## CHAPTER 4

## Direct Adaptive Control System Using FREN

In this chapter, the application of FREN as a direct adaptive controller is presented. Without the need of an exact mathematical model of the controlled plant, the initial FREN's parameters and control rules in IF-THEN format are selected based on users' knowledge about the plant. These parameters are further adjusted during system operation using a method based on the steepest descent technique described in the previous chapter. By using the estimated plant information, the learning rate selection criteria based on Lyapunov's stability condition is also presented. The FREN controller is applied to control various nonlinear systems, for examples, the single invert pendulum plant, the water bath temperature control, and the high voltage direct current transmission system. Computer simulations results indicate that the proposed controller is able to control the target systems satisfactorily.

#### 4.1 Structure of FREN Controller

A general fuzzy inference system can be represented by the IF-THEN rules. For a single input single output system, these rules may be written as,

RULE i: If I is 
$$A_i$$
 Then  $B_i = f_i(\mu_{A_i})$ 

where I denotes the crisp input of this fuzzy system. In direct control applications, the input I can be replaced by the error signal E(k) and the change of error  $\Delta E(k)$ . This rule indicates that if I belongs to the fuzzy set  $A_i$  with the membership value of  $\mu_{A_i}$  then the fuzzy value of the output of this rule, denoted by  $B_i$ , is equal to  $f_i(\mu_{A_i})$ . After all rules have been processed, the crisp output O is calculated using some defuzzification schemes.

Suppose that the fuzzy IF-THEN rules are given by

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RULE 1 IF E is PL THEN U is NL RULE 1 IF \Delta E is PL THEN \Delta U is NL RULE 2 IF E is PM THEN U is NM RULE 3 IF E is NM THEN U is PM RULE 4 IF E is NL THEN U is PL RULE 4 IF \Delta E is NL THEN \Delta U is PL.
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where P, N, M, and L denote positive and negative, medium and large linguistic level, respectively.

From the rules, since there are two input signals thus two FRENS are needed. The first FREN receives the error signal E(k) and computes the nominal control signal U(k). The other receives the change of error  $\Delta E(k)$  and produces the change of control signal  $\Delta U(k)$ . The plant control signal u(k) is then obtained from

$$u(k) = U(k) + \Delta U(k). \tag{4.1}$$

The structure of the control system is shown in Fig. 4.1. The initial setting of FREN's parameters can be performed as mentioned in the previous chapter using the knowledge about the plant operations.

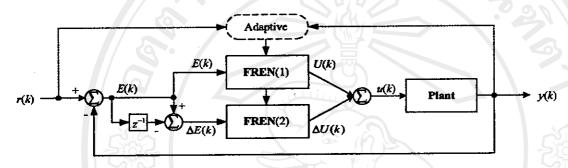


Figure 4.1: Control system using FREN

### 4.1.1 Parameter Adaptation Algorithm

Since the initial setting of FREN parameters are just rough estimation based on a human expert experience. It is necessary to fine tune these values in order to cope with environmental change and also to improve the system performance. In this work, an adaptive technique based on the steepest descent technique is proposed to adjust all parameters, i.e. the shapes of membership functions and linear consequences, during the system operation. Firstly, we define the objective function as

$$\xi(k) = \frac{1}{2} \Big[ r(k) - y(k) \Big]^2, \tag{4.2}$$

where r(k) and y(k) are the reference and the plant's output signal at time k respectively. It is desired to adjust all FREN's parameters to minimize this objective function. Here, the value of parameter  $P_i$  is updated at each time step by

$$P_i^{\text{new}} = P_i + \Delta P_i = P_i - \eta_i \frac{\partial \xi}{\partial P_i}, \tag{4.3}$$

where  $\eta_i$  is called the learning rate of *i*-th parameter. The term  $\partial \xi/\partial P_i$  is calculated from

$$\frac{\partial \xi}{\partial P_{i}} = \frac{\partial \xi}{\partial y} \frac{\partial y}{\partial u} \frac{\partial u}{\partial P_{i}},\tag{4.4}$$

where u is the control signal, i.e. the output of the controller O. Thus

$$\frac{\partial u}{\partial P_i} = \frac{\partial O}{\partial P_i}. (4.5)$$

This term can be analytically obtained since the network structure is already known.

Other terms in Eq.(4.4) are approximated by

$$\frac{\partial y}{\partial u} = Y_p \approx \frac{y(k) - y(k-1)}{u(k) - u(k-1)},\tag{4.6}$$

and

$$\frac{\partial \xi}{\partial y} = y(k) - r(k) = -E(k). \tag{4.7}$$

Finally, Eq (4.3) becomes

$$P_{i}^{\text{new}} = P_{i} + \eta_{i} E(k) Y_{p} \frac{\partial O}{\partial P_{i}}.$$
(4.8)

The difference between Eq.(4.8) and Eq.(3.13) in the previous chapter is  $Y_p$  or plant information signal. This signal is used to estimate the derivative of plant with respect to the control signal.

#### 4.1.2 Learning Rate Selection

As previously mentioned in 3.3.2, a learning rate should be appropriately selected. In the control applications, too large value of the learning rate may result in the system instability. In this subsection, the learning rate selection to guarantee the system stability in Lyapunov's sense is introduced. Consider the following Lyapunov function

$$V(k) = \frac{1}{2} \left( r(k) - y(k) \right)^2 = \frac{1}{2} E^2(k). \tag{4.9}$$

The change of Lyapunov function is given by  $\Delta V(k) \ = \ V(k+1) - V(k)$ 

$$\Delta V(k) = V(k+1) - V(k)$$

$$= \frac{1}{2} \left( E^{2}(k+1) - E^{2}(k) \right)$$

$$= \Delta E(k) \left( E(k) + \frac{1}{2} \Delta E(k) \right), \qquad (4.10)$$

where  $\Delta E(k) = E(k+1) - E(k)$  is the change of error. This can be approximated by

$$\Delta E(k) = \frac{\Delta E(k)}{\Delta P_i} \Delta P_i \approx \frac{\partial E(k)}{\partial P_i} \Delta P_i, \tag{4.11}$$

for small  $\Delta P_i$ .

The term  $\partial E(k)/\partial P_i$  can be calculated by

$$\frac{\partial E(k)}{\partial P_{i}} = \frac{\partial E(k)}{\partial y} \frac{\partial y}{\partial O} \frac{\partial O}{\partial P_{i}} = -Y_{p} \frac{\partial O}{\partial P_{i}}.$$
(4.12)

since  $\partial E(k)/\partial y = -1$  and  $\partial y/\partial O = \partial y/\partial u = Y_p$ .

Using  $\Delta P_i$  from Eq.(4.8), the change of the Lyapunov function can then be written as

$$\Delta V(k) = -\eta_i \left( E(k) Y_p \frac{\partial O}{\partial P_i} \right)^2 \left\{ 1 - \frac{1}{2} \eta_i \left( Y_p \frac{\partial O}{\partial P_i} \right)^2 \right\}. \tag{4.13}$$

According to the stability condition,  $\Delta V(k)$  must be less than zero, this yields

$$0 < \eta_i < 2 \left( Y_p \frac{\partial O}{\partial P_i} \right)^{-2}. \tag{4.14}$$

The learning rate  $\eta_i$  should lie in the range indicated by the above relation in order to guarantee system stability.

## 4.2 Computer Simulation Examples

In this section, FREN controller has been applied to control three types of nonlinear systems, i.e., the single invert pendulum, the water bath temperature control, and the high voltage direct current transmission system. Computer simulations have been performed to investigate the performance of FREN controller.

## 4.2.1 Single Invert Pendulum System

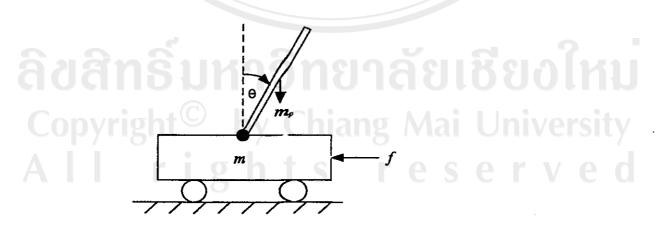


Figure 4.2: Single invert pendulum

The single invert pendulum system to be controlled by the FREN controller is shown in Fig.4.2. The equation governing this system is given by

$$\ddot{\theta} = \frac{mg\sin\theta - \left(f + m_p l\dot{\theta}^2\sin\theta\right)\cos\theta}{4ml/3 - m_p l\cos^2\theta},\tag{4.15}$$

where  $\theta$  is the angle of arm, and  $\dot{\theta} = d\theta/dt$ . Other parameters are:

m the mass of the cart, 1.1 kg.

 $m_p$  the mass of the arm, 0.1 kg.

l the half lenght of the arm, 0.5 m.

g accerelation constant,  $9.8 \text{ m/s}^2$ .

f the control effort in newton unit.

 $\theta_d$  is the desired angle of the arm and the tracking error  $E = \theta_d - \theta$ . The FREN controller is designed based on the follow fuzzy rules:

Fren(1):		Fren(2):		
RULE 1	IF E IS PL THEN U IS NL	Rule 1 If $\Delta E$ is PL Then $\Delta U$ is NL		
Rule 2	IF E IS PM THEN U IS NM	RULE 2 IF $\Delta E$ is PM Then $\Delta U$ is NM		
Rule 3	If $E$ is NM Then $U$ is PM	RULE 3 IF $\Delta E$ is NM Then $\Delta U$ is PM		
Rule 4	If $E$ is NL Then $U$ is PL	RULE 4 IF $\Delta E$ is NL Then $\Delta U$ is PL.		

Here P, N, M, and L denote positive and negative, medium and large linguistic level, respectively.

In this computer simulation, the initial angle  $\theta$  is set to 0.1745 radian and the desired angle  $\theta_d$  is 0 radian. The initial setting of the MF and LC shown in Fig.4.3(a) are selected based on the given fuzzy rules. Note that these initial setting are selected according to our knowledge about the operation of the invert pendulum which is the similar procedure as when designing a fuzzy controller.

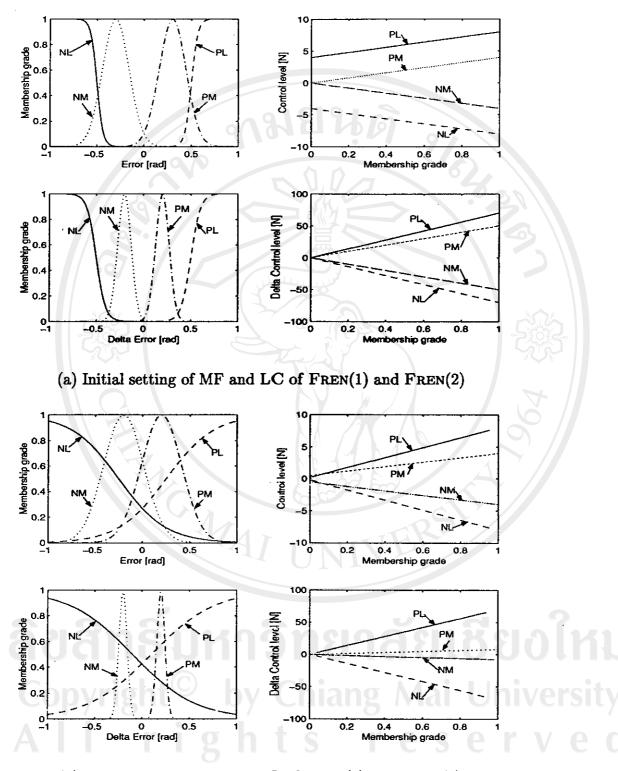
After on-line learning, the final shape of MF and LC become as shown in Fig. 4.3(b). It should be noted that these characteristics are gradually adjusted according to the adaptive algorithm. The plant output response  $\theta$ , the state variables, and the control signal u(k) during the on-line learning are given in Fig.4.4. It can be seen that the FREN controller can force the pendulum arm into stable state. The control effort signal is rather smooth and has a maximum value around 5 newtons.

Fig. 4.4(a) and (b) show the responses of this control system and the control efforts obtained from the FREN controller, a well tuned PID controller, and a neural network trained using the same number of epoches as FREN respectively. In this simulation, one epoch means one learning cycle which is begun and finished at t=0 and t=10 second, respectively.

The responses obtained by FREN and PID controllers are similar but the PID controller requires a well adjusted parameters. While the responses of neural network controller is rather slow.



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(b) Final setting of MF and LC of FREN(1) and FREN(2)

Figure 4.3: FREN's parameters for the invert pendulum control system.

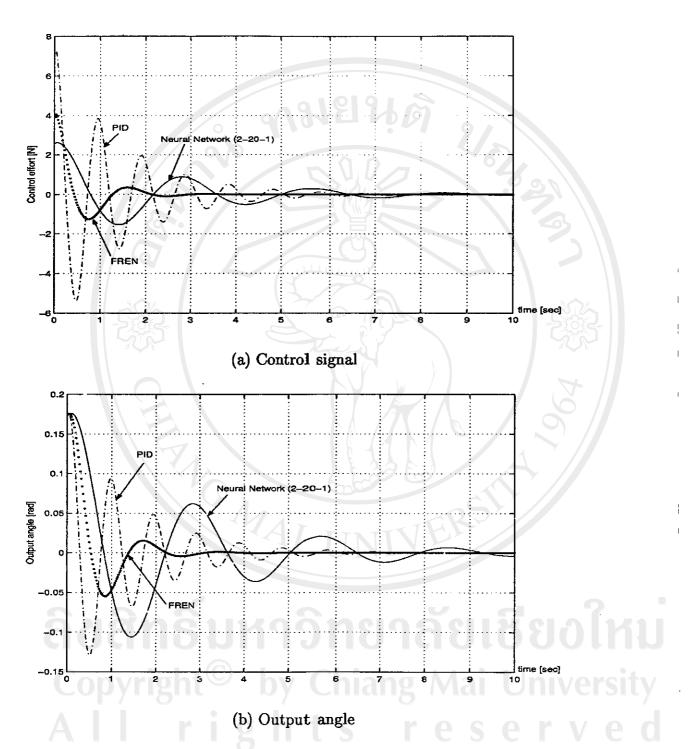


Figure 4.4: Responses of the invert pendulum control system using FREN, PID, and neural network controllers.

#### 4.2.2 Water Bath Temperature Control System

The next nonlinear plant is the water bath temperature control system. This system is highly nonlinear and has been used as a plant controlled by many neuro-fuzzy controllers [2,7]. The plant's equation is given by

$$y(k+1) = Q_{1}(T_{s})y(k) + \frac{Q_{2}(T_{s})}{1 + \exp\left(\frac{1}{2y(k) - \gamma}\right)}u(k) + \left(1 - Q_{1}(T_{s})\right)y_{0},$$

$$Q_{1}(T_{s}) = \exp\left(-aT_{s}\right)$$

$$Q_{2}(T_{s}) = \frac{a}{b}\left(1 - \exp\left(-aT_{s}\right)\right)$$
(4.16)

where y(k) is the temperature in celcius at time index k. u(k) is the control effort at time index k. In this simulation, other parameters are set as follows:  $a = 1.0015 \times 10^{-4}$ ,  $b = 8.67973 \times 10^{-3}$ ,  $\gamma = 40.0$ , and the initial temperature  $y_0 = 25.0$ °C. The plant control signal u(k) is limited between 0 and 5 volts. The sampling period  $T_s$  is 30 seconds. Define r as the desired setting temperature and the error E = r - y. In this simulation, the reference signal is given by

$$r(k) = \begin{cases} 35 ; & 0 < k \le 40, \\ 55 ; & 40 < k \le 80, \\ 75 ; & 80 < k \le 150. \end{cases}$$

The following fuzzy control rules are employed.

FREN(1):		FREN(2):
Rule 1	If $E$ is PL Then $U$ is PL	RULE 1 IF $\Delta E$ is PL THEN $\Delta U$ is PL
Rule 2	If $E$ is PM THEN $U$ is PM	RULE 2 If $\Delta E$ is PM Then $\Delta U$ is PM
Rule 3	If E is NM THEN U is NM	RULE 3 IF $\Delta E$ is NM Then $\Delta U$ is NM
RULE 4	If $E$ is NL Then $U$ is NL,	RULE 4 IF $\Delta E$ is NL Then $\Delta U$ is NL.

These fuzzy rules are also used to design the initial parameters setting of FREN controller as shown in Fig.4.5(a). After on-line learning, the final MF and LC become as shown in Fig.4.5(b).

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control system.

CRITERIA	Fren	NFIN [7]	BPNN [2,7]
Sum of Absolute Error	342.4711	341.7778	344.1059
Network parameters	Light	Medium	Heavy
Network structure	Simple	Complex	Simple
Convergence speed	Fast	Fast	Medium
Computational load	Very light	Light	Heavy
Learning algorithm	Only On-line	On-line and Off-line	Only On-line

The control signal u(k) and the plant output response y(k) after on-line learning are shown in Fig. 4.6. Fren controlled system has fewer parameters comparing with other neuro-fuzzy controllers. It needs only four nodes per hidden layer and the weights between each layers are unity. The structure of Fren is simpler than NFINs's. For this simulation, Fren needs only 7 trials to reduce the sum of absolute error (SAE) from 4000 to 342.4711 while NFIN requires 20 on-line trials to bring SAE from 352 to 341. This shows that even the initial setting are not very good the learning algorithm quickly adjusts Fren parameters to reasonable values. The learning phase of NFIN requires both off-line and on-line but Fren uses only on-line learning to achieve nearly the same performance index. The performance of the proposed controller compared with other controllers are presented in Table 4.1.

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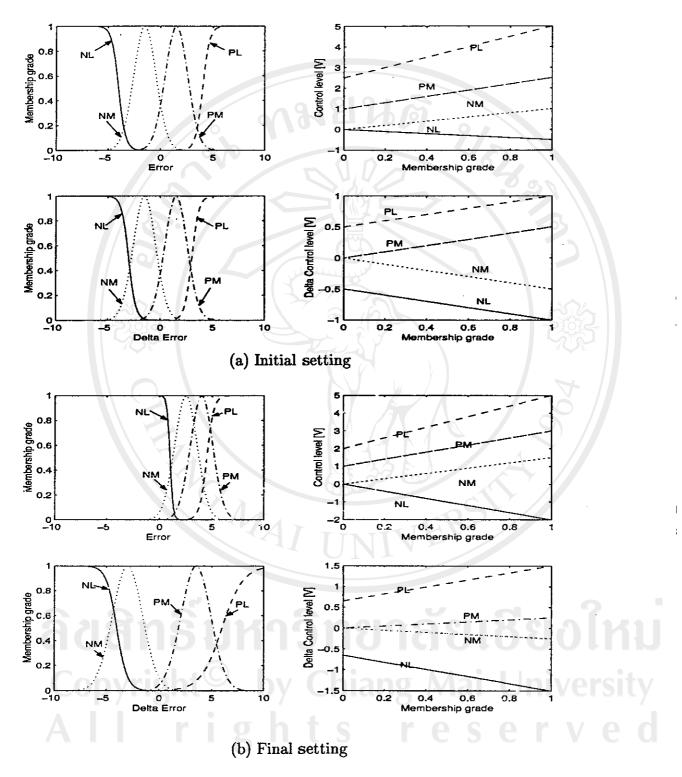


Figure 4.5: Fren's parameters for the water bath temperature control system.

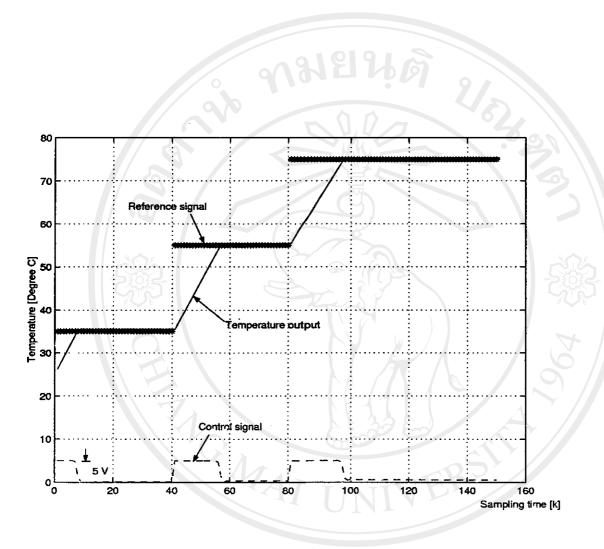


Figure 4.6: Control signal and plant response of the water bath temperature control system.

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#### 4.2.3 HVDC Transmission Control System

The final control system is the HVDC transmission system. This is based on the MATLAB power system toolbox [70]. A 500MW (250kV,2kA) DC is used to transmit power from 315 kV, 5000 MVA AC network. The rectifier and converter transformer are simulated with the simulink models. A 6-pulse rectifier is implemented as the converter. The 300-kilometer transmission line is used to connect through a 0.5 H smoothing reactor. A set of filter (C bank plus 5-th, 7-th and high pass filters: total 320 Mvar) is used to provide the reactive power required by the converter. The DC line fault study can be represented on the rectifier side by a circuit breaker. The firing instants of the thyristors are determined by the current error with a pulse generator. System configuration is shown in Fig. 4.7. This simulation system illustrates the response of the system to a step change command in current and to a DC line to ground fault followed by a load rejection.

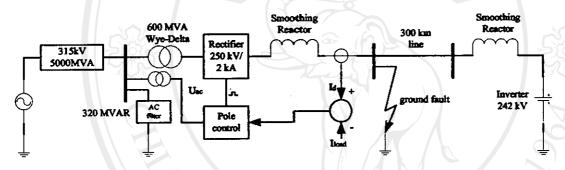


Figure 4.7: HVDC transmission system

Denote U the control signal,  $I_d$  and I be the desired and actual consumption load current respectively. And the error  $E = I_d - I$ . The fuzzy control rules are given by,

RULE 1 IF E IS PL THEN U IS PL

RULE 2 IF E is PM Then U is PM

RULE 3 IF E IS NM THEN U IS NM

RULE 4 IF E IS NL THEN U IS NL,

Note that in this application only one FREN is required. The control system becomes as shown in Fig. 4.8.

The initial setting of membership functions and linear consequences of FREN are selected as shown in Fig. 4.9(a). After on-line learning, the final MF and LC of FREN become as shown in Fig. 4.9(b). Notice that the difference between the initial and final setting of MF and LC is quite large in this case. This indicates that the adaptive technique can adjust the system performances even the system encounters the fault and step change.

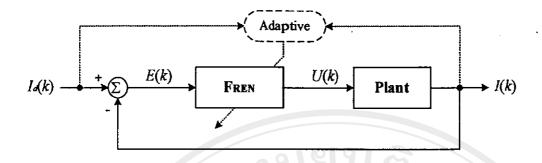


Figure 4.8: Control system using single FREN

The results obtained from the conventional PI controller and the FREN controller are shown in Fig. 4.10(a) and (b) respectively. It can be seen that the overshoot is cancelled and the peak current from the ground fault is reduced by using FREN controller. The output load current can track the setting value satisfactorily.

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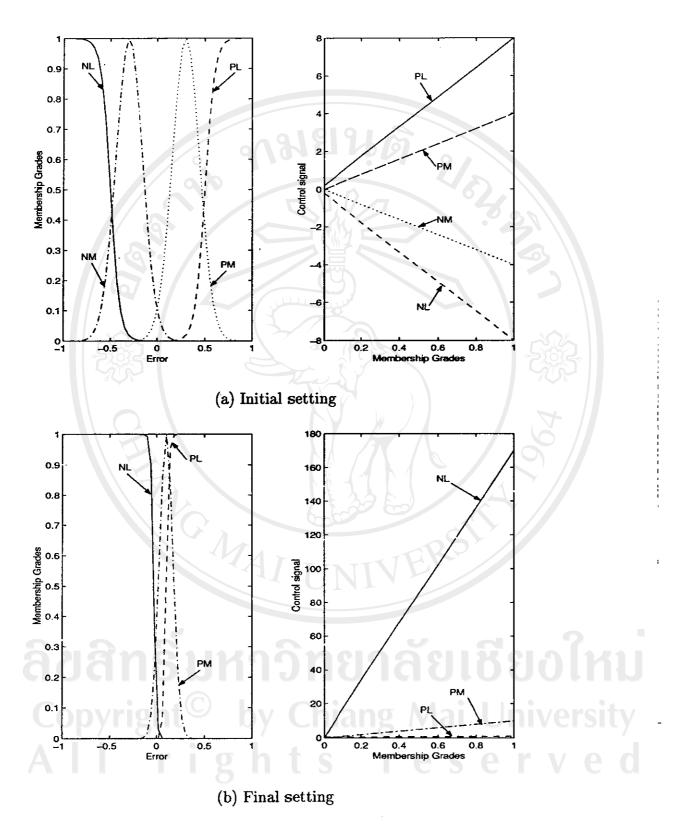


Figure 4.9: Settings of MF and LC in HVDC transmission.

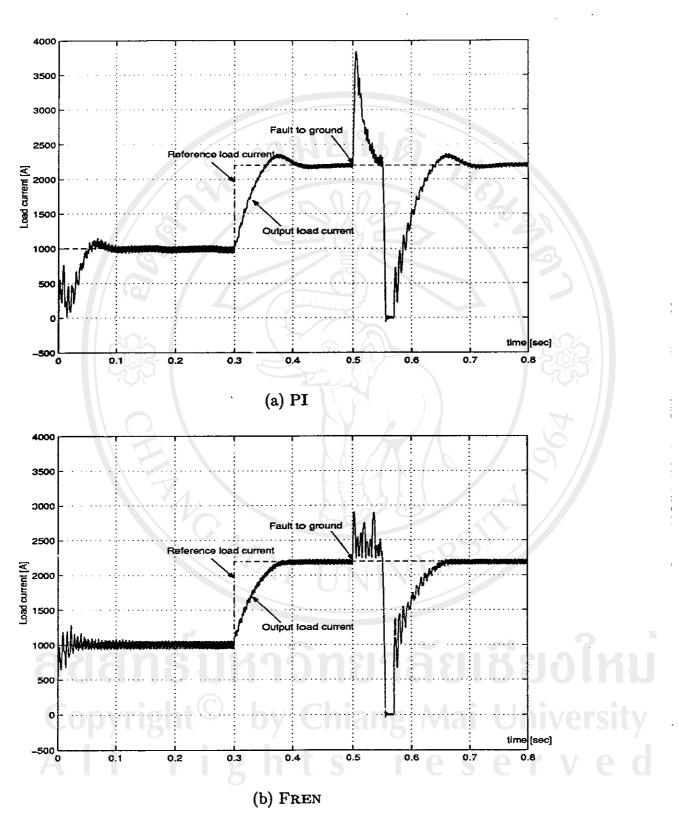


Figure 4.10: HVDC simulation results based on PI and FREN controllers.