CHAPTER 1

INTRODUCTION

The study of fixed point theorem satisfying some contractive conditions has been at the center of vigorous research activity and a number of interesting results have been obtained by various authors.

There have been numerous extension of Banach's principle which asks only that the fixed point be unique and that the Picard iterates of the mapping always converge to the fixed point.

The first such generalization to receive significant attention the following result:

In 1969, Boyd and Wong [2], proved that if M is complete metric space and suppose $f : M \to M$ satisfies

$$d(f(x), f(y)) \leq \psi(d(x, y)) \qquad \text{for each} \qquad x, y \in M$$

where $\psi : R^+ \to [0, \infty)$ is upper semi-continuous from the right and satisfies $0 \leq \psi(t) < t$ for $t > 0$. Then f has a unique fixed point , $\bar{x}$, and $f^n(x)$ converges to $\bar{x}$ for each $x \in M$.

It is well known that in the setting of metric space, strict contractive condition do not ensure the existence of fixed point unless the space is assumed compact or the strict condition are replaced by stronger conditions as in [5] and [9].

In 1986 Jungck [6], introduced the notion of compatible maps. This concept was frequently used to prove existence theorem in common fixed point theory.

In 2002,M.Aamri and D.El Moutawaki [1], gave some new common fixed point theorems under strict contractive conditions for mappings satisfying property (E.A).They proved the following result : If S and T are two weakly compatible selfmappings of a metric space (X, d) such that

(i) T and S satisfy the property (E.A),

(ii) $d(Tx, Ty) < \max\{d(Sx, Sy), [d(Tx, Sx) + d(Ty, Sy)]/2,$
$[d(Ty, Sx) + d(Tx, Sy)]/2\}, \forall x \neq y \in X,$

(iii) $TX \subset SX$.

If SX or TX is a complete subspace of X, then T and S have a unique common fixed point.

The purpose of this study is to extend many concepts and results of fixed point of one selfmapping and common fixed point of two selfmappings in metric spaces.These results generalize those in [1] and [10].

This thesis contains 4 chapters. The introduction of this research is in Chapter 1. In Chapter 2, we give some definitions, notations and results of metric spaces, fixed point of selfmappings and common fixed point of selfmappings in metric spaces. The main result of the research is in Chapter 3 and the conclusion is in Chapter 4.