CHAPTER 1

INTRODUCTION

In [2], Agarwal and Grace studied the asymptotic stability of the equilibrium $x = 0$ of the neutral system

$$x^{'}(t) + Cx'[t-\tau] = Ax(t) + Bx[t-\tau] \qquad (*)$$

where $x(t) \in \Re^n$ is the state vector, τ is a positive constant time-delay, $A \in \Re^{n\times n}$, $B \in \Re^{n\times n}$ and $C \in \Re^{n\times n}$ are constant system matrices. They then have:

Theorem 1.0.1 [2] *The equilibrium $x = 0$ of (*) is asymptotic stability if there exist a positive definite matrix P and a matrix $\Lambda = diag\{\lambda_1, \ldots, \lambda_n\}$, $\lambda_i > 0$, $i = 1, 2, \ldots, n$ such that for any positive constants α, β with $\alpha + \beta = 1$, the following matrix*

$$F = A^TP + PA + \Lambda + \frac{1}{\alpha}K^TH^{-1}K + \frac{1}{\beta}M^TH^{-1}M$$

is negative definite, where $H = \Lambda - B^TPC - C^TPB$ is positive definite, $K = C^TPA$, and $M = B^TP$.

They also studied the asymptotic stability of the equilibrium $x = 0$ of the Hopfiled neural network

$$x^{'}(t) + Cx'[t-\tau] = Ax(t) + BT(x[t-\tau])x[t-\tau] \qquad (**)$$

where $x(t) \in \Re^n$ is the state vector, τ is a positive constant time-delay, $A = diag\{a_1, ..., a_n\}$, $a_i > 0$, $i = 1, 2, ..., n$, $B \in \Re^{n\times n}$ and $C \in \Re^{n\times n}$ are constant system matrices, $T(x) = diag\{\sigma_1(x_1), ..., \sigma_n(x_n)\}$, $\sigma_i(x_i) = \frac{s_i(x_i)}{x_i}$. They then have:

Theorem 1.0.2 [2] *The equilibrium $x = 0$ of (**) is asymptotic stability if there exist a positive definite matrix P and a matrix $\Lambda = diag\{\lambda_1, \ldots, \lambda_n\}$, $\lambda_i > 0$, $i = 1, 2, \ldots, n$ such that for any positive constants α, β with $\alpha + \beta = 1$, the following matrix*

$$F_1 = A^TP + PA + \Lambda + \frac{1}{\alpha}K^T\Lambda^{-1}K + \frac{1}{\beta}W^T\Lambda^{-1}W$$

is negative definite, where $W = PBT(x[t-\sigma])$, and $K = C^TPA$.

In this paper, we study extend (*) to neutral system of the from

$$x^{'}(t) + Cx^{'}[t-\tau] = Ax(t) + Bx[t-\sigma] + f(t, x(t), x[t-\tau], x[t-\sigma]) \qquad (1.1)$$

where $x(t) \in \Re^n$ is the state vector, τ and σ are positive constant time-delays, $A \in \Re^{n\times n}$, $B \in \Re^{n\times n}$ and $C \in \Re^{n\times n}$ are constant system matrices and $f(t, x(t), x[t-\tau], x[t-\sigma])$ is a nonlinear perturbation.
We also study extend (**) to the Hopfiled neural network of the from

$$\begin{aligned} x^{'}(t) + Cx'[t-\tau] &= Ax(t) + BT(x[t-\sigma])x[t-\sigma] \\ &\quad + f(t, x(t), x[t-\tau], x[t-\sigma]) \end{aligned} \qquad (1.2)$$

where $x(t) \in \Re^n$ is the state vector, τ and σ are positive constant time-delays, $A = diag\{a_1, ..., a_n\}$, $a_i > 0$, $i = 1, 2, ..., n$, $B \in \Re^{n\times n}$ and $C \in \Re^{n\times n}$ are constant system matrices, $T(x) = diag\{\sigma_1(x_1), ..., \sigma_n(x_n)\}$, $\sigma_i(x_i) = \frac{s_i(x_i)}{x_i}$ and $f(t, x(t), x[t-\tau], x[t-\sigma])$ is a nonlinear perturbation, where s_i is monotonically increasing for $i = 1, 2, ..., n$.

Therefore, the purpose of this paper is to establish sufficient conditions for the asymptotic stability of the equilibrium $x = 0$ of (1.1) and (1.2).

Moreover, theorems which are obtained will be verified by numerical simulations.