CHAPTER 1
INTRODUCTION

According to the classical beam theory, if $u = u(x)$ denotes the configuration of the deformed beam, then the bending moment satisfies $M = -EIu''$, where E is the Young modulus of elasticity and I is the moment of inertia. If force f load over beam length L, then $f = -v$ and $v = M' = -EIu''$, where v is the shear force. We have the beam equation.

$$EIu^{(4)}(x) = f, \qquad x \in (0, L). \tag{1.1}$$

If we set $L = 1$ and $EI = 1$, then the deformations of an elastic beam in equilibrium state, whose two ends simply supported, are described by the following fourth-order boundary value problem.

$$u^{(4)}(x) = f(x, u(x), u''(x)), \qquad x \in (0, 1). \tag{1.2}$$

$$u(0) = u(1) = u''(0) = u''(1) = 0 \tag{1.3}$$

This problem has been studied by several authors. In [1,8] Aftabizadeh et al. proved the existence of positive solution to (1.2)-(1.3) under some growth conditions of f and a non-resonance condition involving a two-parameter linear eigenvalue problem. All of these results are based upon the Leray-Schauder continuation method and topological degree. Recently Bai [3] and Li [11] showed the existence results with fixed point theory for the problem (1.2)-(1.3)

$$u^{(4)}(x) + \beta u''(x) - \alpha u(x) = f(x, u(x)), \qquad x \in (0, 1). \tag{1.4}$$

with the boundary condition (1.3)

The method of lower and upper solutions has been studied for the fourth-order differential equation with boundary condition (1.3) by several authors; see Cabada [6], Schroder [16] and Ma [12]. Recently, Bai [2] developed the monotone method for the problem (1.2)-(1.3).

The main purpose of this work is to study the boundary value problem (1.2)-(1.3) and we change (1.2) to the equation

$$Lu = u^{(4)}(x) - au''(x) + bu(x) = f_1(x, u(x), u''(x)), \quad x \in (0,1). \tag{1.5}$$

where

$$f_1(x, u(x), u''(x)) = f(x, u(x), u''(x)) - au''(x) + bu(x), \tag{1.6}$$

$$L = D^4 - aD^2 + b \quad , \quad D^n = \frac{d^n}{dx^n} \tag{1.7}$$

We assume the following conditions throughout.

(H1) $f : [0,1] \times \mathbf{R} \times \mathbf{R} \longrightarrow \mathbf{R}$ is continuous.

(H2) $a, b \in \mathbf{R}$, $a > -\pi^2$, $\sqrt{a^2 - 4b} \geq 0$, $\frac{a}{\pi^2} + \frac{b}{\pi^4} + 1 > 0$ and $b \leq 0$ for $-2\pi^2 < a < 0$.

Assumption (H2) involves a two-parameter non-resonance condition. Then we develope the monotone method to show the existence of solutions for the problem (1.5) and (1.3) between a lower solution β and an upper solution α.

It is clear that if β, α are lower and upper solutions of the problem (1.2)-(1.3), respectively, then they are also lower and upper solutions of the problem (1.5) and (1.3) too.