CHAPTER 2

State Estimation and Harmonic State Estimation of Power System

In this chapter a review of state estimation methods applied to power network is given. A historical perspective of estimation theory, and their applications to power systems are presented. In addition, this chapter gives a summary of the formulation of the Harmonic State Estimation (HSE) problem. More detailed theory can be found in [17]. Finally, a basic concept of bad data analysis is presented.

2.1 State Estimation

For many years, estimation theory has been used mainly by astronomers as a means of reducing observations to obtain the orbital elements of minor planets and comets. The first attempt to apply the estimation theory to power systems is introduced by Gauss and Legendre (1800s). In 1960s, state estimation was applied to power systems for fundamental frequency power flow studies in [18]. The basic idea of state estimation is to fine-tune state variables by minimizing the sum of the residual square of the error between estimated and actual values. This is the well-known least squares (LS) method.

State estimation is now an essential part in energy management systems. The advantages of using state estimation are: (1) the state estimation algorithm produces quantities, which are the best possible estimation of the true value, even with measurement error, (2) the ability to detect and identify bad measurement, and (3) the ability to estimate quantities which cannot be measured and telemetered.

In system theory, the way of predicting the value of an unknown system state variable based on the measurements from that system, according to some criteria defined as state estimation. This term is used in electric power engineering to refer to techniques for the calculation and/or approximation of system bus voltage magnitudes, phase angles, and other related quantities. In practice, the state estimation process involves inexact redundant measurements and is based on statistical criteria that finds the true values of the state variables to minimize or maximize the selected criteria. These techniques differ from power flow studies because the latter is a solution of N equations of N state variables to a prescribed tolerance. In general, a state estimator, which may be both static and dynamic, can be used to (1) smooth out small random errors in meter readings, (2) detect and identify gross measurement errors, and (3) fill in nonexistent meter readings due to communication failures.

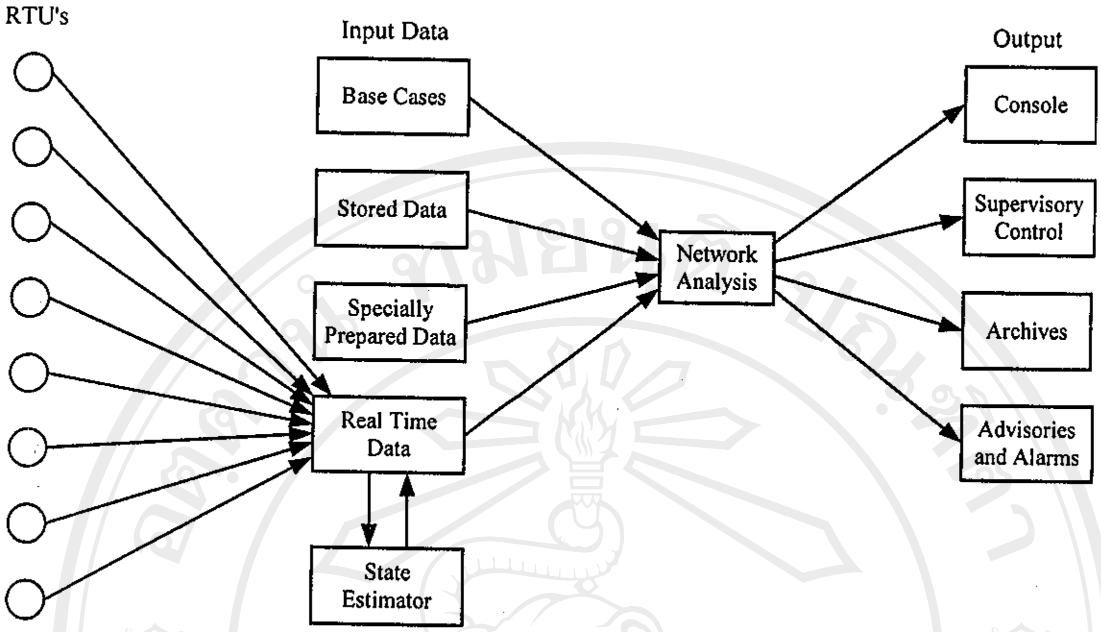

Figure 2.1 Illustration of SCADA system [19].

The principle applications of state estimators in power engineering are in real-time supervisory control and data acquisition (SCADA). Figure 2.1 shows a schematic diagram of the information flow between the various functions of a SCADA. Network analysis (i.e., power flow study, unit commitment analysis, and economic dispatch) requires telemetered or estimated system data such as voltages, currents, active and reactive power measurements, as well as system status data. These data come from remote terminal units (RTUs), that encode measurement transducer outputs and opened/closed relay status information into digital signals that are transmitted to the operations center over communication circuits. The actual field measurements are often a primary source of network data. Because, a measurement of all required data is impractical, and some RTUs may be possibly failed, a state estimator is used. To perform the state estimator, information about the network topology is necessary to know how the transmission lines are connected to the load and generation buses. The state estimation of a power system is formulated from the basic equations of system voltages and line flow. The state variables are the voltage magnitudes and phase angles with respect to a reference (slack) bus, at all busses. Eventually all other quantities such as the line MW-MVAR flows, and currents or voltages at any other point can also be obtained. There are many important points that must be taken into account for assessing the efficiency of any developed algorithm designed to study the state of a system. These points may be summarized as follows:

(i) Number of data measurements used must be as small as possible for economic reason,
(ii) Time needed for estimation calculation, and
(iii) Accuracy of the results.

In short, the function of the state estimator is to smooth out small random errors in telemetered data into a reliable estimate of the transmission network and state by accounting for:

(i) Small random metering and communication errors,
(ii) Uncertainties in system parameter values,
(iii) Bad data due to transients and meter communication failures,
(iv) Errors in the network structure due to faulty circuit breaker status, and
(v) Missing or unmeasured data.

The development of a classical approach for power system estimation was based on a LS technique. The solution of LS problem obtained directly from the normal equations is rather susceptible to round-off error or ill-conditioned. In addition, several other methods have also been suggested to alleviate the ill-conditioning problem of the LS estimator. The following is a list of some of these methods including a brief review of their characteristics and form [19]:

(i) Newton's Method for Ill-Conditioned Power Systems (State Estimation)

The measurements, states and measurement errors are related by the following nonlinear equation:

$$\boldsymbol{Z} = \boldsymbol{H}(x) + \boldsymbol{E} \tag{2.1}$$

where $\boldsymbol{Z}$ is the (M×1) measurement vector, $\boldsymbol{H}(x)$ is the vector of nonlinear functions, x is the (N×1) state variable vector, $\boldsymbol{E}$ is the measurement error vector, M is the number of measurements and N is the number of state variables. When large redundant data (M > N) is the case, the weighted least square (WLS) estimator works very well. However, when M = N and the system is ill-condition (eigenvalues are widely separated or the condition number is much greater than 1), the WLS estimator will oscillate around the solution point of a quadratic criterion function. Tripathy, Cahuhan and Prasad (1987) have suggested the use of Newton's method as a modification to the WLS algorithm. As a result, it was concluded that Newton's method is superior to the WLS in ill-conditioned power systems because of the second-order convergence characteristics of Newton's method.

(ii) Hybrid State Estimator

In this technique, the usual equations used by WLS state estimator is solved iteratively, where the triangular factorisation is carried out using orthogonal transformations. Monticelli, Murari and Wu (1985) have studied this method and it is reported that this hybrid state estimator is numerically robust and stable with the alleviation of the ill-conditioning problem associated with the normal equation.

(iii) Linear Programming Based State Estimators

Based on linear programming, state estimators consist of finding the best estimation of the state that minimizes the sum of the error distances of the solution point to the measurement hyperplanes. Irving (1978) has formulated the linear programming (LP) method with equality constraints, and later Lo (1988) has developed an efficient and reliable LP algorithm for two-level power system state estimation. It shown that this technique had the advantage of bad data rejection, while retaining a useful degree of noise filtering. Convergence of the method when applied to nonlinear network problems is equal to or better than the WLS solution.

(iv) Nonlinear Programming in Power System State Estimation

Similar to LP state estimation, the nonlinear programming (NLP) process estimates the values that lie on the intersection of ρ hyperplanes in ρ-dimensional space. The estimator will select a set of ρ-hyperplanes from available values to minimize the objective function. NLP state estimators preserve the LP properties of combining the automatic bad data rejection with a reliable degree of noise filtering. However, none of NLP methods can be expected to solve all the problems accurately and efficiently. Abbasy and Shahidehpour (1987) have applied the NLP state estimator based on the Powell algorithm. Using this method, it was observed that the solution obtained from using NLP is more accurate than both WLS and LP methods in several applications, e.g. bad data detection and rejection of estimated states not being measured. Also, this method is relatively fast and does not restrict the system studies to Gaussian-distributed noise.

(v) Neural Network Application

Neural network has been developed as a method of using a large number of simple parallel processors to recognize pre-programmed patterns. This approach can be adapted to recognize learned patterns of behavior in electrical networks. The neural network must 'learn' to associate the available power network data patterns with patterns of harmonic source behavior. This behavior can be learned from system operating data and data obtained from temporary harmonic source monitored at known sources. The neural network will then estimate harmonic sources based on experience in the same way an experienced operator infers pseudo-measurements from available data from conventional state estimation. This approach has been investigated by Hartana and Richards [7]. It was concluded that neural networks can be used to perform rough initial estimates of harmonic sources in power systems. The advantages of using a neural network are that no system model is needed and it has the possibility of high-speed calculation if implemented on a parallel-processing computer. However, this is totally unproven and not thoroughly tested. The disadvantages include the fact that the neural network method requires training. The training set can be quite large and even though it is applied off-line, it can be excessive. In addition, a method to synthesize a neural network is presently lacking.

(vi) Kalman Filter Application

During the past decade, many attempts were made to explore the time-varying nature of the state of an electric power system. One is the dynamic state estimation based on extended Kalman filtering technique (1960). In the basic LS fitting, the estimated parameters are assumed constant during the observation period and the measurement is corrupted only by noise. The Kalman filter is a LS estimate in which a state equation is added to allow its application to a dynamic system where the estimated parameters are varying. Its principal feature is the recursive processing of the noise measurement risk. This makes it ideally suitable for on-line estimation of varying parameters. At harmonic frequency, Dash (1988) has used Kalman filtering for estimation of the harmonic content for a bus voltage signal corrupted with noise. Later, Beides and Heydt (1990) have extended the work by including the power network model at various harmonic frequencies.

(vii) Singular Value Decomposition (SVD)

Reference [20] has presented a new fundamental static state estimation algorithm using linear WLS estimation, which is based on SVD rather than normal equations. The simulation study is performed on the IEEE 14-bus test system. The simulation results have shown that the SVD approach can provide a solution even when the system is ill-conditioned, while the normal equation approach failed to give satisfactory results. In addition, the SVD approach can identify that which parts of the network are unobservable island. Furthermore, the SVD approach does not require the whole network system to be observable prior to the estimation, it can provide a solution even if the system under consideration is partially observable.

2.2 Harmonic State Estimation

The task of Harmonic State Estimation (HSE) is to generate the best estimation of the harmonic levels from limited measured harmonic data corrupted with measurement noise. The three issues involved are the choice of state variables, some performance criteria, and the selection of measurement points and quantities to be measured.

Various performance criteria are possible. The most widely used is the WLS based on LS method. This method minimizes the weighted sum of the square of residuals between the estimated harmonic levels and the actual harmonic measurements. Other possible criteria are: Maximum Likelihood, Weighted Least Absolute Value (WLAV), Least Mediar of Squares (LMS), Minimum Variance, and Non-quadratic Estimators.

LS method has become the cornerstone of classical statistics, because of its simplicity. At the time of its invention, there was no computers, and the fact that the LS estimator could be computed explicitly from the data (by means of some matrix algebra) made it the only feasible approach. Even now, most statistical packages still use the same technique because of tradition and computational efficiency.

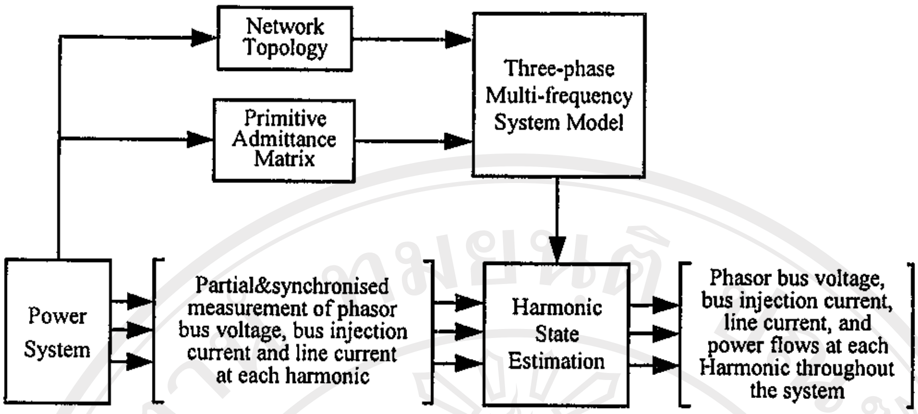

Figure 2.2 Framework for HSE [23].

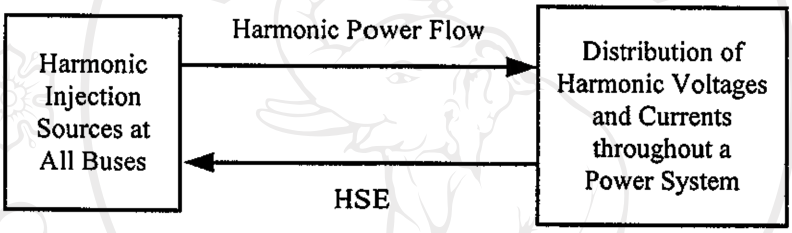

Figure 2.3 HSE and harmonic power flow [17].

The framework of HSE is illustrated in Figure 2.2. It uses a three-phase system model to describe asymmetrical conditions, such as circuit mutual coupling, impedance and current injection imbalances. A partial measurement set consisting of some bus voltages, injection currents and line currents, bus injection Volt-Amperes and line Volt-Amperes, are also needed.

HSE is a reverse process of harmonic power flow as shown in Figure 2.3. Harmonic power flow is used to analyse the response of a power system and harmonic penetration throughout the power network from harmonic injection current sources. On the other hand, HSE uses harmonic measurement at some buses to identify the harmonic sources and also provides information on harmonic penetration throughout the power network.

Recent contributions in [6-13, 21-22] have extended the concept of HSE and identification of harmonic sources. However, a full measurement of the system states, by first recording the voltage and current waveforms at nodes and lines and then deriving their frequency spectra, is prohibited for a large system. Only partial measurement (not necessarily made at the harmonic sources) is practical and, therefore, the measurements must be complemented by using system simulations. If the estimation procedure is sufficiently accurate, it is even possible to identify types as well as locations of the harmonic sources from their harmonic spectrum.

The main applications of state estimation in power quality engineering are [24]:

- Estimation of harmonic signal levels in the network,
- Estimation of harmonic signal levels at points of common coupling,
- Estimation of power quality indices (e.g., Total Harmonic Distortion (THD)) from on-line data,
- Estimation of injected noise and harmonics at bus for the purpose of identifying the location of loads which cause power quality problems,
- Mitigation of the effects of poor power quality on measurements,
- Identifying bad data, and
- Giving a second solution method to supplement measurements.

HSE is a very efficient and economic tool to provide system-wide or partially observable solutions for the assessment of the harmonic contents in a power system. Based on the network topology, the system admittance matrices at harmonic frequencies and the placement of measurement, a system-wide harmonic state estimator can be formulated. The measurements of voltage and current harmonics at selected buses and lines are sent to a central workstation for the estimation of the bus injection currents, bus voltages and line currents spectrum at all or selected positions in the network.

The ability to determine the locations and magnitudes of non-fundamental frequencies injections is important and enables cost-effective solution. The complete harmonic information throughout the power system can then be estimated from a relatively few synchronized, partial, and asymmetric measurements of phasor voltage and current harmonics at selected buses and lines away from the harmonic sources with under-determined system [3, 10-12]. Using harmonic measurements at non-harmonic sources buses, such as those of generators, without loads or linear loads, to estimate the system-wide harmonic levels with under-determined system are presented in [13].

A system-wide or partially observable HSE requires a synchronized measurement of phasor voltage and current harmonics made at different measurement points, as illustrated in Figure 2.4.

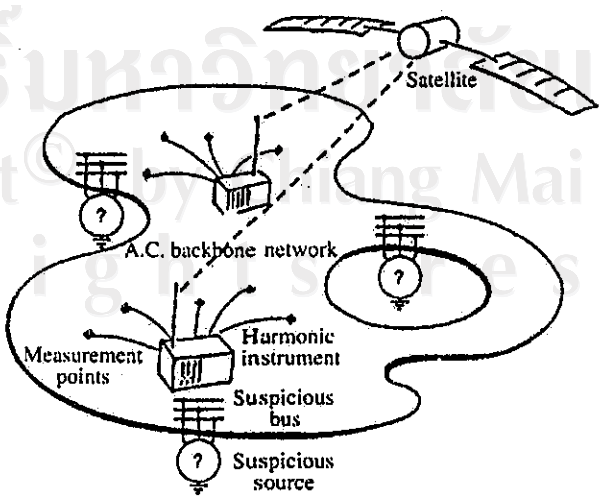

Figure 2.4 System-wide harmonic state estimation [23].

HSE turns multi-point measurement to system-wide measurement in a very economical way. Two important optimisation problems in HSE are the maximum observable subsystem for a given measurement placement and the minimum number of measurement channels needed for the observability of a given system. If the measurement is continuous and the processing speed is fast enough, the HSE can be implemented continuously in real time. Potentially, the harmonic monitoring measurement and estimator can then be integrated into an existing SCADA system.

2.2.1 Harmonic Measurement-State Variable Models [10]

A Three phase power system is modeled as an oriented graph. Let N be a set of all nodes (or bus of each phase) excluding a reference node, B be a set of all branches, and L be a set of all lines connected to the given nodes. The example illustrated in Figure 2.5

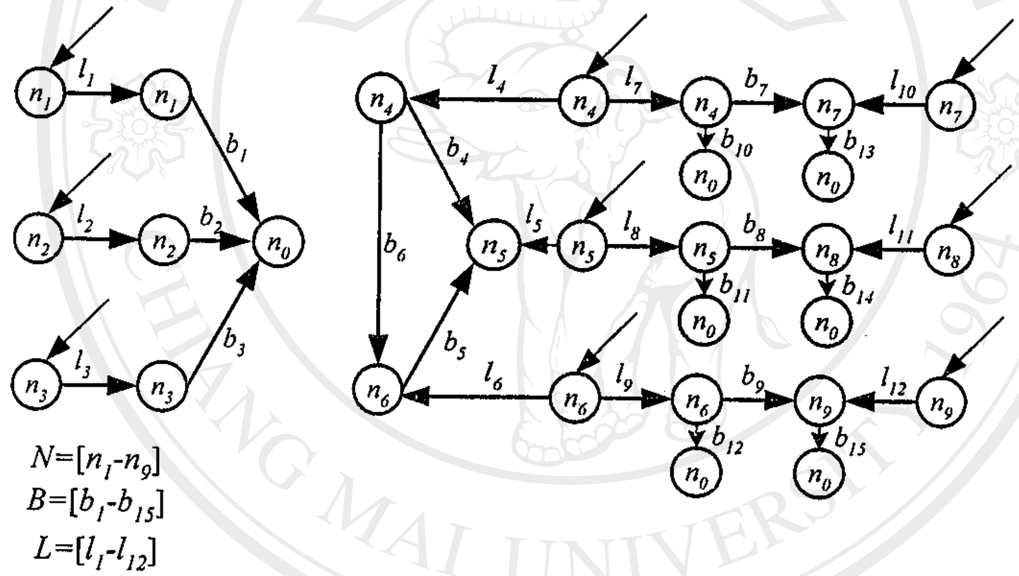

Figure 2.5 Oriented graph of a power system [10].

Let C_{NL}, C_{LB} and C_{BN} be node-line incident matrix, line-branch incident matrix, and branch-node incident matrix, respectively, i.e.

$$C_{NL}(i,j)=\begin{cases}+1 & \text{if line } j \text{ is from node } i \text{ with injection arrow;}\\ -1 & \text{if line } j \text{ is to node } i \text{ with injection arrow;}\\ 0 & \text{otherwise.}\end{cases}$$
$$C_{LB}(i,j)=\begin{cases}+1 & \text{if branch } j \text{ is from line } i;\\ -1 & \text{if branch } j \text{ is to line } i;\\ 0 & \text{otherwise.}\end{cases} \tag{2.2}$$
$$C_{BN}(i,j)=\begin{cases}+1 & \text{if branch } i \text{ is from node } j \text{ with injection arrow;}\\ -1 & \text{if branch } i \text{ is to node } j \text{ with injection arrow;}\\ 0 & \text{otherwise.}\end{cases}$$

Obviously, we have:

$$C_{BN}=\left(C_{NL}C_{LB}\right)^{T} \tag{2.3}$$

where T denotes an operator of vector or matrix transpose.

For each harmonic frequency h, let $V_N(h)$ and $I_N(h)$ be phasor vectors of node voltages and injection currents; $I_L(h)$ be a phasor vector of line currents; $V_B(h)$ and $I_B(h)$ be phasor vectors of branch voltages and currents, and $Y_{BB}(h)$ be a primitive branch admittance matrix. Using Kirchhoff's voltage and current laws, as well as Ohm's law,

$$\begin{aligned}V_B(h)&=C_{BN}V_N(h)\\ I_N(h)&=C_{NL}I_L(h)\\ I_L(h)&=C_{LB}I_B(h)\\ I_B(h)&=Y_{BB}(h)V_B(h)\end{aligned} \tag{2.4}$$

It follows that:

$$\begin{aligned}I_N(h)&=C_{NL}C_{LB}I_B(h)=C_{BN}^{T}Y_{BB}(h)V_B(h)=C_{BN}^{T}Y_{BB}(h)C_{BN}V_N(h),\\ I_L(h)&=C_{LB}Y_{BB}(h)V_B(h)=C_{LB}Y_{BB}(h)C_{BN}V_N(h).\end{aligned} \tag{2.5}$$

Let

$$\begin{aligned}Y_{NN}(h)&=C_{BN}^{T}Y_{BB}(h)C_{BN},\\ Y_{LN}(h)&=C_{LB}Y_{BB}(h)C_{BN}.\end{aligned} \tag{2.6}$$

Then, we have:

$$\begin{aligned}I_N(h)&=Y_{NN}(h)V_N(h),\\ I_L(h)&=Y_{LN}(h)V_N(h).\end{aligned} \tag{2.7}$$

2.2.2 Harmonic State Estimation Algorithm

HSE technique uses few synchronized harmonic measurement data as input to find the harmonic penetration for the whole network. The solution process of HSE is shown in Figure 2.6.

A general mathematical model, which relates the measurements vector $\boldsymbol{Z}$ to the state variable vector $\boldsymbol{X}$ to be estimated, can be formulated as follows:

$$\boldsymbol{Z}(h) = \boldsymbol{H}(h)\boldsymbol{X}(h) + \boldsymbol{E}(h) \tag{2.8}$$

Where $\boldsymbol{Z}(h)$ is a measurement vector,
$\boldsymbol{H}(h)$ is a gain or measurement matrix,
$X(h)$ is the state vector to be estimated,
$\boldsymbol{E}(h)$ is a measurement noise at h^{th} harmonic order.

2.2.2.1 Measurement Vector; $Z(h)$

$Z(h)$ is a vector of available measurements, which consists of measured phasor voltages and injection currents at selected nodes, and phasor currents at selected lines for each harmonic frequency. Branch current measurements are not used because they are either not accessible (e.g. windings of transformers in delta connection) or they do not exist physically (e.g. branches in equivalent Π model of a transmission line).

When real and reactive power measurements are used (instead of current measurements) for branch flows and busbar injection, the measurement equation becomes non-linear. In such a case, the solution must be obtained through an iterative algorithm. This method is used for fundamental frequency state estimation as power measurements are always available for revenue purposes. However, for harmonic frequencies, current measurements are more readily available.

Furthermore, real and reactive power measurements are not used in the present HSE for the following reasons [21]:

- Lack of a reliable method for P and Q meter error correction,
- Lack of uniformity in instrumentation for active and reactive power measurements. For example, some utilities use three phase instrumentation but the others do not, and
- Lack of a generally acceptable definition of reactive power in the presence of waveform distortion.

For HSE, it is assumed that harmonic measurement equipment that can be synchronized is available. The harmonics measured at certain locations can be nodal voltages, nodal currents, or line harmonic currents. The measurement data are then arranged in the measurements vector $\boldsymbol{Z}(h)$.

2.2.2.2 State Vector to be Estimated; $X(h)$

$X(h)$ is a state vector to be estimated. It is a vector of voltage phasors at all nodes for each harmonic frequency, $V(h)$. Once these state variables (which are independent of each other) are known; all the harmonic injection currents, branch currents, line currents, and harmonic power flows can be calculated, given that the network configuration and the primitive admittance matrix are known.

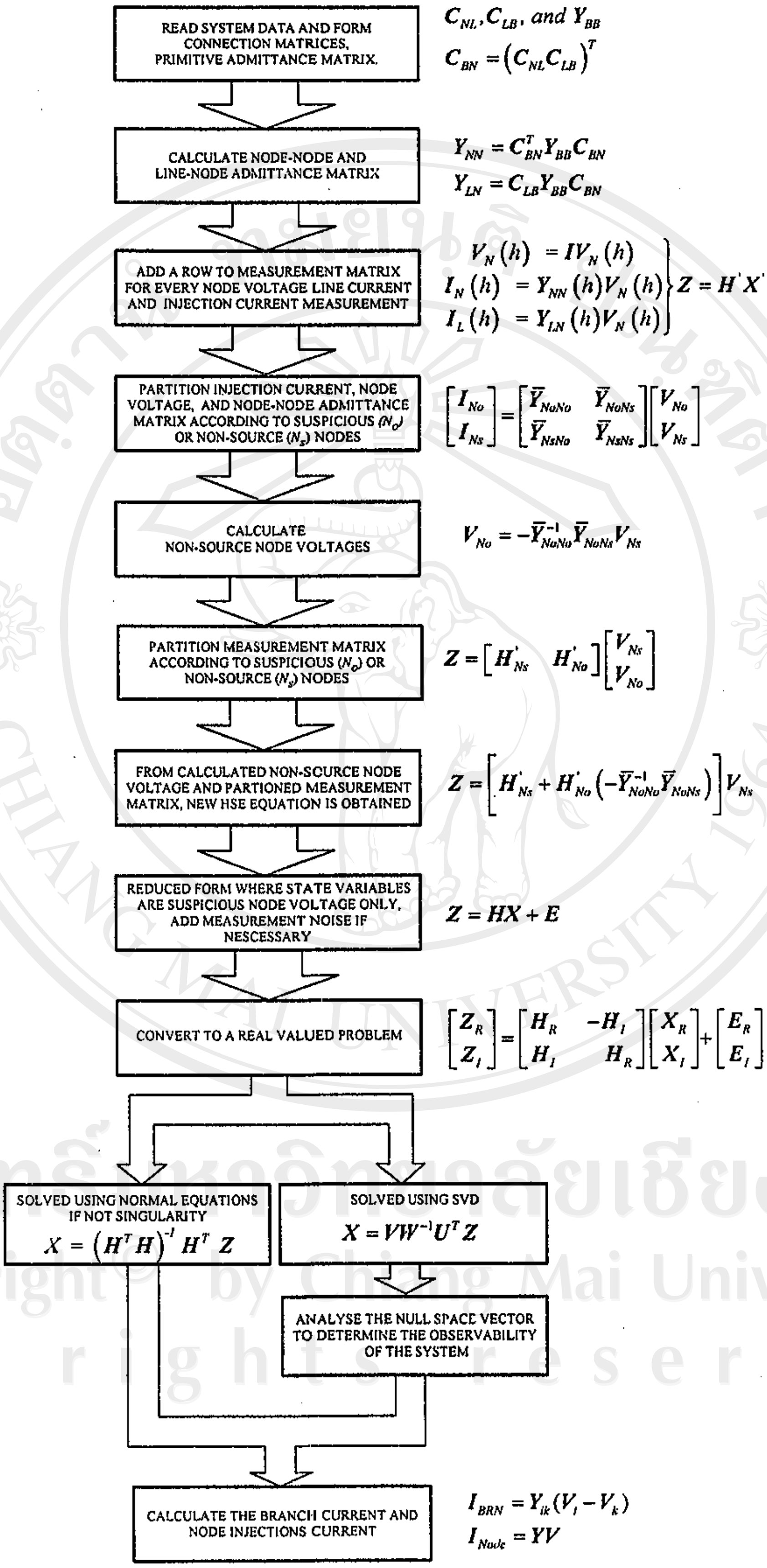

Figure 2.6 The solution process of HSE.

2.2.2.3 Measurement Noise (Uncertainty); *E*(*h*)

Generally, the value ***Z***(*h*) is closed to the true value of the parameter being measured, but differ by some unknown system measurement noise or error, ***E***(*h*), such as transducer errors and data communications problems. Gaussian noise model provides an adequate description for the uncertainties presents [12].

2.2.2.4 Gain or Measurement Matrix; *H*(*h*)

H(*h*) is the gain or measurement matrix which is related to system topological configuration, admittance matrix, and a placement of measurement points for each harmonic frequency *h*. The gain or measurement matrix can be considered as the matrix whose elements relate the measurement vector to the state variable. If the state variable to be estimated is the nodal voltage, then [12],

- For nodal voltage measurement, the relation to the nodal voltage is

$$V_N(h) = IV_N(h) \tag{2.9}$$

where I is identity matrix.

- For nodal current injection measurement (I_N), the relation to the nodal voltage (V_N) and node-node admittance matrix (Y_{NN}) is

$$I_N(h) = Y_{NN}(h)V_N(h) \tag{2.10}$$

- For line current measurement (I_L), the relation to the nodal voltage and line-node admittance matrix (Y_{LN}) is

$$I_L(h) = Y_{LN}(h)V_N(h) \tag{2.11}$$

Since the measurement noises in equation (2.8) do not affect the solvability of HSE, they are ignored in HSE algorithm [10]. In addition, the algorithm considers only one harmonic order, then the variable of harmonic order *h* in the previous equation will be neglected. The system node set *N* is partitioned into two subsets of non-source buses (V_{No}, I_{No}) and suspicious buses (V_{Ns}, I_{Ns}), i.e.

$$V_N = \begin{bmatrix} V_{No} \\ V_{Ns} \end{bmatrix}, \quad I_N = \begin{bmatrix} I_{No} \\ I_{Ns} \end{bmatrix} \tag{2.12}$$

with

$$I_{No} = 0 \tag{2.13}$$

Then, equation (2.10) can be partitioned as follows:

$$\begin{bmatrix} I_{No} \\ I_{Ns} \end{bmatrix} = \begin{bmatrix} \bar{Y}_{NoNo} & \bar{Y}_{NoNs} \\ \bar{Y}_{NsNo} & \bar{Y}_{NsNs} \end{bmatrix} \begin{bmatrix} V_{No} \\ V_{Ns} \end{bmatrix} \tag{2.14}$$

From equations (2.13) and (2.14),

$$V_{No} = -\overline{Y}_{NoNo}^{-1}\overline{Y}_{NoNs}V_{Ns} \tag{2.15}$$

From $\boldsymbol{Z}=\boldsymbol{HX}$, where $\boldsymbol{Z}$, $\boldsymbol{H}$ and $\boldsymbol{X}$ are related to equation (2.9)-(2.11). When $\boldsymbol{X}$ is V_N as in equation (2.12) and $\boldsymbol{H}$ is partitioned into two subset of suspicious and non-source buses $(\boldsymbol{H}_{Ns}, \boldsymbol{H}_{No})$, it follows that:

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{H}_{Ns} & \boldsymbol{H}_{No} \end{bmatrix} \begin{bmatrix} V_{Ns} \\ V_{No} \end{bmatrix} \tag{2.16}$$

Substitute V_{No} of equation (2.15) into equation (2.16), yields

$$\boldsymbol{Z} = \left[\boldsymbol{H}_{Ns} + \boldsymbol{H}_{No}\left(-\overline{Y}_{NoNo}^{-1}\overline{Y}_{NoNs}\right) \right] V_{Ns} \tag{2.17}$$

When V_{Ns} are known, V_{No} can be calculated from equation (2.15). Then all state variables can be solved.

Generally, the problem of solving state estimation equation can be classified as over-determined, completely determined or under-determined; depending on whether the number of independent measurement equations are greater, equal, or less than the number of state variables, respectively. A unique solution can only be obtained from the over or completely determined condition. [23]

In the under-determined case, a unique solution cannot be obtained unless extra information is supplied. Two such pieces of information are defined as pseudo-measurements and virtual measurements. Pseudo-measurements are estimated using historical data. But considering the lack of harmonic data, it is normally not viable for HSE. Virtual measurements provide the kind of information that does not need metering; for example, zero injection at a switching substation. The under-determined HSE problem can be transformed into an over-determined problem using this approach. This is achieved by reducing the number of unknown state variables to include only the variables of known buses. [23]

2.2.3 Solving the Harmonic State Estimation

2.2.3.1 Normal Equation

In some applications, the normal equations of (2.8) that are equivalent to $\boldsymbol{Z}=\boldsymbol{HX}$, are perfectly suitable for the linear least square (LS) problem. The following expression $\boldsymbol{X}$ is obtained.

$$\boldsymbol{X} = \left(\boldsymbol{H}^T\boldsymbol{H}\right)^{-1}\boldsymbol{H}^T\boldsymbol{Z} \tag{2.18}$$

However, this equation is usually under-determined system because of limitation of harmonic instruments. This results in $\left(H^T H\right)$ being singular and a result can not be obtained with normal equation approach. Furthermore, even in completely or over-determined system, the normal equations may be very close to singular or ill-conditioned. Although several methods have been suggested to solve such ill-conditioned problem, observability analysis is still needed prior to estimation. Like

SVD approach, another method that does not require observability analysis before performing HSE is that of orthogonalization.

2.2.3.2 Singular Value Decomposition (SVD)

There exists a very powerful set of techniques for dealing with the sets of equations or matrices that are either singular or numerically very close to singular. In many cases where Gaussian elimination and LU decomposition fail to give satisfactory results, this set of techniques (known as SVD), can diagnose precisely what the problem is. In some cases, SVD will not only diagnose the problem, but also solve it, in the sense of giving a useful numerical answer. It is known that the solution of LS problem obtained directly from the normal equations is rather susceptible to round-off error. It turns out that SVD also fixes the round-off problem, so it is a recommended technique for all. In the case of an over-determined system, SVD produces a solution that is the best approximation in the LS sense. In the case of an under-determined system, SVD produces a solution whose error are smallest in the LS sense. [25]

In the over-determined or completely-determined case, the singularity from the normal equations implies what is known as an unobservable system. In the case of under-determined case, the singularity implies that there is no unique solution to the problem. SVD, however, will provide a particular solution and a null space vector for each singularity.

The SVD method represents and (M×N) matrix $\boldsymbol{H}$ of equation (2.2) as the product of three matrices, i.e.

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{W}\boldsymbol{V}^T \qquad (2.19)$$

where $\boldsymbol{W}$ is a diagonal matrix (N×N) with positive or zero elements, which are the singular values of $\boldsymbol{H}$. Matrices $\boldsymbol{U}$ and $\boldsymbol{V}^T$ are orthogonal matrices, $\boldsymbol{U}$ is a column orthogonal (M×N) matrix and $\boldsymbol{V}^T$ is the transpose of an (N×N) orthogonal matrix.

SVD constructs special orthonormal bases for the null-space and Range of a matrix. Not only are they orthonormal but if $\boldsymbol{H}$ multiplies a column of $\boldsymbol{V}$, a multiple of a column of $\boldsymbol{U}$ is obtained. It can be shown that $\boldsymbol{U}$ is the eigenvector matrix of $\boldsymbol{H}\boldsymbol{H}^T$ and $\boldsymbol{V}$ is the eigenvector matrix of $\boldsymbol{H}^T\boldsymbol{H}$. Moreover, $\boldsymbol{W}\boldsymbol{W}^T$ is a diagonal matrix of eigenvalues. The columns of $\boldsymbol{U}$ corresponding to the non-zero singular values are an orthonormal set of basis vectors that span over the range of $\boldsymbol{H}$. The columns of $\boldsymbol{V}$ corresponding to the zero singular values are an orthonormal set of basis vectors that span over the null space. From equations (2.8) and (2.19) the following expression of $\boldsymbol{X}$ is obtained.

$$\boldsymbol{X} = \boldsymbol{V}\boldsymbol{W}^{-1}\boldsymbol{U}^T\boldsymbol{Z} \qquad (2.20)$$

If some of the singular values (w) are zero or near zero, then a zero is placed in the diagonal element of $\boldsymbol{W}^1$ (instead of $1/w$). This is equivalent to throwing away one linear combination of the set of equations. The condition number of a matrix is the ratio of the largest to smallest singular value. A singularity is considered near zero when its value approaches or below the largest singular value times the machine's precision (e.g. 10^{-6} for single precision and 10^{-12} for double precision) [23].

Although it is still not widely known, the SVD has a fairly long history. The underlying matrix eigenvalue algorithms have been developed by Francis, Rutishauser, and Wilkinson and are presented in Wilkinson's book (1965). Golub and his colleagues Kahan, Businger, and Reinsch (1971) did much of the fundamental work. Recent books by Stewart (1973) and Lawson and Hanson (1974) discuss the SVD as well as other related topics.

A new HSE algorithm, based on singular value decomposition (SVD) method, has been presented in [3, 12]. It can give a solution even if the system under consideration is partially observable. Again, SVD can diagnose precisely what the problem is. In some cases, SVD will not only diagnose the problem, but also solve it, in the sense of giving a useful numerical answer to HSE [25]. Instead of HSE, some contributions [15-16, 26] discuss the issue of applying SVD to detect, locate, and estimate remote harmonics in the presence of high noise contaminating from the voltage or current waveform. In practice, the use of SVD can be significantly slower than solving the normal equations and requires more storage. However, its great advantage more than makes up for the speed disadvantage and it does not require the whole network system to be observable prior to estimation.

2.2.4 Observability Analysis [23]

Observability Analysis (OA) is required in HSE for identifying its solvability. A power system is considered to be observable if the set of available measurements is sufficient to calculate all the state variables of the system uniquely. Observability is dependent on the number, locations, and types of available measurements, network topology, as well as the system admittance matrix. For a different network topology, or same network topology but different measurement placements, an OA is to be performed in each case.

It is important for OA not only to decide whether the system is observable and hence system-wide HSE can be performed, but also to provide information of the observable/unobservable islands as well as redundant measurement points if the system is not completely observable. This allows the re-positioning of measurement points to maximize their usefulness.

A system is observable if a unique solution can be obtained for the given measurements. A unique solution exits if and only if the rank of $\boldsymbol{H}$ is equals to the number of unknown state variables. Therefore, to observable, the number of measurements must not be less than the number of state variables to be estimated. However, this condition is not sufficient because linear dependency may exist among rows of the measurement matrix. The rank of $\boldsymbol{H}$ does not depend on the quality of the measurements and therefore the noise vector can be assumed to be zero.

The existing OAs can be divided into three groups; numerical (floating point calculations), topological, and symbolic methods.

Numerical observability determination is based on assessing the rank of the gain matrix by triangular factorisation. There are several algebraically equivalent ways of expressing the state estimation equations that have good sparsity and numerical stability for large systems. However, due to ill-conditioning and finite precision arithmetic, numerical problems may occur. The factorisation method is simple and uses some of the techniques of the HSE algorithm; however, it can be fail because of numerical round-off errors. This results from performing floating point calculations on the large sets of poorly conditioned equations. For example, matrix elements, that should be zero, are not exactly zero. Therefore, a threshold needs to be applied to those elements. The choice of threshold may not be obvious since it depends on both the network and the precision of the arithmetic used.

This leads to the distinction between algebraic observability and numerical observability. A power system is algebraically observable for a given set of measurements if the rank of the gain matrix is equal to the number of state variables to be estimated. A power system is numerically observable if the measurement model can be solved for the state variables. If a system is numerically observable, then it must also be algebraically observable. However, the converse needs not hold. It is possible for the gain matrix to have the required rank. But it may be ill-conditioned so that cannot be solved numerically. However, for most power systems, algebraic observability would imply numerical observability.

Floating point determination of rank is time-consuming and does not give information on where the problems are.

As the name indicates, the topological approach informs whether a system is topologically observable. Although it is possible for a topologically observable system to be algebraically unobservable, it is unlikely to occur in a practical system as it only happens with a theoretical choice of network admittances. This condition is called parametrically unobservable.

The system is topologically observable if there exists a spanning tree of full rank. In this respect, a tree is any interconnected, loop-free collection of branches of the network and a spanning tree is a tree that is incident to every busbar. The number of possible trees for N busbars is N^{N-2}, which is very large figure even for a small system. To start with, the branches with flow measurements are used to build a tree. All the loops are eliminated, as the flow through any branch (that forms a loop) can be calculated from Kirchoff's voltage law, circuit parameters, and flows in other branches. Hence, such measurements are redundant and do not contribute to the rank of the gain matrix. This leaves several connected pieces or trees, and the resulting unconnected loop-free sub-graph is termed a forest. Then busbar injection information is used. However, the topological method and requires procedures that are not needed to compute the state estimation is combinatorial. Thus a computational effort is considerable.

The symbolic method seeks to overcome the numerical problems associated with floating point operations by replacing them with symbolic calculations, where each entry in the measurement matrix used for OA is either 1 or 0. While being extremely fast and simple, the basic method is not capable of finding all the observable islands and redundant measurements. A second phase has been added to the symbolic method to overcome these deficiencies as well as retain its simplicity and speed.

Unobservable subsystems can be categorized based on the number of additional measurements needed to make them observable. For example, a subsystem is referred to as univariate conditionally-observable if the set of equations is one less than the number of state variables linked, as a one more measurement will make the subsystem observable. Two observable subsystems can be combined to form the observable system even though they do not interconnect (have no state variables in common). However two univariate conditionally-observable subsystems can only be combined to form a larger univariate conditionally-observable subsystem if they have at least one state variable in common. There are thus two process-phases, the first one is searching for the overall observable system and the second one is searching for univariate conditionally-observable subsystems. For a subsystem to be observable, at least one state variable must be known (measured) and it is used as the reference busbar.

2.2.5 Load and Harmonic Source Identification

The harmonic simulation and HSE algorithms are different by means of load treatment. In general, a load bus may contain linear (passive) and non-linear components. These can be modeled in harmonic simulation, which represents the current injections and the passive components separately. HSE, on the other hand, may have no information of the composition of the load and is only capable of estimating the net current flow into or out of the load busbar. [23]

Therefore, the current-injection information supplied to the HSE algorithm is the sum of the harmonic current source and harmonic current flowing in the load. In theory, it should be possible to derive some information on the nature of the load from the estimated harmonic voltages and injected currents at the bus.

The harmonic voltages at the suspicious buses and the harmonic currents injected from the suspicious sources to the backbone are provided by the estimator at the end of HSE. Each suspicious source is classified as a harmonic injector or a harmonic absorber.

A suspicious harmonic source can be considered as a Norton equivalent circuit at each harmonic frequency as shown in Figure 2.7. The following relationship applies for a harmonic of order h:

$$\hat{I}_i(h) - I_i(h) = V_i(h)Y_i(h) \tag{2.21}$$

In equation (2.21), $V_i(h)$ and $I_i(h)$ are the nodal voltage and current injection, respectively, as provided by the estimator, while $\hat{I}_i(h)$ and $Y_i(h)$ are the unknown Norton harmonic current injection and admittance within the suspicious source (i=1, 2, 3). The following two assumptions are made for the suspicious source:

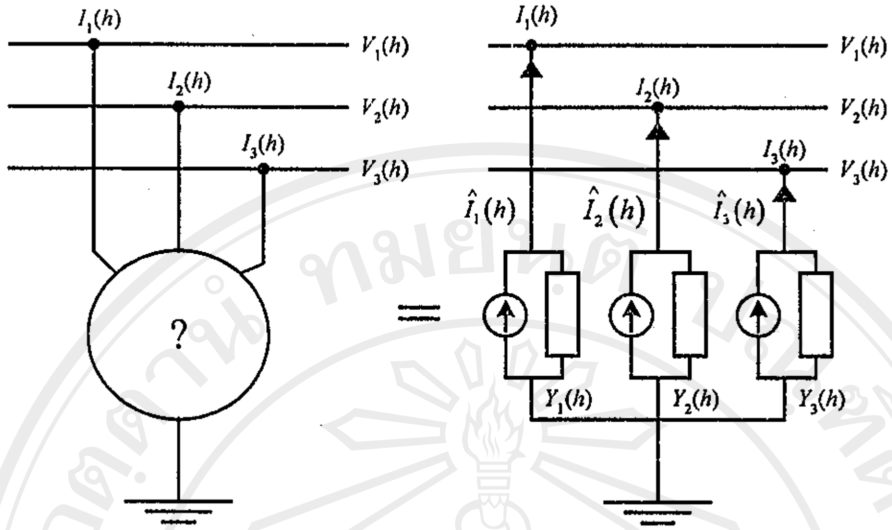

Figure 2.7 Norton equivalent circuit for suspicious harmonic sources [23].

$$Y_i(h) = G_i - jB/h \tag{2.22}$$

$$\left|\hat{I}_i(h)\right| = \delta_i(h, h_0)\left|\hat{I}_i(h_0)\right| \tag{2.23}$$

where G_i and B_i are unknown parameters for node i, h_0 is a chosen reference harmonic (e.g. the 11th harmonic for the cases of 6-pulse and 12-pulse converters), and $\delta_i(h, h_o)$ is a chosen ratio of $|\hat{I}_i(h)|$ to $|\hat{I}_i(h_0)|$. As a result, for any two harmonics h_1 and h_2 which are not h_0, the set of quadratic equations (equations (2.21) - (2.23)) is solvable to obtain the unknown Norton parameters $\hat{I}_i(h)$ and $Y_i(h)$ for each harmonic h of interest.

By sensitivity analysis, it can be shown that the estimated Norton parameters using the above method are very dependent of to the chosen ratio when the suspicious source contains non-zero Norton current injections, and very independent of the chosen ratio when the suspicious source does not contain Norton current injections. Therefore, the above method can at least be used to identify whether a suspicious source is a purely passive load and, in such a case, estimate the equivalent harmonic admittances of the passive load.

The way for harmonic type identification is presented in [6], by comparing the spectrum of harmonic current from HSE with idealized six-pulse or twelve-pulse converter as shown in Figure 2.8 and 2.9, respectively. Idealized data do not effect attenuation in harmonic signal strength due to the network frequency response. Similarly, idealized data do not include commutation characteristics which cause attenuation in high frequency components due to the rounding of the current waves. Stratford (1980) has empirically quantified such phenomena and his guideline attenuation characteristics are sometimes called Stratford's numbers. Note that the difference between the idealized and estimated always happen. Then, the work presented in [6] can be used for harmonic type identification of six-pulse or twelve-pulse converter.

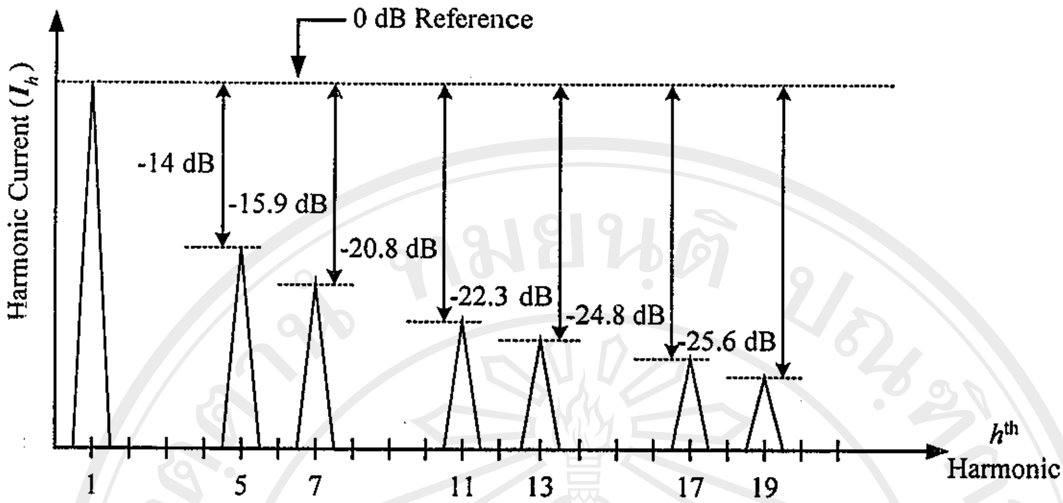

Figure 2.8 Spectrum of harmonic current for an idealized six-pulse converter [6].

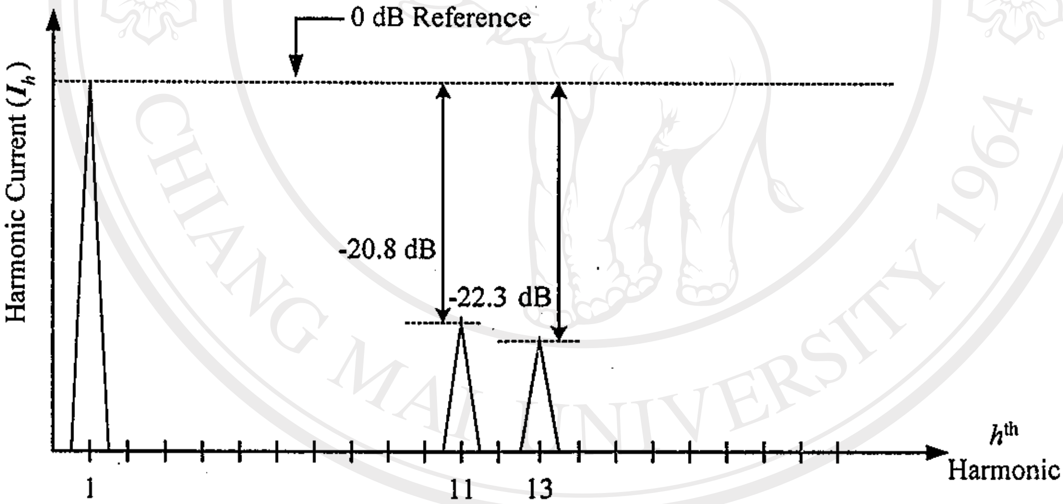

Figure 2.9 Spectrum of harmonic current for an idealized twelve-pulse converter [6].

2.3 Bad Data Analysis

Bad-data can result from erroneous measurement values, incorrect system parameters, or incorrect network topology. At transducer may have been wired incorrectly or the transducer itself may be malfunctioning so that it simply no longer gives accurate readings. Erroneous measurement has been the main focus of bad-data analysis. This can be categorised into three groups: extreme errors, gross errors and normal measurement noise. The presence of bad data degrades the accuracy of the HSE results and the problem is overcome by detection, identification, and removal of the bad-data, or the use of more robust estimator replacing the weighted least squares. [23]

Although a great deal of work has been done on bad-data analysis for fundamental frequency, in particular detecting its presence, identifying which measurements are bad, and eliminating the influence of the bad-data. But this usually requires the system to be over-determined and is therefore of limited applicability to HSE. However, in the presence of bad data, the residual $r_i = Z_i - H_i(x_{est})$ should be large. Where r_i is a residual value, Z_i is a measurement value, H_i is a measurement matrix of Z_i, and x_{est} is an estimated value from HSE. A statistical hypothesis test can thus be applied on the residual values (weighted or normalized version of residual) to identify the presence of bad data. [23]

In any state estimator, the redundancy in a measurement system is very important for three reasons. The first reason is the requirement of accurate state and, consequently, output variable estimates. The second is the ability to detect and identify bad data. The third reason is the ability to correct for parameter inaccuracies.

The main reasons for having redundant measurement information is to provide the capability to identify and locate bad data consisting of gross and/or large modeling errors. Most methods of bad data processing in power system state estimation can reliably identify single and multiple non-interacting bad data. However, the rest can provide a reliable identification in the presence of multiple interacting bad data. Comparatively, the two leading approaches appear to be a combinatorial optimisation identification (COI) method and a hypothesis testing identification (HTI) method have been proposed. Slutsker (1989) proposed a method that attempts to utilize the best features of COI and HTI and compensate for their weaknesses. It combines the COI and the largest normalized residual (LNR) methods for reliable selection of bad data through a sequential removal of measurements with the largest absolute normalized residuals with the HTI method's ability to obtain and statistically analyze the estimate of measurement errors. [19]

WLAV techniques applied to static state estimators is presented in [27]. Simultaneous detection and rejection of bad data are shown to be one of the features of the proposed estimator. Automatic rejection of bad data is a direct consequence of the interpolation property of the WLAV technique. This automatic rejection is conditioned by the availability of an adequate set of local redundant measurements near the point where bad data measurements were located. Several statistical theory required to analyze, detect, and identify bad measurement for state estimation of a power system are described in [28].