00000	0.00000	1.93765	0.00000
7	9	0.00000	0.55000	0.00000	0.00000	0.77000	0.00000	0.00000	1.21000	0.00000
9	10	0.03345	0.41934	0.03647	0.03489	0.58564	0.05112	0.03844	0.91355	0.08063
9	14	0.12502	1.31635	0.12392	0.12245	1.79420	0.17585	0.11188	2.59712	0.28827
10	11	0.08385	0.94628	0.10660	0.08504	1.30333	0.15048	0.08572	1.94885	0.24252
12	13	0.21447	0.98087	0.11484	0.20844	1.34837	0.16226	0.19016	2.00428	0.26233
13	14	0.16083	1.65591	0.17677	0.14999	2.20697	0.25379	0.11428	2.97304	0.43254

Table A.2 Line parameter at each harmonics frequency from TL program (Cont.)

H-Order		13			17			19		
Bus #	Bus #	R (pu)	X (pu)	B (pu)	R (pu)	X (pu)	B (pu)	R (pu)	X (pu)	B (pu)
1	2	0.02009	0.70248	0.72818	0.01922	0.86111	0.98458	0.01835	0.92503	1.12355
1	5	0.02842	2.07073	0.76674	0.00995	2.06759	1.16963	-0.00008	1.92933	1.45925
2	3	0.02917	2.00476	0.64327	0.01690	2.14751	0.93946	0.01009	2.10931	1.13172
2	4	0.04804	1.89726	0.52846	0.03629	2.14652	0.74716	0.02858	2.18918	0.87883
2	5	0.04683	1.93799	0.48678	0.03666	2.21668	0.68402	0.02997	2.27713	0.80107
3	4	0.04870	1.86177	0.49281	0.03591	2.13805	0.69084	0.02864	2.20233	0.80773
4	5	0.01720	0.53868	0.16946	0.01880	0.69673	0.22282	0.01950	0.77358	0.24985
4	7	0.00000	2.71700	0.00000	0.00000	3.55300	0.00000	0.00000	3.97100	0.00000
4	9	0.00000	7.23034	0.00000	0.00000	9.45506	0.00000	0.00000	10.56742	0.00000
5	6	0.00000	3.25260	0.00000	0.00000	4.25340	0.00000	0.00000	4.75380	0.00000
6	11	0.09374	2.32921	0.25353	0.08642	2.81811	0.34527	0.08053	3.00234	0.39585
6	12	0.10281	2.77984	0.34393	0.08013	3.17223	0.48396	0.06512	3.25367	0.56733
6	13	0.07334	1.61621	0.18183	0.07458	2.03784	0.24225	0.07432	2.22772	0.27386
7	8	0.00000	2.28995	0.00000	0.00000	2.99455	0.00000	0.00000	3.34685	0.00000
7	9	0.00000	1.43000	0.00000	0.00000	1.87000	0.00000	0.00000	2.09000	0.00000
9	10	0.04030	1.07436	0.09553	0.04371	1.38773	0.12570	0.04515	1.53952	0.14102
9	14	0.10307	2.90107	0.35092	0.07743	3.27420	0.49695	0.06077	3.33355	0.58519
10	11	0.08438	2.22688	0.29164	0.07704	2.67109	0.39906	0.07087	2.83008	0.45896
12	13	0.17787	2.28085	0.31614	0.14711	2.70639	0.43518	0.12877	2.84767	0.50249
13	14	0.08915	3.15160	0.54256	0.02698	3.05760	0.84371	-0.00764	2.78947	1.06968

Table A.2 Line parameter at each harmonics frequency from TL program (Cont.)

H-Order		23			25		
Bus #	Bus #	R (pu)	X (pu)	B (pu)	R (pu)	X (pu)	B (pu)
1	2	0.01570	1.01819	1.43121	0.01394	1.04629	1.60431
1	5	-0.01964	1.40990	2.48061	-0.02826	1.05161	3.56582
2	3	-0.00402	1.82030	1.68913	-0.01089	1.57938	2.13303
2	4	0.01026	2.10519	1.21714	0.00014	1.98074	1.44607
2	5	0.01392	2.23298	1.09527	0.00494	2.12945	1.28877
3	4	0.01299	2.17577	1.09920	0.00490	2.08559	1.28903
4	5	0.02064	0.92211	0.30481	0.02107	0.99346	0.33280
4	7	0.00000	4.80700	0.00000	0.00000	5.22500	0.00000
4	9	0.00000	12.79214	0.00000	0.00000	13.90450	0.00000
5	6	0.00000	5.75460	0.00000	0.00000	6.25500	0.00000
6	11	0.06425	3.23690	0.51039	0.05404	3.28378	0.57659
6	12	0.02914	3.17709	0.77790	0.00907	3.02093	0.91722
6	13	0.07168	2.55886	0.34070	0.06927	2.69778	0.37633
7	8	0.00000	4.05145	0.00000	0.00000	4.40375	0.00000
7	9	0.00000	2.53000	0.00000	0.00000	2.75000	0.00000
9	10	0.04738	1.83146	0.17222	0.04815	1.97090	0.18815
9	14	0.02149	3.19003	0.81308	-0.00004	2.99091	0.96828
10	11	0.05380	3.00901	0.59669	0.04321	3.02606	0.67780
12	13	0.08687	2.97457	0.66030	0.06377	2.95788	0.75544
13	14	-0.07554	1.87403	1.93439	-0.10508	1.27024	3.00553

Table A.3 Node-line incidence matrix (C_{NL})

	Line#																																								
Bus#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	-1	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	-1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	-1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
5	0	0	0	-1	0	0	0	0	0	-1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-1	0	0	-1
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0

Table A.4 Line-branch incidence matrix (C_{LB})

	Branch#																																		
Line#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1

Table A.5 Non-zero elements of primitive admittance matrix (Y_{BB}) for the 5th harmonic

Branch ii	Y_{BB} (ii,ii)		Branch ii	Y_{BB} (ii,ii)	
	Real	Imag		Real	Imag
1	0.2292	-3.4160	19	0.3175	-4.7414
2	0.1348	0.0000	20	0.0324	0.0000
3	0.1348	0.0000	21	0.0324	0.0000
4	0.0455	-0.9401	22	0.0986	-1.0105
5	0.1268	0.0000	23	0.0773	-0.7945
6	0.1268	0.0000	24	0.1589	-1.5244
7	0.0487	-1.0399	25	0.1890	-2.3697
8	0.1104	0.0000	26	0.0715	-0.7529
9	0.1104	0.0000	27	0.0929	-1.0485
10	0.0778	-1.1588	28	0.2128	-0.9730
11	0.0935	0.0000	29	0.0581	-0.5983
12	0.0935	0.0000	30	0.0000	-0.9569
13	0.0733	-1.1485	31	0.0000	-1.8182
14	0.0866	0.0000	32	0.0000	-1.1354
15	0.0866	0.0000	33	0.0000	-0.3596
16	0.0933	-1.1978	34	0.0000	-0.7994
17	0.0879	0.0000	35	0.0000	0.3165
18	0.0879	0.0000			

Table A.6 Node-node admittance matrix (Y_{NN}) for the 5th harmonic

Bus#	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.536	-0.229	0.000	0.000	-0.045	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	-5.156	3.416	0.000	0.000	0.940	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-0.229	0.854	-0.049	-0.078	-0.073	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3.416	-7.563	1.040	1.159	1.149	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	-0.049	0.340	-0.093	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	1.040	-2.238	1.198	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	-0.078	-0.093	0.702	-0.318	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	1.159	1.198	-8.415	4.741	0.000	0.957	0.000	0.360	0.000	0.000	0.000	0.000	0.000
5	-0.045	-0.073	0.000	-0.318	0.682	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.940	1.149	0.000	4.741	-7.629	0.799	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.335	0.000	0.000	0.000	0.000	-0.099	-0.077	-0.159	0.000
	0.000	0.000	0.000	0.000	0.799	-4.129	0.000	0.000	0.000	0.000	1.010	0.795	1.524	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.957	0.000	0.000	-3.911	1.135	1.818	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	1.135	-1.135	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.261	-0.189	0.000	0.000	0.000	-0.072
	0.000	0.000	0.000	0.360	0.000	0.000	1.818	0.000	-4.984	2.370	0.000	0.000	0.000	0.753
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.189	0.282	-0.093	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.370	-3.418	1.049	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	-0.099	0.000	0.000	0.000	-0.093	0.192	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	1.010	0.000	0.000	0.000	1.049	-2.059	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.000	-0.077	0.000	0.000	0.000	0.000	0.000	0.290	-0.213	0.000
	0.000	0.000	0.000	0.000	0.000	0.795	0.000	0.000	0.000	0.000	0.000	-1.768	0.973	0.000
13	0.000	0.000	0.000	0.000	0.000	-0.159	0.000	0.000	0.000	0.000	0.000	-0.213	0.430	-0.058
	0.000	0.000	0.000	0.000	0.000	1.524	0.000	0.000	0.000	0.000	0.000	0.973	-3.096	0.598
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.072	0.000	0.000	0.000	-0.058	0.130
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.753	0.000	0.000	0.000	0.598	-1.351

Note:

Bus#		1
1	Real part	0.536
	Imaginary part	-5.156

Table A.7 Line-node admittance matrix (Y_{LN}) for the 5[th] harmonic

	Bus#													
Line#	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.364	-0.229	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	-3.416	3.416	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-0.229	0.094	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3.416	-3.416	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.172	0.000	0.000	0.000	-0.045	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	-0.940	0.000	0.000	0.000	0.940	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	-0.045	0.000	0.000	0.000	-0.081	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.940	0.000	0.000	0.000	-0.940	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.159	-0.049	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	-1.040	1.040	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	-0.049	-0.062	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	1.040	-1.040	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.171	0.000	-0.078	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	-1.159	0.000	1.159	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	-0.078	0.000	-0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	1.159	0.000	-1.159	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.160	0.000	0.000	-0.073	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	-1.149	0.000	0.000	1.149	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	-0.073	0.000	0.000	-0.013	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	1.149	0.000	0.000	-1.149	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	-0.093	0.181	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	1.198	-1.198	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.005	-0.093	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	-1.198	1.198	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.000	0.000	0.000	0.350	-0.318	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	-4.741	4.741	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	-0.318	0.285	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	4.741	-4.741	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.099	0.000	0.000	0.000	0.000	-0.099	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	-1.010	0.000	0.000	0.000	0.000	1.010	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.000	-0.099	0.000	0.000	0.000	0.000	0.099	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	1.010	0.000	0.000	0.000	0.000	-1.010	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.000	-0.077	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	-0.795	0.000	0.000	0.000	0.000	0.000	0.795	0.000	0.000
18	0.000	0.000	0.000	0.000	0.000	-0.077	0.000	0.000	0.000	0.000	0.000	0.077	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.795	0.000	0.000	0.000	0.000	0.000	-0.795	0.000	0.000
19	0.000	0.000	0.000	0.000	0.000	0.159	0.000	0.000	0.000	0.000	0.000	0.000	-0.159	0.000
	0.000	0.000	0.000	0.000	0.000	-1.524	0.000	0.000	0.000	0.000	0.000	0.000	1.524	0.000
20	0.000	0.000	0.000	0.000	0.000	-0.159	0.000	0.000	0.000	0.000	0.000	0.000	0.159	0.000
	0.000	0.000	0.000	0.000	0.000	1.524	0.000	0.000	0.000	0.000	0.000	0.000	-1.524	0.000
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.189	-0.189	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.370	2.370	0.000	0.000	0.000	0.000

Table A.7 Line-node admittance matrix ($\boldsymbol{Y}_{LN}$) for the 5th harmonic (Cont.)

	Bus#													
Line#	1	2	3	4	5	6	7	8	9	10	11	12	13	14
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.189	0.189	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.370	-2.370	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.072	0.000	0.000	0.000	0.000	-0.072
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.753	0.000	0.000	0.000	0.000	0.753
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.072	0.000	0.000	0.000	0.000	0.072
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.753	0.000	0.000	0.000	0.000	-0.753
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.093	-0.093	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.049	1.049	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.093	0.093	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.049	-1.049	0.000	0.000	0.000
27	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.213	-0.213	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.973	0.973	0.000
28	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.213	0.213	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.973	-0.973	0.000
29	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.058	0.058
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.598	-0.598
30	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.058	-0.058
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.598	0.598
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	-0.957	0.000	0.000	0.957	0.000	0.000	0.000	0.000	0.000	0.000	0.000
32	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.957	0.000	0.000	-0.957	0.000	0.000	0.000	0.000	0.000	0.000	0.000
33	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	-1.818	0.000	1.818	0.000	0.000	0.000	0.000	0.000
34	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	1.818	0.000	-1.818	0.000	0.000	0.000	0.000	0.000
35	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	-1.135	1.135	0.000	0.000	0.000	0.000	0.000	0.000
36	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	1.135	-1.135	0.000	0.000	0.000	0.000	0.000	0.000
37	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	-0.360	0.000	0.000	0.000	0.000	0.360	0.000	0.000	0.000	0.000	0.000
38	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.360	0.000	0.000	0.000	0.000	-0.360	0.000	0.000	0.000	0.000	0.000
39	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	-0.799	0.799	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
40	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.058	-0.058
	0.000	0.000	0.000	0.000	0.799	-0.799	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
41	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.317	0.000	0.000	0.000	0.000	0.000

Table A.8 Harmonic source spectra

H-order	Six-Pulse HVDC		Delta Conn ected TCR	
	Mag(pu)	Angle(deg)	Mag(pu)	Angle(deg)
1	1.0000	-49.56	1.0000	46.92
5	0.1941	-67.77	0.0702	-124.40
7	0.1309	11.90	0.0250	-29.87
11	0.0758	-7.13	0.0136	-23.75
13	0.0586	68.57	0.0075	71.50
17	0.0379	46.53	0.0062	77.12
19	0.0329	116.46	0.0032	173.43
23	0.0226	87.47	0.0043	178.02
25	0.0241	159.32	0.0013	-83.45
29	0.0193	126.79	0.0040	-80.45

Table A.9 The physical geometry of the IEEE 14-bus test system using TL program

Bus #	Bus #	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	2	52.00	21	4	1	15.0	31	1	1	3.00	0	3.00	3.5	3.00	7.00	2.00	12.00
1	5	96.31	15	4	0.128	15.0	31	1	1	1.00	0	11.00	8	1.00	16.00	3.00	18.00
2	3	85.00	15	4	0.128	15.0	31	1	1	11.00	0	0	0	-11.00	0	1.00	5.00
2	4	75.40	42	2	0.150	15.0	31	1	1	4.00	0	7.00	6	4.00	12.00	2.00	16.00
2	5	72.82	42	2	0.150	15.0	31	1	1	8.00	0	0	0	-8.00	0	0	5.00
3	4	69.46	11	2	0.190	15.0	31	1	1	5.00	0	8.00	7.00	5.00	14.00	2.00	18.00
4	5	21.75	18	2	0.700	15.0	31	1	1	5. .00	0	5.00	5.00	5. .00	10.00	2.00	15.00
6	11	56.50	21	1	0	12.0	0	1	9	5.83	0	0	0	-5.83	0	-	-
6	12	73.18	21	1	0	12.0	0	1	9	5.50	0	0	0	-5.50	0	-	-
6	13	39.34	21	1	0	12.0	0	1	9	3.83	0	0	0	-3.83	0	-	-
9	10	22.94	20	1	0	12.0	0	1	9	8.13	0	0	0	-8.13	0	-	-
9	14	75.72	21	1	0	12.0	0	1	9	6.48	0	0	0	-6.48	0	-	-
10	11	59.20	20	1	0	12.0	0	1	9	3.40	0	0	0	-3.40	0	-	-
12	13	62.60	26	1	0	12.0	0	1	9	1.88	0	0	0	-1.88	0	-	-
13	14	102.06	21	1	0	12.0	0	1	9	4.60	0	0	0	-4.60	0	-	-

Note: A: Length of line (km)
B: Conductor type (Table A.10)
C: Number of conductor in bundle
D: Bundle spacing (m)
E: Average height of lowest conductor above the ground
F: Earth wire type (0: no earth wire) (Table A.10)
G: Circuit type (1: single circuit, 2: double circuit)
H: Number of earth wire (1 or 2, 9: line has earth wire only at the ends)
I: Phase 1 horizontal distance from the tower axis (m)
J: Phase 1 vertical distance from the average height (m)
K: Phase 2 horizontal distance from the tower axis (m)
L: Phase 2 vertical distance from the average height (m)
M: Phase 3 horizontal distance from the tower axis (m)
N: Phase 3 vertical distance from the average height (m)

Table A.10 Conductor types

Type	Conductor description	Brand name	Diameter (cm)	Geometric Mean Radius (cm)	Resistance per km (at 20 °C)	Inside diameter (cm)
1	84/3.70 + 19/2.22 ACSR	CHUKAR	40.69	16.34	0.03134	11.10
2	54/4.36 + 19/2.62 ACSR	SPECIAL	39.24	15.76	0.03819	13.10
3	76/3.72 + 7/2.89 ACSR	SPECIAL	38.40	15.39	0.03511	8.76
4	54/3.90 + 19/2.34 ACSR	PHEASANT	35.10	14.20	0.04491	11.70
5	54/3.18 + 7/3.18 ACSR	ZEBRA	28.58	11.55	0.07009	9.54
6	30/3.71 + 7/3.71 ACSR	GOAT	25.96	10.70	0.09010	11.13
7	30/3.00 + 7/3.00 ACSR	PANTHER	20.98	8.76	0.13790	9.06
8	30/2.59 + 7/2.59 ACSR	WOLF	18.14	7.47	0.18450	7.77
9	26/2.57 + 7/2.00 ACSR	PARTRIDGE	16.31	6.61	0.21320	6.00
10	26/2.54 + 7/1.91 ACSR	COYOTE	15.88	6.40	0.22120	5.73
11	16/2.86 + 19/2.48 ACSR	BRAHMA	18.14	3.17	0.29170	12.40
12	7/4.39 + 7/1.93 ACSR	HYENA	14.58	2.48	0.30700	5.79
13	6/4.72 + 1/4.72 ACSR	HARE	14.17	2.44	0.30800	4.72
14	6/4.72 + 7/1.57 ACSR	DOG	14.17	2.44	0.30570	4.71
15	6/4.25 + 1/4.25 ACSR	PIGEON	12.75	1.83	0.33800	4.25
16	12/2.59 + 7/2.59 ACSR	SKUNK	12.95	1.83	0.45690	7.77
17	6/3.66 + 1/3.66 ACSR	MINK	10.97	1.49	0.47290	3.66
18	19/2.57 COPPER	COPPER	12.83	4.85	0.18400	
19	37/1.83 COPPER	COPPER	12.80	4.91	0.18390	
20	7/4.22 COPPER	COPPER	12.65	4.57	0.18310	
21	19/2.34 COPPER	COPPER	11.68	4.42	0.22190	
22	7/3.45 COPPER	COPPER	10.36	3.75	0.27260	
23	7/3.25 COPPER	COPPER	9.75	3.54	0.29950	
24	7/2.77 COPPER	COPPER	8.31	2.99	0.43020	
25	19/1.63 COPPER	COPPER	8.13	3.08	0.45850	
26	7/2.64 COPPER	COPPER	7.92	2.87	0.46630	
27	7/2.34 COPPER	COPPER	7.01	2.53	0.60100	
28	7/2.03 COPPER	COPPER	6.10	2.23	0.79470	
29	7/3.71 GEHSS	GEHSS	11.13	3.00E-06	3.50000	
30	7/3.68 GEHSS	GEHSS	11.05	3.00E-06	3.50000	
31	7/3.18 GEHSS	GEHSS	9.52	3.00E-06	4.00000	
32	7/3.05 GEHSS	GEHSS	9.14	3.00E-06	5.00000	
33	7/2.64 GEHSS	GEHSS	7.92	3.00E-06	8.00000	
34	7/2.59 GEHSS	GEHSS	7.77	3.00E-06	8.50000	
35	7/3.71 ALUMOWELD	AL/WELD	11.13	0.72	4.00000	
36	7/3.05 ALUMOWELD	AL/WELD	9.14	0.59	5.50000	
37	19/2.87 GEHSS	GEHSS	14.35	3.00E-06	6.00000	
38	37/3.66 COPPER	COPPER	25.60	9.83	0.04600	
39	61/2.62 COPPER	COPPER	23.55	9.09	0.05450	
40	37/2.62 COPPER	COPPER	18.31	7.03	0.09000	
41	7/4.62 COPPERWELD	CU/WELD	13.87	0.20	0.37830	
42	7/4.72 H.D. ALUMINIUM	WEKE	14.16	5.14	0.23280	
43	19/4.22 H.D. ALUMINIUM	COCKROACH	21.10	7.99	0.10830	
44	72/4.41 + 7/2.94 ACSR	KIWI	44.07	17.37	0.02956	
45	6/3.35 + 1/3.35 ACSR	RABBIT	10.06	1.37	0.54040	
46	6/3.00 + 1/3.00 ACSR	ROBIN	9.02		0.67400	

APPENDIX B

Three Phase Transmission Line Parameters (TL) Program [30]

B.1 Introduction

This program may be used to calculate the following electrical characteristics of a three phase transmission line, at arbitrary frequency: series impedance and shunt admittance matrices in actual phase quantities, and series impedance and shunt admittance matrices in sequence component.

From the lines geometry, conductor type and the earth resistively. Up to four mutually coupled three phase lines maybe handled with up to four earth wires. All circuits and earth wires may be of different types and need have no special symmetry. The program was written to provide transmission line parameters for the three phase power flow analysis at arbitrary frequencies

The program calculates the unbalanced three phase parameters, including mutual coupling terms between double circuit lines and electrically coupled single circuit lines, and outputs a full record of the input data and the parameters, in both sequence components and the phase components.

B.2 Details of the model

This program which is based primarily on the method outlined in Dommel, uses Carson's equations and correction factors to calculate the series impedance (***Z***) and shunt admittance (***Y***) matrices. To explain the method, the example of a single circuit line with twin bundle conductors and two ground wires, as shown in Figure B.1 will be used. Conductors 1, 2 are bundled into phase A, no. 3, 4 into phase B, 5, 6 into phase C and the earth wires conductors 7, 8.

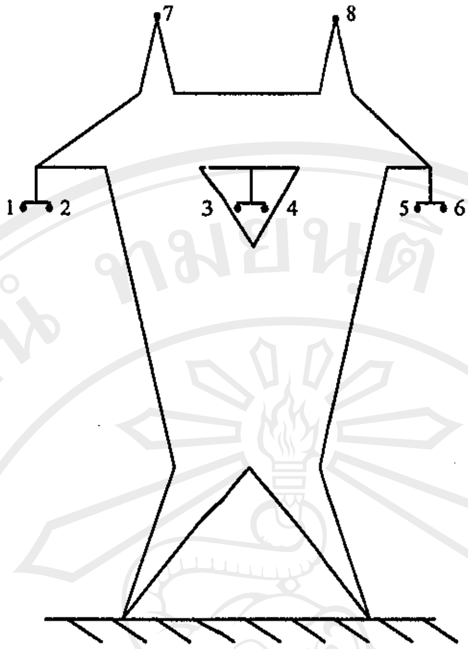


Figure B.1 Example of tower for TL program.

B.2.1 Series impedance

The series impedance of the 8 conductors are described in the form of an 8×8 impedance matrix $\boldsymbol{Z}$. Its diagonal element Z_{ii} is the series impedance per unit length of conductor i with ground serving as the return pate, and its off-diagonal element Z_{ik} is the series mutual impedance per unit length between conductor i and k. The matrix is symmetric, $Z_{ik} = Z_{ki}$. The values of the matrix elements are computed with Carson's formula,

$$Z_{ii} = \left(R_i + \Delta R_{ii}\right) + j\left(2\omega \times 10^{-4} \times \ln\left(\frac{2h_i}{GMR_i}\right) + \Delta X_{ii}\right) \quad \Omega/\mathrm{km} \qquad (\mathbf{B}.1)$$

$$Z_{ik} = Z_{ki} = \Delta R_{ik} + j\left(2\omega \times 10^{-4} \times \ln\left(\frac{D_{ik}}{d_{ik}}\right) + \Delta X_{ik}\right) \quad \Omega/\mathrm{km} \qquad (\mathbf{B}.2)$$

with

R_i is resistance of conductor i in Ω/km,

h_i is average height above ground of conductor i,

D_{ik} is distance between conductor i and the image of conductor k (m) as shown in Figure **B**.2,

d_{ik} is distance between conductor i and k (m),

GMR_i is Geometric Mean Radius of conductor i (m),

ω is angular frequency

ΔR, ΔX is Carson's correction terms which account for ground return effects.

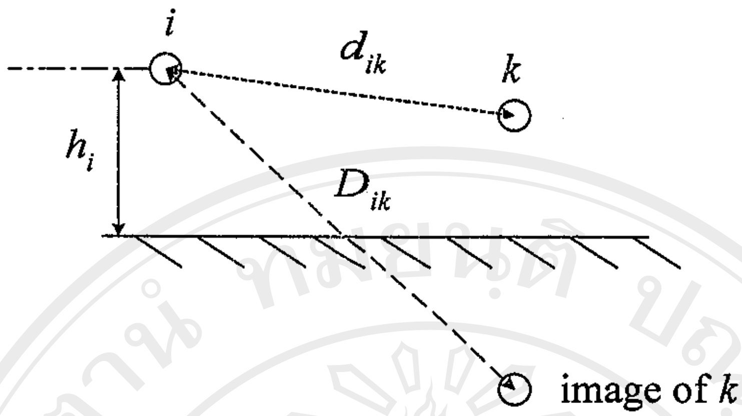


Figure B.2 Image of conductor for TL program.

The steady state voltage drop per unit length along the 8 conductors can then be written as

$$\begin{bmatrix} V_1 \\ \dots \\ V_8 \end{bmatrix} = \begin{bmatrix} Z_{1,1} & \dots & Z_{1,8} \\ \dots & Z_{2,2} & \dots \\ Z_{8,1} & \dots & Z_{8,8} \end{bmatrix} \begin{bmatrix} I_1 \\ \dots \\ I_8 \end{bmatrix} \qquad \textbf{(B.3)}$$

B.2.2 Shunt capacitance

The capacitance between the 8 conductors and ground are described in the form of 8×8 capacitance matrix ***C***. Its diagonal elements C_{ii} is the sum of the capacitance per unit length from conductor *i* to all other conductors as well as to ground; its off-diagonal element C_{ik} is the negative value of the shunt capacitance per unit length between conductor *i* and *k*. Again, ***C*** is symmetric, $C_{ik}=C_{ki}$. The capacitance matrix cannot be computed directly. Instead, the matrix ***P*** of Maxwell's potential coefficients is formed, and ***C*** is then found by matrix inversion,

$$\boldsymbol{C} = \boldsymbol{P}^{-1} \qquad \textbf{(B.4)}$$

The elements of ***P*** are computed from the tower geometry. If $r << h$, with r being the radius of the conductor, then

$$P_{ii} = 18\times10^6 \times \ln\left(\frac{2h_i}{r_i}\right) \text{ km/F} \qquad \textbf{(B.5a)}$$

$$P_{ik} = P_{ki} = 18\times10^6 \times \ln\left(\frac{D_{ik}}{d_{ik}}\right) \text{ km/F} \qquad \textbf{(B.5b)}$$

where r_i is external radius of conductor *i*.

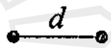
The exact value of the factor in equation (**B**.5) is $2c^2\times10^{-4}$, with c being the velocity of light in air in km/s. Maxwell's potential coefficients relate line-to-ground voltages of the 8 conductors to the charges on the conductors,

$$\begin{bmatrix} V_1 \\ \dots \\ V_8 \end{bmatrix} = \begin{bmatrix} P_{1,1} & \dots & P_{1,8} \\ \dots & P_{2,2} & \dots \\ P_{8,1} & \dots & P_{8,8} \end{bmatrix} \begin{bmatrix} Q_1 \\ \dots \\ Q_8 \end{bmatrix} \qquad \textbf{(B.6)}$$

B.2.3 Elimination of bundling and ground wires

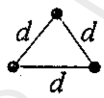
The reduction process is simple for connection conductors into a bundle. For a two conductor bundle the geometric mean distance (*GMD*), of the 2,3 or 4 conductors in the bundle reduces the bundle to a single equivalent conductor,

$$GMD = \sqrt{GMR \times d}$$



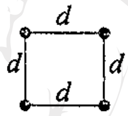
Similarly for three conductors

$$GMD = \sqrt[3]{GMR \times d^2}$$



Similarly for four conductors

$$GMD = \sqrt[4]{GMR \times d^3}$$



For the series impedance matrix the radius *R* is replaced by the geometric mean radius of effective geometric mean distance as it is sometimes called and similar equivalent conductor relationships are used.

Equation (**B**.3) and (**B**.6) for all 5 conductors are too detailed if the phase quantities are required, as they are for power flow or harmonic studies.

They can be reduced to an equation per phase per circuit for a total of three. The reduction process is easier to explain if the inverse forms of equation (**B**.3) and (**B**.6) are used, rather than their original forms. Computation is faster, however of the reduction process is applied to the original form. Assume that the inverse relationship of equation (**B**.6) has been found by inverting ***P***.

$$\begin{bmatrix} Q_1 \\ \dots \\ Q_5 \end{bmatrix} = \begin{bmatrix} C_{1,1} & \dots & C_{1,5} \\ \dots & C_{2,2} & \dots \\ C_{5,1} & \dots & C_{5,5} \end{bmatrix} \begin{bmatrix} V_1 \\ \dots \\ V_5 \end{bmatrix} \qquad \textbf{(B.7)}$$

Since the voltage on the ground wires is zero (assuming that they are not insulated from the towers), one can set V_4=0 and V_5=0 in equation (**B**.7), and omit the rows for Q_4 and Q_5. This means that wires 7 and 8 are eliminated by simply scratching the last two rows and columns in equation (**B**.7). Their effect on the line performance is contained, however, in the first 3 equations of equation (**B**.7). The 8×8 matrix has been replaced by a 3×3 matrix with one row and column per phase as required by power flow and harmonic studies.

Similar, equation (**B**.3) can be reduced to

$$\begin{bmatrix} Q_a \\ Q_b \\ Q_c \end{bmatrix} = \begin{bmatrix} C_{a,a} & C_{a,b} & C_{a,a} \\ C_{b,a} & C_{b,b} & C_{b,c} \\ C_{c,a} & C_{c,b} & C_{c,c} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{B.8}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_{a,a} & Z_{a,b} & Z_{a,a} \\ Z_{b,a} & Z_{b,b} & Z_{b,c} \\ Z_{c,a} & Z_{c,b} & Z_{c,c} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{B.9}$$

The reduction process for the impedance works the save way as for ***C*** if $\boldsymbol{Z}^{1}$ is worked with rather than ***Z***.

B.2.4 Equivalent Π circuits

For steady state solutions, an M-phase line can be represented by and equivalent M-phase Π circuit, which exactly describes the conditions at the line terminals for specific frequency and length.

The equivalent M-phase Π circuit is a generalization of the well known single phase equivalent Π circuit in Figure **B**.3 (sometimes called the lone line representation), where the series impedance Z_{series} and the shunt admittance Y_{shunt} is calculated from

$$Z_{Series} = \sqrt{\frac{Z}{Y}} \times sinh\left(l\sqrt{ZY}\right) \tag{B.10}$$

$$\frac{1}{2}Y_{Shunt} = \frac{tanh\left(\frac{l}{2}\sqrt{ZY}\right)}{\sqrt{\frac{Z}{Y}}} \tag{B.11}$$

Z is series impedance per unit length
Y is shunt admittance per unit length
l is length of line

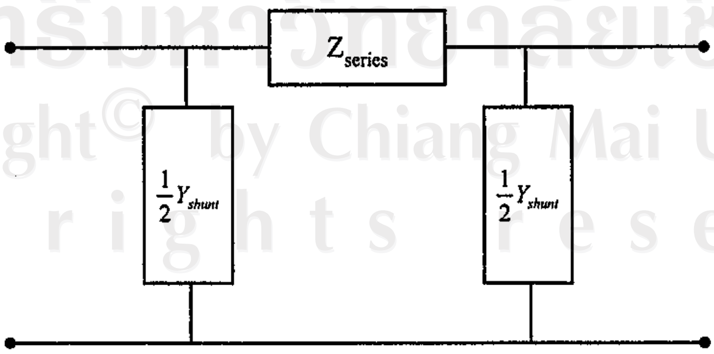


Figure B.3 The single phase Π equivalent circuit for TL program.

APPENDIX C

M-File of Proposed Measurement Placement Algorithm

C3.1 Placement.m (Proposed Measurement Placement Algorithm)

```
%Proposed measurement placement method
%Using minimum condition number analysis
%Base on sequential elimination
%%=====================================================================
echo off
clear all
clc
close all
sum_bus=14;          %This system, IEEE 14-bus test system, has 14 busbars.
sum_node=14;          %This system has 14 nodes.
sum_line=41;          %This system has 41 lines.
sum_branch=35;          %This system has 35 branch.
sum_state_variable=10; %This system has 10 state variables.
H=[];

%Step 1: Form H (Using flowchart in Figure 2.6)
%=============================================================
%=============================================================
load H05;  %Load matrix H from this file.
Y_new=h;

%%Placement from possible location
%%=====================================================================
%Case I; Possible location = 2N+L
Node_number      =[1 2 3 4 5 6 7 8 9 10 11 12 13 14]';
IN_meas_position =[1 1 1 1 1 1 1 1 1 1 1 1 1 1]';    %1 is measured, 0 is unmeasured.
VN_meas_position =[1 1 1 1 1 1 1 1 1 1 1 1 1 1]';
Line_number      =[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40 41]';
IL_meas_position =[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1]';
%Case II; Possible location = 2N+L-2Ns
Node_number      =[1 2 3 4 5 6 7 8 9 10 11 12 13 14]';
IN_meas_position =[0 0 1 1 1 0 0 1 1 1 1 1 1 1]';    %1 is measured, 0 is unmeasured.
VN_meas_position =[0 0 1 1 1 0 0 1 1 1 1 1 1 1]';
Line_number      =[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40 41]';
IL_meas_position =[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1]';
%4 sites fully, busbar 4,6,7-8, and 9, total 28 possible locations
Node_number      =[1 2 3 4 5 6 7 8 9 10 11 12 13 14]';
IN_meas_position =[0 0 0 1 0 1 1 1 1 0 0 0 0 0]';    %1 is measured, 0 is unmeasured.
VN_meas_position =[0 0 0 1 0 1 1 1 1 0 0 0 0 0]';
Line_number      =[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40 41]';
IL_meas_position =[0 0 0 0 0 0 0 1 0 0 1 0 1 1 0 0 1 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 1 1 1 1 1
1 1 1 0 1 1]';
```

```
sum_IN_meas=sum(IN_meas_position);
sum_VN_meas=sum(VN_meas_position);
sum_IL_meas=sum(IL_meas_position);
sum_meas=sum_IN_meas+sum_VN_meas+sum_IL_meas;
Poss_Loca=sum_meas;

meas_position=ones(sum_meas,1);

%ith iteration
%================================================================
%================================================================
for Eliminate=1:Poss_Loca
  for meas=1:Poss_Loca
    if meas_position(meas,1)==1
      %Step 2: Each row of H is temporary remove, one at a time.
      %Then calculated condition number of each H.
      %================================================================
      %================================================================
      meas_position(meas,1)=0;   %Each row of H is temporary remove, to be unmeasured.
      sum_meas=sum(meas_position);
      %Deduct unmeasued row of Y, then save to H (H1,H2,...Hn).
      jj=1;
      for ii=1:sum_meas
        for kk=jj:Poss_Loca
          if meas_position(kk)==1
            for nn=1:sum_state_variable
              H(ii,nn)=Y_new(kk,nn);
            end
            jj=jj+1;
            break
          else jj=jj+1;
          end
        end
      end

      CondN=cond(H); %Calculated condition number of each H (H1,H2,...Hn).
      MinCondN(meas,1)=CondN; %Keep value of CondN in each iteration.
      H_last=H;
      H=[];
      meas_position(meas,1)=1;   %Returned row of H that has been remove for this iteration, to be
measured.
    end %if meas_position(meas,1)==1
  end %for meas=1:Poss_Loca
  %Protection for the moving of the same as the past moved.
  for meas=1:Poss_Loca
    if meas_position(meas,1)==0;
      MinCondN(meas,1)=1e600;
    end
  end
  [Value MeasMove]=min(MinCondN);
  if meas_position(MeasMove,1)==0 % Breake if MeasMove same as the past moved
    sprintf('%s','MeasMove same as the past moved')
    break
  end
  MeasMoveKeep(:,Eliminate)=MeasMove;
  MinCondNKeep(Eliminate)=Value;
  MinCondNKeep_All(:,Eliminate)=MinCondN;
```

```
   %Step 3: Remove this measurement placement location for the next iteration.
   %==========================================================
   %==========================================================
   meas_position(MeasMove,1)=0;
   sum_meas=sum(meas_position);
   meas_positionKeep(:,Eliminate)=meas_position;
   MeasMoveKeep(:,Eliminate)=MeasMove;
   Eliminate   %Display ith iteration of elimination.
   MeasMove    %Display the measurement placement location.
end %for Eliminate=1:Poss_Loca, end of this iteration
sprintf('%s','Done')
MeasMoveKeep=MeasMoveKeep';
Sequential=[MeasMoveKeep MinCondNKeep'];
```

APPENDIX D

Singular Value Decomposition (SVD) [25]

D.1 Introduction

There exists a very powerful set of techniques for dealing with sets of equations or matrices that are either singular or else numerically very close to singular. In many cases where Gaussian elimination and LU decomposition fail to give satisfactory results, this set of techniques, known as singular value decomposition, or SVD, will diagnose for you precisely what the problem is. In some cases, SVD will not only diagnose the problem, it will also solve it, in the sense of giving you a useful numerical answer, although, as we shall see, not necessarily "the" answer that you thought you should get.

SVD is also the method of choice for solving most linear least-squares problems. SVD methods are based on the following theorem of linear algebra, whose proof is beyond our scope: Any M×N matrix $\boldsymbol{A}$ whose number of rows M is greater than or equal to its number of columns N, can be written as the product of an M×N column-orthogonal matrix $\boldsymbol{U}$, an N×N diagonal matrix $\boldsymbol{W}$ with positive or zero elements (the singular values), and the transpose of an N×N orthogonal matrix $\boldsymbol{V}$. The various shapes of these matrices will be made clearer by the following tableau:

$$\left(\begin{array}{c} \\ \\ \boldsymbol{A} \\ \\ \\ \end{array} \right) = \left(\begin{array}{c} \\ \\ \boldsymbol{U} \\ \\ \\ \end{array} \right) \left(\begin{array}{cccc} w_1 & & & \\ & w_2 & & \\ & & \cdot & \\ & & & \cdot \\ & & & & w_N \end{array} \right) \left(\begin{array}{c} \\ \\ \boldsymbol{V}^T \\ \\ \\ \end{array} \right) \tag{D.1}$$

The matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ are each orthogonal in the sense that their columns are orthonormal,

$$\sum_{i=1}^{M} U_{ik} U_{in} = \delta_{kn} \qquad \begin{array}{c} 1 \le k \le N \\ 1 \le n \le N \end{array} \tag{D.2}$$

$$\sum_{i=1}^{N} V_{jk} V_{jn} = \delta_{kn} \qquad \begin{array}{c} 1 \le k \le N \\ 1 \le n \le N \end{array} \tag{D.3}$$

or as a tableau,

$$\begin{pmatrix} & & \\ & U^T & \\ & & \end{pmatrix}\begin{pmatrix} & & \\ & U & \\ & & \end{pmatrix} = \begin{pmatrix} & & \\ & V^T & \\ & & \end{pmatrix}\begin{pmatrix} & & \\ & V & \\ & & \end{pmatrix} \tag{D.4}$$

$$= \begin{pmatrix} & & \\ & 1 & \\ & & \end{pmatrix}$$

Since V is square, it is also row-orthonormal, $VV^T = 1$.

The SVD decomposition can also be carried out when M<N. In this case the singular values w_j for j = M + 1;…; N are all zero, and the corresponding columns of U are also zero. Equation (D.2) then holds only for k; n ≤ M.

The decomposition of equation (D.1) can always be done, no matter how singular the matrix is, and it is "almost" unique. That is to say, it is unique up to (i) making the same permutation of the columns of U, elements of W, and columns of V (or rows of V^T), or (ii) forming linear combinations of any columns of U and V whose corresponding elements of W happen to be exactly equal. An important consequence of the permutation freedom is that for the case M<N, a numerical algorithm for the decomposition need not return zero wj's for j = M + 1; : : :; N; the N –M zero singular values can be scattered among all positions j = 1;2;…; N.

D.2 SVD of a Square Matrix

If the matrix A is square, N×N say, then U, V, and W are all square matrices of the same size. Their inverses are also trivial to compute: U and V are orthogonal, so their inverses are equal to their transposes; W is diagonal, so its inverse is the diagonal matrix whose elements are the reciprocals of the elements w_j. From equation (D.1) it now follows immediately that the inverse of A is

$$A^{-1} = V\left[diag\left(\frac{1}{w_j}\right)\right]U^T \tag{D.5}$$

The only thing that can go wrong with this construction is for one of the w_j's to be zero, or (numerically) for it to be so small that its value is dominated by roundoff error and therefore unknowable. If more than one of the w_j's have this problem, then the matrix is even more singular. So, first of all, SVD gives you a clear diagnosis of the situation.

Formally, the condition number of a matrix is defined as the ratio of the largest (in magnitude) of the w_j's to the smallest of the w_j's. A matrix is singular if its condition number is infinite, and it is ill-conditioned if its condition number is too large, that is, if its reciprocal approaches the machine's floating-point precision (for example, less than 10^{-6} for single precision or 10^{-12} for double).

For singular matrices, the concepts of nullspace and range are important. Consider the familiar set of simultaneous equations

$$Ax = b \tag{D.6}$$

where A is a square matrix, b and x are vectors. Equation (D.6) defines A as a linear mapping from the vector space x to the vector space b. If A is singular, then there is some subspace of x, called the nullspace, that is mapped to zero, $Ax = 0$. The dimension of the nullspace (the number of linearly independent vectors x that can be found in it) is called the nullity of A.

Now, there is also some subspace of b that can be "reached" by A, in the sense that there exists some x which is mapped there. This subspace of b is called the range of A. The dimension of the range is called the rank of A. If A is nonsingular, then its range will be all of the vector space b, so its rank is N. If A is singular, then the rank will be less than N. In fact, the relevant theorem is "rank plus nullity equals N."

What has this to do with SVD? SVD explicitly constructs orthonormal bases for the nullspace and range of a matrix. Specifically, the columns of U whose same-numbered elements w_j are nonzero are an orthonormal set of basis vectors that span the range; the columns of V whose same-numbered elements w_j are zero are an orthonormal basis for the nullspace.

Now let's have another look at solving the set of simultaneous linear equations (D.6) in the case that A is singular. First, the set of homogeneous equations, where $b = 0$, is solved immediately by SVD: Any column of V whose corresponding w_j is zero yields a solution.

When the vector b on the right-hand side is not zero, the important question is whether it lies in the range of A or not. If it does, then the singular set of equations does have a solution x; in fact it has more than one solution, since any vector in the nullspace (any column of V with a corresponding zero w_j) can be added to x in any linear combination.

If we want to single out one particular member of this solution-set of vectors as a representative, we might want to pick the one with the smallest length $|\mathbf{x}|^2$. Here is how to find that vector using SVD: Simply replace 1= w_j by zero if $w_j = 0$. (It is not very often that one gets to set $\infty = 0$!) Then compute (working from right to left)

$$x = V\left[diag\left(\frac{1}{w_j}\right)\right]\left(U^T b\right) \tag{D.7}$$

This will be the solution vector of smallest length; the columns of V that are in the nullspace complete the specification of the solution set.

Proof: Consider $|x + x'|$, where x' lies in the nullspace. Then, if W^{1} denotes the modified inverse of W with some elements zeroed,

$$\begin{aligned} |x + x'| &= \left|VW^{-1}U^T b + x'\right| \\ &= \left|V\left(W^{-1}U^T b + V^T x'\right)\right| \\ &= \left|W^{-1}U^T b + V^T x'\right| \end{aligned} \tag{D.8}$$

Here the first equality follows from equation (D.7), the second and third from the orthonormality of V. If you now examine the two terms that make up the sum on the right-hand side, you will see that the first one has nonzero j components only where

$w_j \neq 0$, while the second one, since **x0** is in the nullspace, has nonzero j components only where $w_j = 0$. Therefore the minimum length obtains for $\boldsymbol{x}' = 0$, q.e.d.

If $\boldsymbol{b}$ is not in the range of the singular matrix $\boldsymbol{A}$, then the set of equations (D.6) has no solution. But here is some good news: If $\boldsymbol{b}$ is not in the range of $\boldsymbol{A}$, then equation (D.7) can still be used to construct a "solution" vector $\boldsymbol{x}$. This vector $\boldsymbol{x}$ will not exactly solve $\boldsymbol{Ax} = \boldsymbol{b}$. But, among all possible vectors $\boldsymbol{x}$, it will do the closest possible job in the least squares sense. In other words equation (D.7) finds

$$\boldsymbol{x} \text{ which minimizes } \boldsymbol{r} \equiv \left|\boldsymbol{Ax} - \boldsymbol{b}\right| \tag{D.9}$$

The number r is called the residual of the solution.

The proof is similar to equation (D.8): Suppose we modify $\boldsymbol{x}$ by adding some arbitrary $\mathbf{x}'$. Then $\boldsymbol{Ax}-\boldsymbol{b}$ is modified by adding some $\mathbf{b}'=\mathbf{Ax}'$. Obviously $\mathbf{b}'$ is in the range of $\boldsymbol{A}$. We then have

$$\begin{aligned} \left|\boldsymbol{Ax} - \boldsymbol{b} + \boldsymbol{b}'\right| &= \left|\left(\boldsymbol{UWV}^T\right)\left(\boldsymbol{VW}^{-1}\boldsymbol{U}^T\boldsymbol{b}\right) - \boldsymbol{b} + \boldsymbol{b}'\right| \\ &= \left|\left(\boldsymbol{UWW}^{-1}\boldsymbol{U}^T - 1\right)\boldsymbol{b} + \boldsymbol{b}'\right| \\ &= \left|\left(\boldsymbol{WW}^{-1} - 1\right)\boldsymbol{U}^T\boldsymbol{b} + \boldsymbol{U}^T\boldsymbol{b}'\right| \end{aligned} \tag{D.10}$$

Now, $(\boldsymbol{WW}^{1} - 1)$ is a diagonal matrix which has nonzero j components only for $w_j = 0$, while $\boldsymbol{U}^T\boldsymbol{b}'$ has nonzero j components only for $w_j \neq 0$, since $\boldsymbol{b}'$ lies in the range of $\boldsymbol{A}$. Therefore the minimum obtains for $\mathbf{b}' = 0$, q.e.d.

In the discussion since equation (D.6), we have been pretending that a matrix either is singular or else isn't. That is of course true analytically. Numerically, however, the far more common situation is that some of the w_j's are very small but nonzero, so that the matrix is ill-conditioned. In that case, the direct solution methods of LU decomposition or Gaussian elimination may actually give a formal solution to the set of equations (that is, a zero pivot may not be encountered); but the solution vector may have wildly large components whose algebraic cancellation, when multiplying by the matrix A, may give a very poor approximation to the right-hand vector $\boldsymbol{b}$. In such cases, the solution vector $\boldsymbol{x}$ obtained by zeroing the small w_j's and then using equation (D.7) is very often better (in the sense of the residual $|\boldsymbol{Ax}-\boldsymbol{b}|$ being smaller) than both the direct-method solution and the SVD solution where the small w_j's are left nonzero.

It may seem paradoxical that this can be so, since zeroing a singular value corresponds to throwing away one linear combination of the set of equations that we are trying to solve. The resolution of the paradox is that we are throwing away precisely a combination of equations that is so corrupted by roundoff error as to be at best useless; usually it is worse than useless since it "pulls" the solution vector way off towards infinity along some direction that is almost a nullspace vector. In doing this, it compounds the roundoff problem and makes the residual $|\boldsymbol{Ax}-\boldsymbol{b}|$ larger.

SVD cannot be applied blindly, then. You have to exercise some discretion in deciding at what threshold to zero the small w's, and/or you have to have some idea what size of computed residual $|\boldsymbol{Ax}-\boldsymbol{b}|$ is acceptable.

D.3 SVD for Fewer Equations than Unknowns

If you have fewer linear equations M than unknowns N, then you are not expecting a unique solution. Usually there will be an N–M dimensional family of solutions. If you want to find this whole solution space, then SVD can readily do the job.

The SVD decomposition will yield N–M zero or negligible w_j's, since M < N. There may be additional zero w_j's from any degeneracies in your M equations. Be sure that you find this many small w_j's, and zero them, which will give you the particular solution vector $\boldsymbol{x}$. As before, the columns of $\boldsymbol{V}$ corresponding to zeroed w_j's are the basis vectors whose linear combinations, added to the particular solution, span the solution space.

D.4 SVD for More Equations than Unknowns

In tableau, the equations to be solved are

$$\begin{pmatrix} & & \\ & \boldsymbol{A} & \\ & & \end{pmatrix} \begin{pmatrix} \boldsymbol{x} \end{pmatrix} = \begin{pmatrix} \\ \boldsymbol{b} \\ \\ \end{pmatrix} \tag{D.11}$$

The proofs that we gave above for the square case apply without modification to the case of more equations than unknowns. The least-squares solution vector $\boldsymbol{x}$ is given by equation (D.7), which, with non square matrices, looks like this,

$$\begin{pmatrix} \boldsymbol{x} \end{pmatrix} = \begin{pmatrix} \\ \boldsymbol{V} \\ \\ \end{pmatrix} \begin{pmatrix} \\ diag\left(\frac{1}{w_j}\right) \\ \\ \end{pmatrix} \begin{pmatrix} & \boldsymbol{U}^T & \end{pmatrix} \begin{pmatrix} \\ \boldsymbol{b} \\ \\ \end{pmatrix} \tag{D.12}$$

In general, the matrix $\boldsymbol{W}$ will not be singular, and no w_j's will need to be set to zero. Occasionally, however, there might be column degeneracies in $\boldsymbol{A}$. In this case you will need to zero some small w_j values after all. The corresponding column in $\boldsymbol{V}$ gives the linear combination of $\boldsymbol{x}$'s that is then ill-determined even by the supposedly overdetermined set.

Sometimes, although you do not need to zero any wj's for computational reasons, you may nevertheless want to take note of any that are unusually small: Their corresponding columns in $\boldsymbol{V}$ are linear combinations of $\boldsymbol{x}$'s which are insensitive to your data. In fact, you may then wish to zero these w_j's, to reduce the number of free parameters in the fit.

D.5 Constructing an Orthonormal Basis

Suppose that you have N vectors in an M-dimensional vector space, with N ≤ M. Then the N vectors span some subspace of the full vector space. Often you want to construct an orthonormal set of N vectors that span the same subspace. The textbook way to do this is by Gram-Schmidt orthogonalization, starting with one vector and then expanding the subspace one dimension at a time. Numerically, however, because of the build-up of roundoff errors, naïve Gram-Schmidt orthogonalization is terrible.

The right way to construct an orthonormal basis for a subspace is by SVD: Form an M ≤ N matrix $\boldsymbol{A}$ whose N columns are your vectors. The columns of the matrix $\boldsymbol{U}$ (which in fact replaces $\boldsymbol{A}$ on output are your desired orthonormal basis vectors.

You might also want to check the output w_j's for zero values. If any occur, then the spanned subspace was not, in fact, N dimensional; the columns of $\boldsymbol{U}$ corresponding to zero w_j's should be discarded from the orthonormal basis set.

D.6 Approximation of Matrices

Note that equation (D.1) can be rewritten to express any matrix A_{ij} as a sum of outer products of columns of $\boldsymbol{U}$ and rows of $\boldsymbol{V}^T$, with the "weighting factors" being the singular values w_j,

$$A_{ij} = \sum_{k=1}^{N} w_k U_{ik} V_{jk} \qquad \text{(D.13)}$$

APPENDIX E

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Power system state estimation using singular value decomposition

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bstract

This paper presents a new fundamental static state estimation algorithm using weighted least square (WLS) estimation, which is ased on singular value decomposition (SVD) rather than the normal equations. In addition, this paper also uses linear WLS stimation, which is quicker than non-linear. The SVD approach does not require the whole network system to be observable prior o estimation. It can provide a solution even if the system under consideration is partially observable. The simulation study was erformed on the IEEE 14-bus test system. The simulation results, both linear and non-linear WLS, have shown that the SVD pproach can provide a solution even when ill-conditioned occurred while the normal equation approach failed to give satisfactory esults. In addition, the SVD approach can identify which parts of the network are unobservable islands.

eywords: Singular value decomposition (SVD); State estimation; Weighted least square (WLS)

. Introduction

State estimation processes a set of redundant meaurements to estimate the state of the power system. nalog and logic measurements are telemeters to the ontrol center. Logic measurements are used in topology rocessor to determine the system configuration. The tate estimator uses a set of analog measurements along ith the system configuration supplied by the topology rocessor, network parameters such as line impedance nd perhaps some pseudo measurements as its input. In bservability analysis, if the set of measurements is ıfficient to make state estimation possible, thus the etwork is observable. Usually a system is designed to e observable prior to the state estimation for most peration conditions. Temporary unobservability may ill occur due to unanticipated network topology ıanges or failures in the telecommunication systems.

In the conventional state estimation e.g. [1], real and active power measurement are used, instead of current measurements, for branch flows and busbar injection, the measurement equation is non-linear. In such case, the solution must be obtained through an iterative algorithm. The phasor measurement unit (PMU) is used in state estimation as shown in [2]. With an advent of satellite clock synchronization, phasor metering achieved a level of precision that made phasor telemetry a valuable source of measurement data [2]. From [3], bus injection measurements are considered to be more important than line flow measurements. This is due to the fact that a bus injection measurement is tightly related to more state variables than a line measurement. In order to improve the measurement redundancy, which is key to bad data identification, the method could easily extend to include line current measurements if necessary.

A conventional state estimation methods based on weighted least square (WLS) method using the normal equation approach may fail to provide solution when gain matrix is ill-conditioned due to temporary unobservability and if it does not have re-observability analysis. Even various methods have been suggested to solve the numerical ill-conditioned problem [4,5], it still needs observability analysis prior to estimation. In [6], observability with orthogonalization may be neglected

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›y extra computational effort for deletion of unobser-'able buses. There exists a very powerful set of echniques for dealing with sets of equations or matrices hat are either singular or else numerically very close to ingular, known as singular value decomposition (SVD). [he SVD has already been used in [7] for harmonic state ·stimation with under-determined system. To solve the ınder-determined case, the SVD needs to be applied, ince convention techniques fail to solve such equations. n [8] used the SVD approach to detect and estimate ıarmonic component in power system from simulated vaveform. In this paper, by using the busbar phasor 'oltages as state variables and measuring busbar phasor 'oltages and injection phasor currents make the mea-ıurement equation linear. When the measurement :quation is linear, the estimation algorithm is direct, :o linear equation can be faster than non-linear.

In practical fundamental state estimation the number ›f actual measurements is far greater than the number of ;tate variable. As a result, there are many more equation o solve than there are unknown state variable, the ;ystem always is an over-determined. In addition, to :ompare the SVD method and the normal equation, the ınder-determined system can not be used. Then this ›aper will use the completely and/or over-determined ;ystem.

First of all this paper reviews the static state estima-:ion of power system in Section 2. Then the SVD is ›resented in Section 3. After that, test system and ;imulation results carried on the IEEE 14-bus test ;ystem is discussed in Section 4. The state estimation ısing the normal equation approach comparing with the SVD approach to solve the linear and non-linear WLS ›f the two approaches will be presented. Finally, the :oncluding remarks are made in Section 5.

2. Static state estimation

The task of the state estimation is to generate the best ɜstimation of the state variables from measured data, with corrupted with measurement noise or error. The :hree issues involved are the choice of state variables, performance criteria and selection of measurement points and quantities to be measured. Various perfor-mance criteria are possible, the most widely used is the WLS. For a given measurement set and system topol-ogy, the basic circuit laws lead to the following measurement equation.

$$z = Hx + \varepsilon \tag{1}$$

where z and x are the vectors of measurements and state variables, respectively. H is the measurement matrix, and ε is the measurement noise vector, which is assumed to be made of independent random variables with Gaussian distribution.

The WLS estimate is, therefore, the vector x that minimizes the weighted sum of the squares of the residuals ($r = z - Hx$) between the actual measurements and estimated levels, i.e.

$$\text{Minimise } J(x) = (z - Hx)^T R^{-1} (z - Hx) \tag{2}$$

where R^{-1} is the inverse of the covariance matrix.

Matrix R is diagonal and contains the covariances of the measurements (if they are known). This permits applying higher weighting to measurements that are known to be more accurate. R is replaced by the identity matrix if the same instrumentation is used to obtain them.

The solution to Eq. (1) in the WLS sense is obtained by solving the following equation.

$$(H^T R^{-1} H)x = (H^T R^{-1})z \tag{3}$$

Real and reactive power measurements are used in the conventional state estimation, the measurement equa-tion is non-linear. In such case, the solution for Eq. (2) must be obtained through an iterative algorithm. How-ever, the measurement equation can be linear by choosing the phasor busbar voltages as state variables and measuring phasor busbar voltages and phasor injection currents. However, to improve the measure-ment redundancy, it can easily extend to include line current measurements if necessary. It is important to emphasize that the estimation algorithms for this case is direct (not iterative). In this case, the task of estimating busbar voltage (v) by given measurement vector (z) in the presence of noise (ε) are expressed as:

$$\begin{bmatrix} z_v \\ z_I \end{bmatrix} = \begin{bmatrix} I & 0 \\ Y_{IM} & Y_{IC} \end{bmatrix} \begin{bmatrix} v_M \\ v_C \end{bmatrix} + \begin{bmatrix} \varepsilon_v \\ \varepsilon_I \end{bmatrix} \tag{4}$$

where I is identity matrix, z_v, z_I are voltage and injection current measurement subvector, respectively, Y_{IM}, Y_{IC} are the node admittance matrix of measure-ment and calculated busbar voltage that related to z_I, respectively, v_M, v_C are measurement and calculated (unmeasured) busbar voltage subvector, respectively, ε_v, ε_I are voltage and injection current measurement noise subvector, respectively.

3. Singular value decomposition (SVD)

The SVD is a very powerful set of techniques for dealing with sets of equations or matrices that are either singular or else numerically very close to singular. In some cases where Gaussian elimination and LU decom-position fail to provide satisfactory results, the SVD will diagnose precisely what the problem is. In some cases, the SVD will not only diagnose the problem, but it will also solve it, in the sense of giving a useful numerical answer. The SVD is also the method of choice for solving most linear least-squares problems.

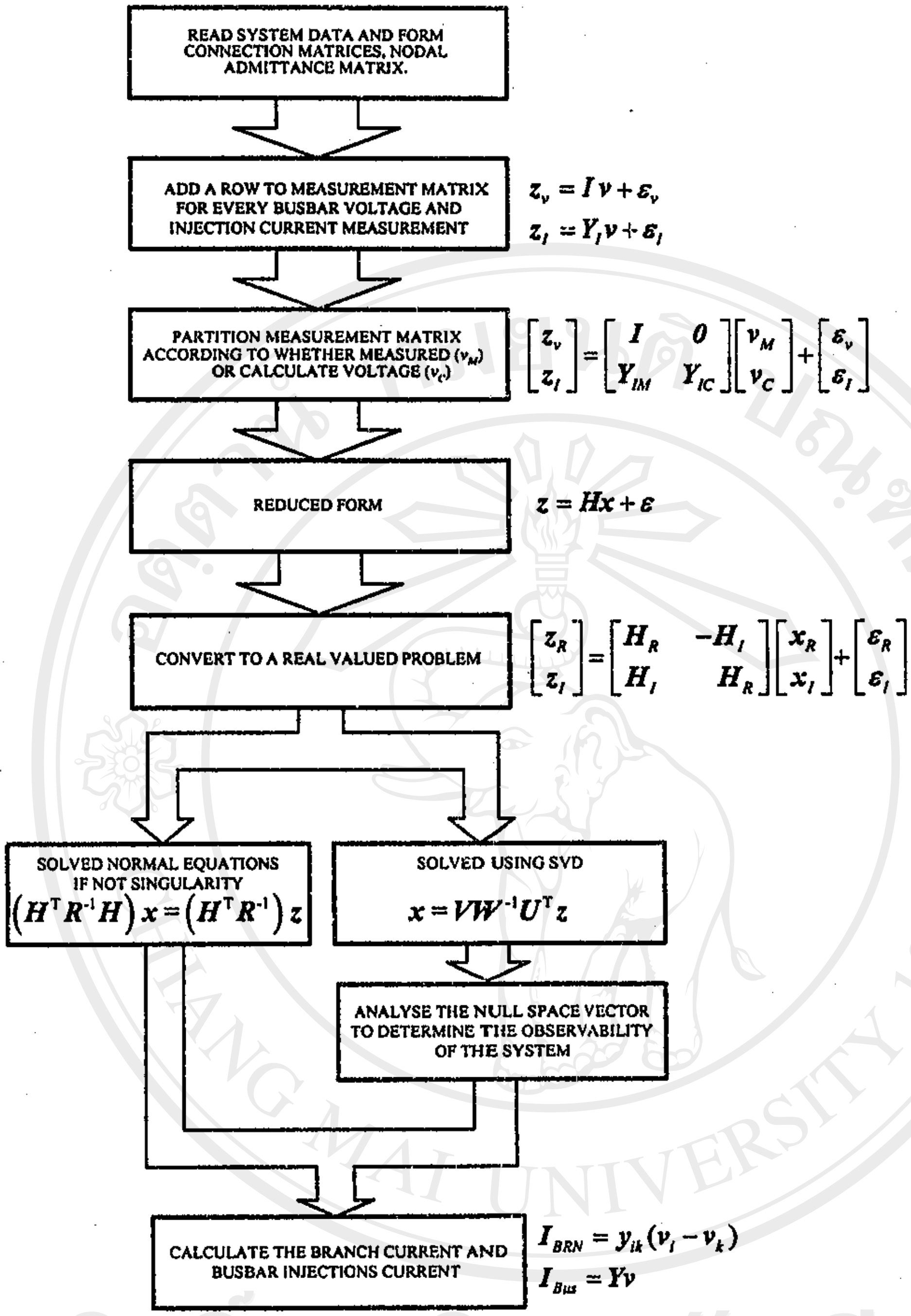

Fig. 1. The algorithm of linear WLS state estimation.

The direct solution of WLS problem from the normal equations is rather susceptible to round-off error. That is one or more of measurement equations are a linear combination of the others, or a zero pivot or a very small pivot element may be encountered during the solution of the linear equations, in which case it has no solution at all. These numerical difficulties can happen even in case of non-linear WLS, as mentioned before. It turns out that the SVD also fixes the round-off problem. So it is a recommended technique for all [9].

In the case of over-determined system, the SVD provides a solution that is the best approximation in the WLS sense. In the case of under-determined system, the SVD provides a solution whose values are the smallest in the WLS sense. In the over-determined or completely-determined case, the singularity from the normal equations implies what is known as an unobservable system. In the case of under-determined case, the singularity implies that there is no unique solution to the problem. However, the SVD will provide a particular solution and a null space vector for each singularity.

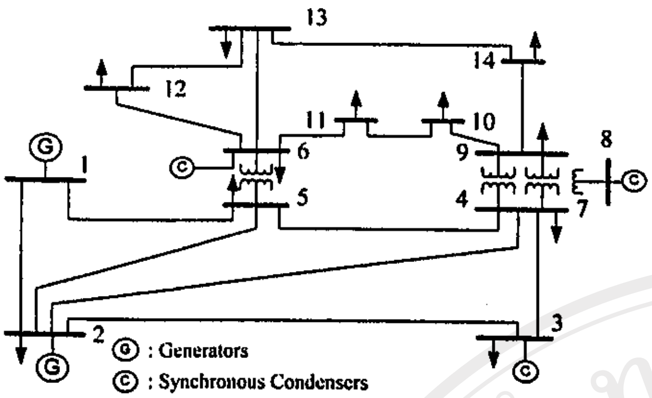

Fig. 2. IEEE 14-bus test system.

The SVD method represents the matrix H (m × n) of Eq. (1) as the product of three matrices [9]. When m is the number of measurement placement, and n is the number of state variable.

$$H = UWV^T \tag{5}$$

W is a diagonal matrix (n × n) with positive or zero elements, which are the singular values of H. Matrices U and V^T are orthogonal matrices, U being a column orthogonal (m × n) matrix and V^T is the transpose of a (n × n) orthogonal matrix.

From Eqs. (1) and (5) the following expression of x is obtained [9], i.e.

$$x = VW^{-1}U^Tz \tag{6}$$

The algorithm of state estimation in this paper, was modified from [10], is shown in Fig. 1. The algorithm does not require the whole network system to be observable prior to estimation and also can identify observable island in such case.

4. Test system and simulation results

The IEEE 14-bus is shown in Fig. 2 is chosen to verify the algorithm and compare the performance of the SVD approach with the normal equation approach in linear WLS. The parameters are included in Appendices A, B and C. The simulation assumed that measurement equipment that can be synchronized is available and uses the same type, then R in Eq. (3) is a unit matrix. For complex measurement, voltage and current values can be assumed that they have PMUs in which the standard deviations are better than 0.15° confirming high accuracy of metering hardware [2]. The examples of random measurement placements are shown in Table 1.

In normal condition, a system is designed to be observable prior to the state estimation. If temporary unobservability occurs and has only measurement placement as in Table 1, the simulation will test whether the SVD approach and the normal equation approach will provide a solution for state estimation or not.

The methodology used to illustrate state estimation is as follows. First, the system has partial 'measured values' as in Table 1 and is equal to their corresponding 'true values' obtained from IEEE 14-bus test system for busbar voltage and injection current can calculate from $I = Yv$. Then, estimated values throughout the test system are obtained from the partial 'measured values' using the estimator. Finally, all the estimated values are compared with the corresponding 'true values'. Since the measurement noises do not affect the observability of state estimation [11], they are ignored in this illustration.

From Table 1, the SVD approach can provide a direct solution for an observable island even singularity occurred in H while the normal equation approach failed to give satisfactory results. [9] describes variables corresponding to non-zero elements in the null space vectors (found in column of V) which relate to zero singular value of H (diagonal element of W) are the unobservable region.

The example in case 1 has shown that, the measurement matrix H afterwards converts to real value

Table 1
Example of measurements placement and performance of the SVD and the normal equation approach in each case

Case	Measurement placement		H, zero at column of bus #	SVD shows unobservable at bus #	Normal equation
	Busbar voltage #	Busbar injection current #			
	1–5, 9, 10, 14	1–5, 9, 10, 14	8, 12	8, 12	Singularity
	1, 2, 4, 5, 8, 9, 13, 14	1, 2, 4, 5, 8, 9, 13, 14	11	11	Singularity
	1, 2, 4, 5, 8, 9, 11, 13	1, 2, 4, 5, 8, 9, 11, 13	–	–	Fine
	1–5, 9, 10, 12, 14	1–5, 9, 10, 12 14	8	8	Singularity
	3, 4, 6, 7, 9, 14	1–3, 5, 6, 10–13	8	8	Singularity
	4–10	1–3, 10–14	–	–	Fine

Table 2
True value of busbar voltage, estimated voltage, and errors between true and estimated voltage

Bus #	Voltage (true)		Voltage (estimated)		Error (true–estimated)	
	Magnitude (pu)	Angle (°)	Magnitude (pu)	Angle (°)	Magnitude (pu)	Angle (°)
1	1.060	0.00	1.060	0.00	0.00	0.00
2	1.045	−4.980	1.045	−4.980	0.00	0.00
3	1.010	−12.720	1.010	−12.720	0.00	0.00
4	1.019	−10.330	1.019	−10.330	0.00	0.00
5	1.020	−8.780	1.020	−8.780	0.00	0.00
6	1.070	−14.220	1.070	−14.220	0.00	0.00
7	1.062	−13.370	1.062	−13.370	0.00	0.00
8	1.090	−13.360	0.00	0.00	1.090	−13.360
9	1.056	−14.940	1.056	−14.940	0.00	0.00
10	1.051	−15.100	1.051	−15.100	0.00	0.00
11	1.057	−14.790	1.057	−14.790	0.00	0.00
12	1.055	−15.070	0.00	0.00	1.055	−15.070
13	1.050	−15.160	1.050	−15.160	0.00	0.00
14	1.036	−16.040	1.036	−16.040	0.00	0.00

problem is zero at columns bus 8 and 12, as shown in Appendix D. Therefore, this matrix is singular. By the SVD approach, W^t of Eq. (5) as shown in Appendix E shows the singular values of column 25–28 are zero. After that, checking at column 25–28 of V from Eq. (5) as shown in Appendix F, which is null space vectors, bus 8 and 12 are non-zero. Hence, bus 8 and 12 are unobservable islands. It means that, the SVD approach can identify which parts of the network are unobservable.

The estimated value of unobservable regions of the SVD approach, bus 8 and 12, are incorrect. The SVD result shows significant error (between actual and estimated value). Table 2 provides the result of case 1 as described above. However, the SVD can provide a correct answer at least 12 buses while the normal equation approach fail to give any results. The singularity of the normal equation approach in each measurement placement implies what is known as not fully observable system.

For the conventional state estimation, non-linear WLS, the state variable of IEEE-14 bus system are 14 buses voltage magnitude and 13 buses voltage angle. Then, measurement number for completely-determined is 27. If measurement placement are 13 buses injected active power at 1, 2, 4–14, and 11 buses injected reactive power at 1, 2, 5–7, 9–14, and three buses voltage at 1, 13, 14. Both of the normal equation and the SVD can provide a correct answer for all state variables. If measurement place-ment are injected active power, bus injected reactive power, and bus voltage at 1, 2, 4, 5, 7, 8, 11, 13, 14, only the SVD can provide a correct answer while the normal equation approach failed to provide satisfactory results.

The use of the SVD approach can be significantly slower than solving the normal equations and requires more storage for three matrices in Eq. (5). However, it is less susceptible to round-off error. Moreover, it is theoretically foolproof reliability more than making up for the speed disadvantage [10]. Even with the use of the SVD approach, there is a speed problem. Fundamental frequency state estimation is normally used in on-line supervisory control and data acquisition (SCADA) systems where speed is a critical issue. But, of course computer speeds are doubling every 3 years and hence the SVD approach will be just as fast as very soon.

The further study should include the consideration of optimal measurement placement for the SVD approach and bad data analysis.

5. Conclusion

The development of a new algorithm for static state estimation based on the SVD method has been presented. In this paper, the linear WLS is used. The measurement equation will be linear by choosing the complex busbar voltages as state variables and measuring complex busbar voltages and injection currents, the estimation algorithm is direct (not iterative). However, the non-linear WLS also be tested. From the simulation study performed on IEEE 14-bus test system, it is found that the SVD approach can provide a solution even singularity occurred while the normal equation approach failed to provide satisfactory results. In addition, the SVD approach can identify which parts of the network are unobservable. The SVD is a highly reliable and computationally stable technique to solve matrices that are either singular or very close to singular. In such case, the SVD will indicate the problem and result in a useful numerical answer.

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Appendix A: Bus voltages of IEEE 14-bus test system and calculated injection current [10]

Bus #	Voltage (IEEE) Magnitude (pu)	Angle (°)	Injection current (calculated) Magnitude (pu)	Angle (°)
1	1.060	0.00	2.198	4.130
2	1.045	−4.980	0.334	−63.130
3	1.010	−12.720	0.934	169.830
4	1.019	−10.330	0.491	157.210
5	1.020	−8.780	0.354	98.500
6	1.070	−14.220	0.351	−116.520
7	1.062	−13.370	0.112	−101.580
8	1.090	−13.360	0.159	−102.980
9	1.056	−14.940	0.293	143.850
0	1.051	−15.100	0.103	132.040
1	1.057	−14.790	0.036	138.450
2	1.055	−15.070	0.059	149.350
3	1.050	−15.160	0.145	140.640
4	1.036	−16.040	0.150	146.490

Appendix B: Line parameters of IEEE 14-bus test system [10]

Bus #	Bus #	R (pu)	X (pu)	B (pu)
	2	0.01938	0.05917	0.0528
	5	0.05403	0.22304	0.0492
	3	0.04699	0.19797	0.0438
	4	0.05811	0.17632	0.0374
2	5	0.05695	0.17388	0.0340
3	4	0.06701	0.17103	0.0346
4	5	0.01335	0.04211	0.0128
4	7	0.00	0.20912	0.00
4	9	0.00	0.55618	0.00
5	6	0.00	0.25202	0.00
6	11	0.09498	0.19890	0.00
6	12	0.12291	0.25581	0.00
6	13	0.06615	0.13027	0.00
7	8	0.00	0.17615	0.00
7	9	0.00	0.11001	0.00
9	10	0.03181	0.08450	0.00
9	14	0.12711	0.27038	0.00
10	11	0.08205	0.19207	0.00
12	13	0.22092	0.19988	0.00
13	14	0.17093	0.34802	0.00

Appendix C: Bus admittance matrix of IEEE 14-bus test system

us	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	6.0250 − 19.4471i	−4.9991 + 15.2631i	0	0	−1.0259 + 4.2350i	0	0	0	0	0	0	0	0	0
	−4.9991 + 15.2631i	9.5213 − 30.2707i	−1.1350 + 4.7819i	−1.6860 + 5.1158i	−1.7011 + 5.1939i	0	0	0	0	0	0	0	0	0
	0	−1.1350 + 4.7819i	3.1210− 9.8115i	−1.9860 + 5.0688i	0	0	0	0	0	0	0	0	0	0
	0	−1.6860 + 5.1158i	−1.9860 + 5.0688i	10.5130 − 38.3007i	−6.8410 + 21.5786i	0	0 + 4.7819i	0	0 +1.7980i	0	0	0	0	0
	−1.0259 + 4.2350i	−1.7011 + 5.1939i	0	−6.8410 + 21.5786i	9.5680 − 34.9274i	0 +3.9679i	0	0	0	0	0	0	0	0

Appendix C *(continued)*

6	0	0	0	0	0 +3.9679i	6.5799 − 17.3407i	0	0	0	0	−1.9550 +4.0941i	−1.5260 +3.1760i	−3.0989 +6.1028i	0
7	0	0	0	0 +4.7819i	0	0	0 − 19.5490i	0 + 5.6770i	0 +9.0901i	0	0	0	0	0
8	0	0	0	0	0	0	0	0 + 5.6770i	0 −5.6770i	0	0	0	0	0
9	0	0	0	0 +1.7980i	0	0	0 + 9.0901i	0	5.3261 − 24.0925i	−3.9020 + 10.3654i	0	0	0	−1.4240 +3.0291i
10	0	0	0	0	0	0	0	0	−3.9020 + 10.3654i	5.7829 − 14.7683i	−1.8809 +4.4029i	0	0	0
11	0	0	0	0	0	−1.9550 + 4.0941i	0	0	0	−1.8809 + 4.4029i	3.8359 − 8.4970i	0	0	0
12	0	0	0	0	0	−1.5260 + 3.1760i	0	0	0	0	0	4.0150 − 5.4279i	−2.4890 +2.2520i	0
13	0	0	0	0	0	−3.0989 + 6.1028i	0	0	0	0	0	−2.4890 +2.2520i	6.7249 − 0.6697i	−1.1370 +2.3150i
14	0	0	0	0	0	0	0	0	−1.4240 + 3.0291i	0	0	0	−1.1370 +2.3150i	2.5610 − 5.3440i

Appendix D. Matrix *H* of case 1

Bus #	1	2	3	4	5	9	10	14	6	7	8	11	12	13	1	2	3	4	5	9	10	14	6	7	8	11	12	13
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	6	−5	0	0	−1	0	0	0	0	0	0	0	0	0	19	−15	0	0	−4	0	0	0	0	0	0	0	0	0
2	−5	10	−1	−2	−2	0	0	0	0	0	0	0	0	0	−15	30	−5	−5	−5	0	0	0	0	0	0	0	0	0
3	0	−1	3	−2	0	0	0	0	0	0	0	0	0	0	0	−5	10	−5	0	0	0	0	0	0	0	0	0	0
4	0	−2	−2	11	−7	0	0	0	0	0	0	0	0	0	0	−5	−5	38	−22	−2	0	0	0	−5	0	0	0	0
5	−1	−2	0	−7	10	0	0	0	0	0	0	0	0	0	−4	−5	0	−22	35	0	0	0	−4	0	0	0	0	0
9	0	0	0	0	0	5	−4	−1	0	0	0	0	0	0	0	0	0	−2	0	24	−10	−3	0	−9	0	0	0	0
10	0	0	0	0	0	−4	6	0	0	0	0	−2	0	0	0	0	0	0	0	−10	15	0	0	0	0	−4	0	0
14	0	0	0	0	0	−1	0	3	0	0	0	0	0	−1	0	0	0	0	0	−3	0	5	0	0	0	0	0	−2
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	−19	15	0	0	4	0	0	0	0	0	0	0	0	0	6	−5	0	0	−1	0	0	0	0	0	0	0	0	0
	15	−30	5	5	5	0	0	0	0	0	0	0	0	0	−5	10	−1	−2	−2	0	0	0	0	0	0	0	0	0
	0	5	−10	5	0	0	0	0	0	0	0	0	0	0	0	−1	3	−2	0	0	0	0	0	0	0	0	0	0
	0	5	5	−38	22	2	0	0	0	5	0	0	0	0	0	−2	−2	11	−7	0	0	0	0	0	0	0	0	0
	4	5	0	22	−35	0	0	0	4	0	0	0	0	0	−1	−2	0	−7	10	0	0	0	0	0	0	0	0	0
	0	0	0	2	0	−24	10	3	0	9	0	0	0	0	0	0	0	0	0	5	−4	−1	0	0	0	0	0	0
0	0	0	0	0	0	10	−15	0	0	0	0	4	0	0	0	0	0	0	0	−4	6	0	0	0	0	−2	0	0
4	0	0	0	0	0	3	0	−5	0	0	0	0	0	2	0	0	0	0	0	−1	0	3	0	0	0	0	0	−1

ppendix E. Matrix *W* of case 1

ow #	Column # 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
	61.43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	61.43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

3	0	0	44.07	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	44.07	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	33.47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	33.47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	20.14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	20.14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	11.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	11.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	11.07	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	11.07	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	6.07	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	6.07	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.87	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.87	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.86	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.86	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.52	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.52	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.36	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.36	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.16	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.16	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix F. Matrix V of case 1

Bus #	Column # 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**−0.99**	**0.11**	**0**	**0**
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00	0.00	0.00	0.00
12	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0.11**	**0.99**	**0**	**0**
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1.00**	**0**
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1.00**
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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An Optimal Measurement Placement Method for Power System Harmonic State Estimation

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Abstract--**This paper focuses on a new technique for optimal measurement placement for power system Harmonic State Estimation (HSE). The solution provides the optimal number of measurements and the best positions to place them, in order to identify the locations and magnitudes of harmonic sources. The minimum condition number of the measurement matrix is used as the criteria in conjunction with sequential elimination to solve this problem. Two different test systems are provided to validate the measurement placement algorithm. A three-phase asymmetric power system has been tested using the New Zealand test system, while the IEEE 14-bus test system has been used for testing a balanced power system.**

Index Terms— **Harmonic State Estimation (HSE), Optimal Measurement Placement, Power Quality.**

I. Introduction

THE problem of harmonic pollution in the power networks has been widely recognized. Standards for limiting this pollution have been set in many countries [1]. The increase of harmonics in the power system threatens the quality of the electricity supplied to consumers. The problem of identifying the location and magnitude of harmonic sources has become more important in power system engineering in order to ensure compliance with the standards.

The Harmonic State Estimation (HSE) is a reverse process of harmonic simulation, which analyzes the response of a power system to the given injection current sources. The HSE uses the harmonic measurements at selected busbars to identify the location and magnitude of harmonic sources. In addition, HSE is capable of providing information on harmonic at locations not monitored.

The design of a measurement system to perform HSE is a very complex problem. Among the reasons for its complexity are the system size, conflicting requirements of estimator accuracy, reliability in the presence of transducer and data communication failures, adaptability to changes in the network topology and cost minimization. In particular, the number of harmonic instruments available is always limited due to cost, and the quality of the estimates is a function of the number and location of the measurement points. Therefore, a systematic procedure is needed to design the optimal measurement placement.

A measurement placement algorithm for harmonic component identification is presented in [2], based on sequential solution and minimum variance criteria. However, it addressed the problem of selecting the best location to place a measurement to identify harmonic sources rather than the optimization of number of measurements and the estimation of the exact values of harmonic magnitudes. Furthermore, line current measurements are not considered. In addition, the optimal procedure in [2] needs load and generation data at each harmonic order for all busbars, which is usually not available.

In [3] a new symbolic method for observability analysis (OA) is presented. This method identifies redundant measurements thus giving the minimum number of measurements that are needed to perform HSE. However, the algorithm uses the initial measurement placement add extra measurements at alternative locations. The method assumes that all voltage measurements are not redundant then considers the number of unknown state variables and the number of equations linking these state variables in each identified group. It should be noted that this method cannot detect cases when there are two dependent measurement equations (such as when currents at both ends of a line are measured) because the actual values are lost.

Therefore, this paper mainly focus on a new technique for optimal measurement placement for HSE in terms of the optimal number of measurements and the best locations to place them in order to identify the location and magnitude of harmonic sources.

A new HSE algorithm, based on singular value decomposition (SVD) method, has been presented in [3], [4]. The SVD algorithm does not require the whole system network to be observable prior to estimation. It can give a solution even if the system under consideration is partially observable. The SVD will diagnose precisely what the problem is. In some cases, the SVD will not only diagnose the problem, but also solve it, in the sense of giving a useful numerical answer to HSE [5]. Instead of using HSE, [6]–[8] discuss the issue of applying SVD to detect, locate, and estimate remote harmonics in the presence of high noise contaminating from voltage or current waveform. The use of SVD is significantly slower than solving the normal equations and requires more storage, but is less susceptible to round-off

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error. Moreover, it's theoretically foolproof reliability more than makes up for the speed disadvantage.

First of all, this paper reviews the use of HSE in a power system (sections II and III). The proposed optimal measurement placement algorithm is presented in section IV and applied to the lower South Island of New Zealand and IEEE 14-bus test systems in section V. Finally, concluding remarks are made in section VI.

II. Harmonic State Estimation

The complete harmonic information throughout the power system can be estimated from a relatively small number of synchronized, partial and asymmetric measurements of phasor voltage and current harmonics at selected busbars and lines, which are distant from the harmonic sources [3], [4], [9], [10]. Using harmonic measurements at non-harmonic source busbars (such as those of generator busbars with no loads connected) to estimate the system-wide harmonic levels with under-determined system is presented in [11]. A framework of HSE can be found in [12]. A system-wide or partially observable HSE requiring synchronized measurement of phasor voltage and current harmonics made at different measurement points is described in [12]. Like recent HSE algorithms, the present work uses voltage and current rather than real and reactive power as the observed quantities, for reasons outlined in [13].

A general mathematical model relating the measurement vector $\boldsymbol{Z}$ to the state variable vector $\boldsymbol{X}$, to be estimated, can be formulated as follows:

$$\boldsymbol{Z}(h) = \boldsymbol{H}(h)\boldsymbol{X}(h)+\boldsymbol{E}(h), \tag{1}$$

where $\boldsymbol{Z}(h)$ is a measurements vector, $\boldsymbol{H}(h)$ is a measurement matrix, $\boldsymbol{X}(h)$ is a state vector to be estimated, $\boldsymbol{E}(h)$ is the measurement noise at h^{th} harmonic order.

The measurement matrix can be considered as the matrix whose elements relate the measurement vector to the state variable. If the state variable to be estimated is the nodal voltage, then:

- For nodal current injection measurement ($\boldsymbol{I}_N$), the relation to the nodal voltage ($\boldsymbol{V}_N$) and node-node admittance matrix ($\boldsymbol{Y}_{NN}$) is:

$$\boldsymbol{I}_N(h) = \boldsymbol{Y}_{NN}(h)\boldsymbol{V}_N(h), \tag{2}$$

- For nodal voltage measurement, the relation to the nodal voltage is:

$$\boldsymbol{V}_N(h) = \boldsymbol{I}^*\boldsymbol{V}_N(h), \tag{3}$$

where $\boldsymbol{I}$ is identity matrix,

- For line current measurement ($\boldsymbol{I}_L$), the relation to the nodal voltage and line-node admittance matrix ($\boldsymbol{Y}_{LN}$) is:

$$\boldsymbol{I}_L(h) = \boldsymbol{Y}_{LN}(h)\boldsymbol{V}_N(h). \tag{4}$$

Since the measurement noise in (1) do not affect the solvability of HSE, they may be ignored [9]. As a result, the proposed algorithm considers only one harmonic order at a time, and the variable of h^{th} harmonic order in the previous equation will be left. The system node set N is partitioned into two subsets of non-source busbars $(\boldsymbol{V}_{No}, \boldsymbol{I}_{No})$ and suspicious busbars $(\boldsymbol{V}_{Ns}, \boldsymbol{I}_{Ns})$, i.e.

$$\boldsymbol{V}_N = \begin{bmatrix} \boldsymbol{V}_{No} \\ \boldsymbol{V}_{Ns} \end{bmatrix}, \quad \boldsymbol{I}_N = \begin{bmatrix} \boldsymbol{I}_{No} \\ \boldsymbol{I}_{Ns} \end{bmatrix} \tag{5}$$

$$\text{with} \qquad \boldsymbol{I}_{No} = 0 \tag{6}$$

Then (2) can be partitioned as follows:

$$\begin{bmatrix} \boldsymbol{I}_{No} \\ \boldsymbol{I}_{Ns} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{Y}}_{NoNo} & \bar{\boldsymbol{Y}}_{NoNs} \\ \bar{\boldsymbol{Y}}_{NsNo} & \bar{\boldsymbol{Y}}_{NsNs} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_{No} \\ \boldsymbol{V}_{Ns} \end{bmatrix} \tag{7}$$

From (6) and (7), it can be found that:

$$\boldsymbol{V}_{No} = -\bar{\boldsymbol{Y}}_{NoNo}^{-1}\bar{\boldsymbol{Y}}_{NoNs}\boldsymbol{V}_{Ns} \tag{8}$$

From $\boldsymbol{Z}$=$\boldsymbol{HX}$, while $\boldsymbol{Z}$, $\boldsymbol{H}$ and $\boldsymbol{X}$ are related to (2)–(4). When $\boldsymbol{X}$ is $\boldsymbol{V}_N$ as in (5) and $\boldsymbol{H}$ is partitioned into two subsets of suspicious and non-source busbars $(\boldsymbol{H}_{Ns}, \boldsymbol{H}_{No})$, hence:

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{H}_{Ns} & \boldsymbol{H}_{No} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_{Ns} \\ \boldsymbol{V}_{No} \end{bmatrix} \tag{9}$$

Substitute $\boldsymbol{V}_{No}$ from (8) into (9), it yields:

$$\boldsymbol{Z} = \left[\boldsymbol{H}_{Ns} + \boldsymbol{H}_{No}\left(-\bar{\boldsymbol{Y}}_{NoNo}^{-1}\bar{\boldsymbol{Y}}_{NoNs}\right) \right] \boldsymbol{V}_{Ns} \tag{10}$$

When $\boldsymbol{V}_{Ns}$ are known, $\boldsymbol{V}_{No}$ can be calculated from (8). Then all state variables can be solved.

III. Solving The Harmonic State Estimation

A. Normal Equation

In some applications, the normal equations of (10) that are equivalent to $\boldsymbol{Z}$=$\boldsymbol{HX}$, are perfectly suitable for the linear least square (LS) problem. The following expression $\boldsymbol{X}$ is obtained.

$$\boldsymbol{X} = \left(\boldsymbol{H}^T\boldsymbol{H}\right)^{-1}\boldsymbol{H}^T\boldsymbol{Z} \tag{11}$$

However, this equation is usually under-determined system because of limitation of harmonic instruments. This results in $\left(\boldsymbol{H}^T\boldsymbol{H}\right)$ being singular and a result can not be obtained with normal equation approach. Furthermore, even in completely or over-determined system, the normal equations may be very close to singular or ill-conditioned. Although several methods have been suggested to solve such ill-conditioned problem, e.g. [14], [15], observability analysis is still needed prior to estimation. Like SVD approach, another method that does not require observability analysis before performing HSE is that of orthogonalization [16].

B. Singular Value Decomposition (SVD) [5], [12]

To solve the HSE problem for the under-determined case, when only observable islands exist, SVD needs to be applied, since standard techniques for solving such equations will fail [12].

Under normal condition, a system is designed to be completely observable prior to HSE. If the system has temporarily unobservable regions because of unanticipated network topology changes or failures in the telecommunication systems, the SVD still can provide correct answers for the part of the system forming the observable regions. In addition using SVD removes the need for observability.

The SVD method represents the matrix $\boldsymbol{H}$ (m×n) of (1) as a product of three matrices, i.e.

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{W}\boldsymbol{V}^T , \tag{12}$$

where $\boldsymbol{W}$ is a diagonal matrix (n×n) with positive or zero elements, which are the singular values of $\boldsymbol{H}$. Matrices $\boldsymbol{U}$ and $\boldsymbol{V}^T$ are orthogonal matrices. $\boldsymbol{U}$ is a column orthogonal (m×n) matrix and $\boldsymbol{V}^T$ is the transpose of an (n×n) orthogonal matrix.

SVD constructs special orthonormal bases for the null space and range of a matrix. Not only are they orthonormal but, if $\boldsymbol{H}$ multiplies a column of $\boldsymbol{V}$, a multiple of a column of $\boldsymbol{U}$ is obtained. It can be shown that $\boldsymbol{U}$ is the eigenvector matrix of $\boldsymbol{H}\boldsymbol{H}^T$ and $\boldsymbol{V}$ is the eigenvector matrix of $\boldsymbol{H}^T\boldsymbol{H}$. Moreover, $\boldsymbol{W}\boldsymbol{W}^T$ is a diagonal matrix of eigenvalues. The column of $\boldsymbol{U}$, corresponding to the non-zero singular values are an orthonormal set of basis vectors that span over the range of $\boldsymbol{H}$. The column of $\boldsymbol{V}$, corresponding to the zero singular values are an orthonormal set of basis vectors that span over the null space.

From (1) and (12), the following expression of $\boldsymbol{X}$ is obtained.

$$\boldsymbol{X} = \boldsymbol{V}\boldsymbol{W}^{-1}\boldsymbol{U}^T\boldsymbol{Z} \tag{13}$$

The solution process for an under-determined system using SVD can be found in [12]. When performing HSE using SVD, if all the singular values of $\boldsymbol{H}$ are non-zero, then the power system is fully observable and HSE yields the correct answer of all node voltages [17].

IV. Optimal Measurement Placement Algorithm

There is a limitation to the number of instruments a utility can afford to place in a power system. The more sensors connected to the system, the more accurate the parameter estimation, but the higher the cost. A proper methodology is needed for selecting optimal sites for the measuring devices.

The new solution technique presented in this paper provides optimal number and the best positions to place harmonic instruments with a limited number of observations, in order to identify the location and magnitude of harmonic sources. The minimum condition number criteria of the measurement matrix, based on sequential elimination, is utilized to solve this problem.

The condition number of a matrix is the ratio of the largest (in magnitude) to the smallest singular value. A matrix is singular if its condition number is infinite, and it would be considered ill-conditioned if its condition number is too large. That is if its reciprocal approaches the machine's floating-point precision (for example, less than 10^{-6} for single precision or 10^{-12} for double precision).

A brute-force method may be used to compute a comparative measure for all possible combinations of sensor placement [2]. The procedure exhausts all possibilities and yields the true optimal solution for the problem. For an N-bus system, M possible locations with a limited P measuring devices to be placed, $\binom{M}{P}$ possible combinations must be computed in order to determine the best locations for placing instruments. For example a 27 busbar system, 141 possible locations with 9 measuring devices for 9 suspicious busbars (N_s), the possible combinations are $\binom{141}{9}$ or 4.68×10^{13}. Hence the number of possibilities is usually large. The initial simulations on realistic models of power systems indicate that the location procedure could be performed in a sequential fashion. The methodology for sequential elimination is the best (M+1) measurement locations containing the best M locations (for all M). The sequential procedure has proven itself to be valid in many cases and is always near optimal [2].

The benefits gained from using the sequential procedure are dramatic because of the reduction in the number of possible combinations (as compared to complete enumeration). The sequential procedure need not be repeated from the beginning when increasing or decreasing the number of sensors. In general, for N-bus system, M possible locations with P measuring devices are to be placed. The sequential procedure needs only to compute P(2M+1−P)/2 combinations to determine the best, near optimal instrument locations [2]. Hence the amount of computation required by the sequential procedure is small compared with complete enumeration of a realistic size system. For example, a 27 busbar system, 141 possible locations with 9 measuring device, the sequential procedure requires 1,233 combinations to be computed, instead of the 4.68×10^{13} combinations required by complete enumeration.

The placement of measurement points is normally assumed to be symmetrical (e.g. either three or no phases measured at a location). However, this requirement restricts the search for the optimal placement of measurement points in three-phase asymmetrical power systems. As a result, all possible measurement locations for an N-bus system in this paper include all injection currents (N locations), all node voltages (N locations), and all line currents (L locations, both sending and receiving ends). In fact, the measurement placement at non-harmonic source busbar (N_0 locations) yields less useful information than those of suspicious busbars. However, the proposed measurement placement algorithm will be tested for both the case of all possible locations (2N+L locations) and the case where the injection currents and node voltages at non-harmonic source busbars are not included (2N+L−2N_0 locations).

Fig. 1 shows a flowchart of optimal measurement placement algorithm. From all possible locations, the measurement matrix can be formulated using (2)–(4). The objective function is the condition number of the measurement matrix.

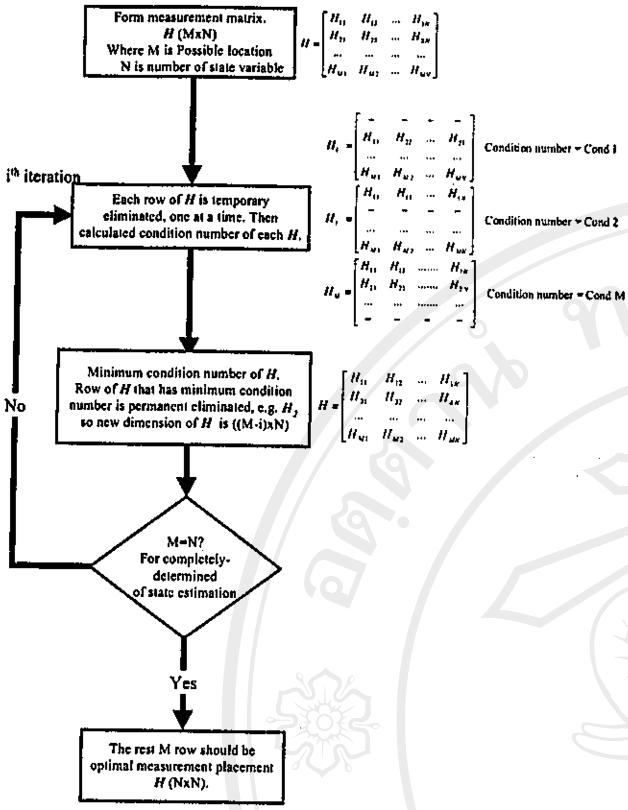

Fig. 1. Flowchart of proposed algorithm for measurement placement.

Due to cost the number of available harmonic instruments is always limited so that the measuring devices (P) have to be minimized. However, to improve the measurement redundancy (which is key to bad data identification), therefore virtual and pseudo measurements should be included in the measurement matrix. Virtual measurements provide the kind of information that does not need metering (e.g. zero harmonic current injections at switching substation and at non-harmonic source bus). To obtain a unique solution (i.e. completely observable system), the minimum required number of harmonic instruments has to be equal to the number of state variables. As a result, for N state variables, in order to minimize P, M has to be minimized as well. Therefore the algorithm needs to iterate until M=N for to ensure a completely observable system. It means that the number of computations needed is M–N iterations.

In each iteration each possible location is temporarily eliminated one at a time and then the condition number of the corresponding measurement matrix is calculated (step 2), yield Cond 1 (1st row of H was eliminated), Cond 2,...,Cond M (Mth row of H was eliminated). The location that has a minimum condition number from step 2 will be eliminated sequentially to reduce the number of M for the next iteration (step 3). This means that the condition number of a new measurement matrix in step 3, after eliminating location that has minimum condition number from step 2, will have the best (minimum) condition number (for example, the harmonic instrument in the 2nd row of the corresponding H in fig. 1 will be removed). The minimum condition number of the measurement matrix H, the ill-conditioned of the measurement matrix, will be minimum as well. As a result, the measurement matrix of this proposed algorithm is always not singular that ensures system solvability. Again, in such a case, all state variables can be obtained when all singular values of the measurement matrix are non-zero [17]. The iterative procedure is performed until M=N (step 4), that is, a row of the measurement matrix H will be eliminated by every iteration. The number of possible locations will be reduced, from M to M–1, M–2,..., M–(M–P). The remaining locations after sequential elimination, base on minimum condition number, should be optimal or near optimal for the measurements [2].

Because load information is not available prior to performing HSE, the loads are not represented in (2) but their current is part of the estimated (or measured) harmonic current injection. The methodology of HSE, for testing the measurement location is; (i) Assume that the partial 'measured values' from the measurement points are equal to their corresponding 'true values' plus some random noises generated with Gaussian distribution (if necessary), (ii) Estimate the values for all state variables using the estimator from the partial 'measured values', (iii) Compare estimated values with the corresponding 'true values' (results of complete simulation).

V. Application Example

Two test systems are used to test the proposed measurement placement algorithm. These test systems also illustrate the complexity of designing a measurement system to perform HSE. The first test system is the Lower South Island of New Zealand system, which is a three-phase asymmetric power system. The second is the IEEE 14-bus test system and this is a balanced system hence single-phase representation is adequate. All nodes or busbars with loads connected are treated as suspicious nodes. The remaining nodes are non-harmonic source nodes. It is assumed that the suitable measurement equipment capable taking synchronized measurements is available. The proposed algorithm for measurement placement and HSE is written using MATLAB®.

A. Test System I (The New Zealand Test System)

The proposed algorithm is tested using the 220 kV interconnected transmission grid below Roxburgh in the South Island of New Zealand. Three-phase modeling is applied to take into account imbalances and the coupling between phases at harmonic frequencies. This is achieved by using a transmission line parameter to calculate the electrical parameters of the lines from their physical geometry. Fig. 2 shows the three-phase diagram of the test network. The system includes 8 transmission lines represented by the equivalent Π model. The three synchronous generators are modeled as shunt branches and generate no harmonic currents. The five transformers are connected in star-delta. In the test system, there are 27 nodes, 111 branches, and 87 lines. Three loads are connected at Tiwai 220 kV (nodes 1–3), Invercargill 33 kV (nodes 22–24) and Roxburgh 33 kV (nodes 4–6). The actual harmonic sources are twelve-pulse rectifiers at Tiwai. Because a three-phase system is used, each busbar includes three nodes. Therefore, there are 18 non-harmonic source nodes (N_0) and 9 suspicious nodes (N_s) in the test system. There are 141 possible measurement locations (M), given that there are 27 locations for injection current measurements, 27 locations for node voltage measurements and 87 locations for line current measurements.

B. Test System II (The IEEE 14-bus Test System)

A schematic of the IEEE 14-bus test system is shown in Fig. 3. There are 14 busbars, 35 branches, and 41 lines. The equivalent Π model is used to represent each transmission line, with the electrical parameters being calculated from the physical geometry using a transmission line parameter program [18]. As the physical geometry is not available for the IEEE 14-bus test system a trial and error procedure is used to obtain a physical geometry that gives, as close as possible, the correct positive sequence impedance (R and X) and susceptance (B) at fundamental frequency. For all short lines, the susceptance is not modeled (as set to zero in the IEEE 14-bus system). For all long lines it is possible to model all R, X, and B values with an absolute error less than 9×10^{-4}.

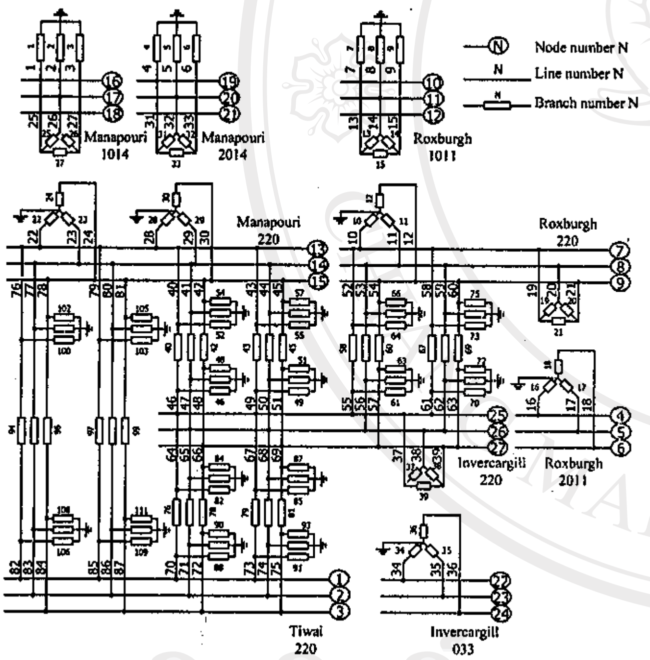

Fig. 2 The New Zealand test system.

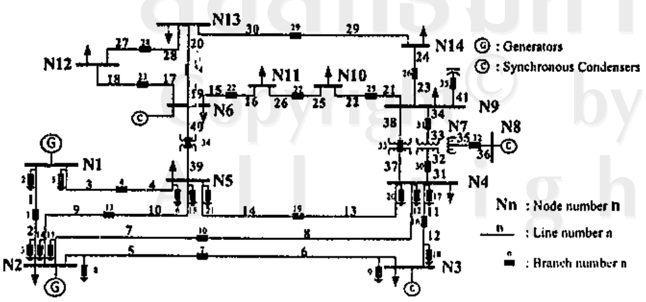

Fig. 3. IEEE 14-bus test system.

The system consists of 10 loads connected at busbars 3–5, and 8–14 [19]. There are 4 non-harmonic source nodes (N_0) and 10 suspicious nodes (N_s) in the test system. The two harmonic current sources are a twelve-pulse HVDC terminal at busbar 3 and an SVC at busbar 8. The source spectra are provided in Table 1.4 of [19]. There are 69 possible measurement locations (M), given that there are 14 injections current measurements, 14 busbars voltage measurements and 41 lines current measurements.

Actually the state variable of the test system I and II are 27 and 14, respectively. Using HSE algorithm as described in section II, the number of state variable can be reduced to the number of suspicious nodes. There are 9 and 10 for the test system I and II, respectively.

C. Test Results

To obtain a unique solution for an N-bus system, the minimum required numbers of harmonic instruments (P), for all possible locations (M), has to be equal to the number of state variables. As a result, the optimal number of harmonic instruments is equal to the number of state variables. In the proposed algorithm, the measurement matrix of each harmonic order is considered one at a time with the objective of minimizing the number of measurements. Two cases are considered: Case I is starting from all possible locations (2N+L locations); while in Case II, harmonic current injections and busbar voltage at non-harmonic source busbars are not included (2N+L–2N_0 locations). The measurement placements obtained by using this algorithm, which make the two test systems full observable, are shown in Tables I and II.

TABLE I
MEASUREMENT PLACEMENT: TEST SYSTEM I

Harmonic order	Case	Injection current	Node voltage	Line current
5	I	22,24	No	16,18,21-24,35
	II	6,24	No	16,17,22-24,34,35
7	I	4,22,24	2	11,17,19,35,73
	II	4,23,24	No	11,17,21,34,73,75
11	I	4	22-24	56,61,63,76,81
	II	No	22-24	12,17,55,56,79,81
13	I	No	4-6,22-24	22,74,75
	II	No	2,4-6,22-24	54,58
17	I	No	5,6,8,22-24	12,16,24
	II	No	1,3-6,22-24	26
19	I	No	4-6,22-27	No
23	I	No	4-6,22-24	54,65,67
	II	No	4-6,22-24	52,66,68
25	I	No	3-6,22-24	34,42
	II	22	3-6,22-24	42

TABLE II
MEASUREMENT PLACEMENT: TEST SYSTEM II
(CASE II ONLY)

Harmonic order	Injection current	Busbars voltage	Line current
5	No	3,8-14	7,40
7	No	3,4,9,10,12-14	3,16,36
11	No	3,5,8,10-12,14	7,20,34
13	No	3,5,8,10-12,14	7,20,34
17	No	4,8,10-12,14	3,5,20,34
25	No	4,5,8,10-12,14	5,20,34

When the system is fully observable (as shown in Tables I and II), both normal equation and SVD can be used to solve the problem. Unfortunately, it was found that the test systems are not fully observable with some measurement placements that have not been shown in Tables I and II. To solve HSE directly (without any extra computation effort) in such cases requires SVD. This yields correct answer at all observable busbars [17].

It should be noted that the network configurations of the two test systems are completely different. In addition, the ratio of state variables to possible locations is quite different. There are 9 state variables for 141 possible locations in test system I, while test system II has 10 state variables for 69 possible locations. When the number of state variables is quite high compare with the number of possible locations (in Case I of the test system II) measurement placement solution resulted in all 10 suspicious busbar voltages.

Moreover the measurement placements are different among harmonic orders, but all of the measurement placements from all harmonic orders as shown in Table I and Table II are sufficient to uniquely calculate all state variables for all harmonic orders of the system correctly. Example, harmonic instruments location from harmonic order 5 in Case I can be used to calculate all state variables for all harmonic orders. In such a case, both normal equation and SVD can be used to solve the problem.

However, minimizing the number of channels (harmonic instrument) does not necessarily result in lower cost because the predominant cost is in the base unit (site), while the incremental cost for additional channels is relative small. An optimal measurement placement of this proposed method is to minimize the number of sites and also to minimize the number of total harmonic instruments (to be equal to the number of state variables) thus reducing the monitoring costs attached to HSE. At the same time, using minimum condition number of the measurement matrix with sequential elimination simutaneously increases the HSE solvability.

To minimize the number of site, a trial and error procedure base on condition number analysis will be used.

1. Fully measurement placement at all possible locations at each site should be considered (shown in Table III). The site that has minimum condition number and measurement matrix is not singular should be selected in the first priority. Next, perform HSE with this fully measurement placement, if it is enough to solve all state variables (all singular value of a measurement matrix are non-zero), then use the proposed algorithm to reduce the number of harmonic instruments is applied. On the other hand, if one site is not enough to solve the problem, more sites may be added using condition number analysis, one by one.
2. If the numbers of possible locations in each site quite small compare with the number of state variables (such as test system II), the number of possible harmonic instruments in each site should be considered. The more possible locations, the more harmonic instruments could be placed. For example, busbars 4, 7–9 (same site) of test system II, which has a dominant number of possible location (22 locations) compare with the other sites, should be selected first (shown in Table IV).

TABLE III
FULLY MEASUREMENT PLACEMENT IN EACH LOCATION: TEST SYSTEM I

Site	Injection current and Node voltage	Line current	Possible locations	Condition number
Tiwai	1-3	70-75,82–87	18	3.16×10^{16}
Roxburgh	4-12	7-21,52-54, 58-60	39	7.45×10^{16}
Manapouri	13-21	1-6,22-33, 40-45,76-81	48	1.37×10^{17}
Invercargill	22-27	34-39,46-51, 55-57,61-69	36	5.46×10^{9}

From the simulation result using fully placement be shown in Table III, only fully placement at Invercargill that measurement matrix is not singular (double precision). It should be noted that, the condition number quite large (some singular value of a measurement matrix near zero). So, a measurement matrix may be not sufficient to solve all state variables correctly. HSE has to be performed to test solvability. Form HSE we know that fully placement at this site can be solved all state variable. Then the proposed algorithm with those possible locations is employed. The optimal measurement placements of this system, using the measurement matrix of the 5th harmonic, are node voltages at busbars 22, 25–27 and line currents at lines 56, 61, 63, 64 & 68.

TABLE IV
FULLY MEASUREMENT PLACEMENT IN DOMINANT LOCATION: TEST SYSTEM II

Site	Injection current and Node voltage	Line current	Possible locations	Condition number
Busbars 4, 7-9	Busbars 4, 7-9	8,11,13,21,23, 31-38,41	22	Infinity
Busbars 5-6	Busbars 5-6	4,10,14,15,17, 19,39,40	12	Infinity
Busbar 2	Busbar 2	2,5,7,9	6	Infinity
Busbars 4, 7-9 and Busbars 5-6	Busbars 4-9	4,8,10,11, 13-15,17,19, 21,23,31-41	34	16.11
Busbars 4, 7-9 and Busbar 2	Busbars 2,4,7-9	2,5,7,8,9,11,13, 21,23,31-38,41	28	Infinity

To minimize the number of sites for test system II, the earlier guideline is considered. From the network configuration, the process should start from busbars 4, 7–9. The value of the condition number of busbars 4, 7–9 (infinity; first line of Table IV) indicates that the measurement matrix is singular. Hence more sites have to be added, i.e. busbars 5 & 6 (both at the same site), which gives many possible measurement locations (12 locations). From the condition number of these two sites, it is known that fully placement at busbars 4–9 are sufficient to solve all state variables (all singular value of a measurement matrix are non-zero). Again the proposed algorithm is employed. The optimal measurement placements of this system, using the measurement matrix of the 5th harmonic, are node voltages at busbars 4–9 and line currents in lines 4, 8, 17 & 23. To solve HSE directly (without any extra computation effort) in such a case, the measurement matrix is singular (condition number is 7.45×10^{16}), only SVD can be used and yield partially correct answer at observable busbars. However, to make the measurement matrix fully observable, more harmonic instruments have to be added using condition number analysis. The previously removed harmonic measurements, by sequential elimination, have to be added. These are line currents in lines 15 and 21. On the other hand, if the measurement matrix of the 17th harmonic is used, the optimal measurement placement will be the busbar voltage at busbar 8 and line currents in lines 4, 8, 11, 15, 17, 19, 21, 23 & 34, resulting in the fully observable system.

Typical results obtained by using the HSE algorithm for node or busbar injection currents for up to the 25 harmonic order of the test systems are shown in Figs.4–5 while node or busbar voltages, and line currents throughout the test system could be found in [3]–[4], [9]–[12].

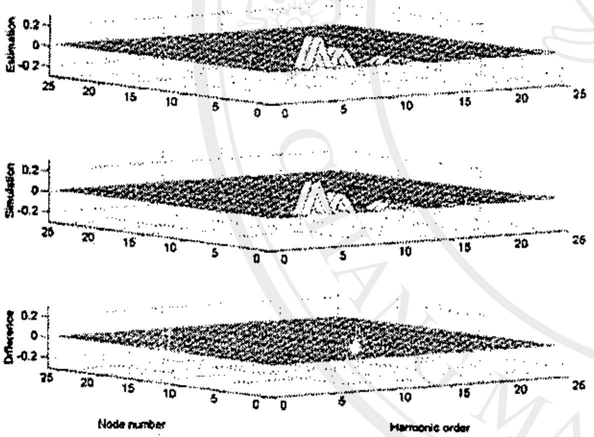

Fig. 4 Node harmonic injection currents of the New Zealand test system.

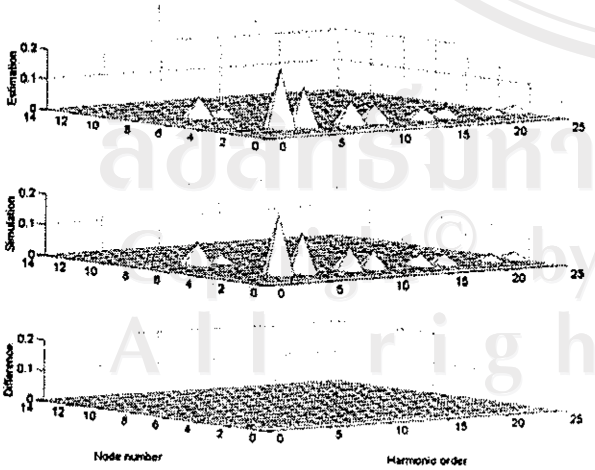

Fig. 5 Node harmonic injection currents of the IEEE 14-bus test system.

Generally the estimation for phase angle is less accurate than the estimation of the magnitudes of the same quantity. However, it does not affect the identification of harmonic source location, since it is able to identify the harmonic source with sufficient magnitude for each harmonic of interest.

The type of harmonic sources can also be identified. In the test system I, in evident that a six-pulse converter exists at Tiwai busbar because the injection currents at the 5th, 7th, 11th, 13th, 17th, 19th, 23rd, and 25th harmonics have been identified.

In the test system II, the injection currents at the 5th, 7th, 11th, 13th, 17th, 19th, 23rd, and 25th harmonics have been identified at busbar 3 and busbar 8 by performing the HSE. It is found that the harmonic sources exist at busbars 3 and 8.

VI. Conclusion

A new technique for optimal measurement placement for power system Harmonic State Estimation (HSE) has been presented. The minimum condition number of the measurement matrix is used as a criterion in conjunction with sequential elimination to reach the near optimal measurement placement. It is found that, the algorithm can yield a solution for the measurement placement that makes the power system completely observable.

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