CHAPTER 4

CONCLUSION

This research extend the results obtained by Brian Fisher[4-5], and D.El Moutawaki[1]. The new strict contractive condition are given for selfmappings in complete metric space X to guarantee that selfmappings on X have a unique common fixed point and the iterative methods for computing their common fixed point is given. The followings are the main result of this study.

1. Let (X, d) be a complete metric space and let $T : X \to X$. Suppose that there exists a mapping $\Phi : X \to \mathbb{R}^+$ such that

(i) $d(x, Tx) \leq \Phi(x) - \Phi(Tx), \forall x \in X,$

(ii) $d(Tx, Ty) < \max\{d(x, y), c_1 d(x, Ty) + c_2 d(y, Tx)\}, \forall x \neq y \in X,$

where $c_1 > 0, c_2 > 0$ and $c_1 + c_2 = 1$ Then T has a unique fixed point.

2. Let S and T be two weakly compatible selfmapping of a metric space (X, d) such that

(1) T and S satisfy the property(E.A),

(2) $d(Tx, Ty) < \max\{d(Sx, Sy), c_1 d(Tx, Sx) + c_2 d(Ty, Sy), d(Tx, Sy)\},$
$\forall x \neq y \in X$, where $0 \leq c_1 < 1$ and $0 \leq c_2 < 1$.

(3) $TX \subset SX$.

If SX or TX is a complete subspace of X, then T and S have a unique commom fixed point.

3. Let S and T be two weakly compatible selfmapping of a metric space (X, d) such that

(1) T and S satisfy the property(E.A),

(2) $d(Tx,Ty) < \max\{d(Sx,Sy), c[d(Tx,Sx)+d(Ty,Sy)], d(Tx,Sy)\}$,
$\forall x \neq y \in X$, where $0 \leq c < 1$.

(3) $TX \subset SX$.

If SX or TX is a complete subspace of X, then T and S have a unique commom fixed point.

4. Let S and T be two weakly compatible selfmapping of a metric space (X,d) such that

(1) T and S satisfy the property(E.A),

(2) $d(Tx,Ty) < \max\{d(Sx,Sy), cd(Tx,Sx), d(Tx,Sy)\}$,
$\forall x \neq y \in X$, where $0 \leq c < 1$.

(3) $TX \subset SX$.

If SX or TX is a complete subspace of X, then T and S have a unique commom fixed point.

5. Let S and T be two weakly compatible selfmapping of a metric space (X,d) such that

(1) T and S satisfy the property(E.A),

(2) $d(Tx,Ty) < \max\{d(Sx,Sy), cd(Ty,Sy), d(Tx,Sy)\}$,
$\forall x \neq y \in X$, where $0 \leq c < 1$.

(3) $TX \subset SX$.

If SX or TX is a complete subspace of X, then T and S have a unique commom fixed point.

6. Let (X,d) be a complete metric space and let $S,T : X \to X$ are commuting mappings satisfying the inequality

$$d(Sx,Sy) \leq F(\max\{d(Tx,Ty), d(Tx,Sx), d(Ty,Sy), d(Ty,Sx)\}), \forall x,y \in X$$

where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is a nondecreasing continuous function such that $F(t) < t$ for each $t > 0$.

If $SX \subset TX$ and T is continuous then S and T have a unique common fixed point. Moreover, if x is the common fixed point of S and T, then for any $x_0 \in X,\ Sx_n \to x$ and $Tx_n \to x$ where (x_n) is the sequence given by $Sx_n = Tx_{n+1},\ n = 0, 1, 2, \ldots$

7. Let (X, d) be a complete metric space and let $S, T : X \to X$ are commuting mappings satisfying the inequality

$$d(Sx, Sy) \leq c \cdot \max\{d(Tx, Ty), d(Tx, Sx), d(Ty, Sy), d(Ty, Sx)\}), \forall x, y \in X,$$

where $0 \leq c < 1$. If $SX \subset TX$ and T is continuous then S and T have a unique common fixed point.

8. Let S be selfmapping of a complete metric space (X, d) satisfying the inequality

$$d(Sx, Sy) \leq F(\max\{d(x, y), d(x, Sx), d(y, Sy), d(y, Sx)\}), \forall x, y \in X$$

where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is a nondecreasing continuous function such that $F(t) < t$ for each $t > 0$. Then S has a unique fixed point. Moreover, for any $x_0 \in X$, (Sx_n) converges to the fixed point of S where $x_{n+1} = Sx_n, n = 0, 1, 2, \ldots$

9. Let (X, d) be a complete metric space and let $S, T, I, J : X \to X$ and S and I be commuting mappings and T and J be commuting mappings satisfying the inequality

$$d(Sx, Ty) \leq F(\max\{d(Ix, Jy), d(Ix, Sx), d(Jy, Ty)\}), \forall x, y \in X,$$

where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is nondecreasing continuous function such that $F(t) < t$ for each $t > 0$. If $TX \subset IX$ and $SX \subset JX$ and if one of S, T, I and J is continuous, then S, T, I and J have a unique common fixed point.

10. Let (X, d) be a complete metric space and let $S, T, I : X \to X$ and S and I be commuting mappings and T and I be commuting mappings satisfying the inequality

$$d(Sx, Ty) \leq F(\max\{d(Ix, Iy), d(Ix, Sx), d(Iy, Ty)\}), \forall x, y \in X,$$

where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is nondecreasing continuous function such that $F(t) < t$ for each $t > 0$. If $TX \subset IX$ and $SX \subset IX$ and if one of S, T and I is continuous, then S, T and I have a unique common fixed point.

11. Let S and T be mappings of a complete metric space (X, d) into itself satisfying the inequality

$$d(Sx, Ty) \leq F(\max\{d(x, y), d(x, Sx), d(y, Ty)\}), \forall x, y \in X,$$

where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is nondecreasing continuous function such that $F(t) < t$ for each $t > 0$. Then S and T have a unique common fixed point.

12. Let I and J be mappings of a complete metric space (X, d) onto itself satisfying the inequality

$$d(x, y) \leq F(\max\{d(Ix, Jy), d(Ix, x), d(Jy, y)\}), \forall x, y \in X,$$

where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is nondecreasing continuous function such that $F(t) < t$ for each $t > 0$. Then I and J have a unique common fixed point.

13. Let (X, d) be a complete metric space and let $S, T, I, J : X \to X$ and S and I be commuting mappings and T and J be commuting mappings satisfying the inequality

$$d(Sx, Ty) \leq F(\max\{d(Ix, Jy), d(Ix, Sx), d(Jy, Ty), \frac{1}{2}d(Ix, Ty), \frac{1}{2}d(Jy, Sx)\}),$$

for all $x, y \in X$, where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is nondecreasing continuous function such that $F(t) < t$ for each $t > 0$. If $TX \subset IX$ and $SX \subset JX$ and if one of S, T, I and J is continuous, then S, T, I and J have a unique common fixed point.

14. Let S and T be mappings of a complete metric space (X, d) into itself satisfying the inequality

$$d(Sx, Ty) \leq F(\max\{d(x, y), d(x, Sx), d(y, Ty), \frac{1}{2}d(x, Ty), \frac{1}{2}d(y, Sx)\}),$$

for all $x, y \in X$ where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is nondecreasing continuous function such that $F(t) < t$ for each $t > 0$. Then S and T have a unique common fixed point.

15. Let I and J be mappings of a complete metric space (X, d) onto itself satisfying the inequality

$$d(x,y) \leq F(\max\{d(Ix,Jy), d(Ix,x), d(Jy,y), \frac{1}{2}d(Ix,y), \frac{1}{2}d(Jy,x)\}),$$

for all $x, y \in X$ where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is nondecreasing continuous function such that $F(t) < t$ for each $t > 0$. Then I and J have a unique common fixed point.

16. Let (X, d) and (Y, d') be a complete metric space. If $T : X \to Y$ and $S : Y \to X$ satisfying the inequalities

$$d'(Tx, TSy) \leq F(\max\{d(x, Sy), d'(y, Tx), d'(y, TSy)\})$$

and

$$d(Sy, STx) \leq F(\max\{d'(y, Tx), d(x, Sy), d(x, STx)\})$$

for all $x \in X$ and for all $y \in Y$, where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is nondecreasing continuous function such that $F(t) < t$ for each $t > 0$ and if there is $x \in X$ such that the sequence (y_n), define by $y_n = T(ST)^{n-1}x$ converges, then ST has a unique fixed point in X and TS has a unique fixed point in Y.

17. Let (X, d) be a complete metric space. If $S, T : X \to Y$ satisfying the inequalities

$$d(Tx, TSy) \leq F(\max\{d(x, Sy), d(y, Tx), d(y, TSy)\})$$

and

$$d(Sy, STx) \leq F(\max\{d(y, Tx), d(x, Sy), d(x, STx)\})$$

for all $x \in X$ and for all $y \in Y$, where $F : \mathbb{R}^+ \to \mathbb{R}^+$ is nondecreasing continuous function such that $F(t) < t$ for each $t > 0$ and if there is $x \in X$ such that the sequence (y_n), define by $y_n = T(ST)^{n-1}x$ converges then ST has a unique fixed point and TS has a unique fixed point. Further if fixed point of ST is fixed point of TS, then S and T has a unique fixed point.