CHAPTER 5

CONCLUSION

In this work, we study conditions on the parameters that make the equilibrium points of (3.1), (3.2) asymptotically stable and conditions that make the equilibrium points of (3.1), (3.2) unstable. The results are summarized as follows:

1. The Perturbed Lü Chaotic Dynamical System.

Theorem 3.1.1 The equilibrium point $E_1 = (0, 0, 0)$ is

(i) asymptotically stable if $a > c$ and $b > c$.

(ii) unstable if $a > c$ and $b < c$.

Theorem 3.1.2 The equilibrium point $E_2 = (\beta, \beta, c)$ is

(i) asymptotically stable if $a > 4c$.

(ii) unstable if $2c > a$, $c > b$ and $a + b > c$.

Theorem 3.1.3 The equilibrium point $E_3 = (-\beta, -\beta, c)$ is

(i) asymptotically stable if $a > 4c$.

(ii) unstable if $2c > a$, $c > b$ and $a + b > c$.

Theorem 3.1.4 The equilibrium point $E_1 = (0, 0, 0)$ is

(i) asymptotically stable if $a > c$ and $b > c$.

(ii) unstable if $a > c$ and $b < c$.

Theorem 3.1.5 The equilibrium point $E_2 = (x_1, x_1, c)$ is

(i) asymptotically stable if $a > 2c$, $b > \sqrt{x_1^2 + x_1 dcos(x_1)}$ and $b > c$.

(ii) unstable if $2c > a$, $b < \sqrt{x_1^2 + x_1 dcos(x_1)}$ and $b < c$.

Theorem 3.1.6 The equilibrium point $E_3 = (x_2, x_2, c)$ is

(i) asymptotically stable if $a > 2c$, $b > \sqrt{x_2^2 + x_2 dcos(x_2)}$ and $b < c$.

(ii) unstable if $2a > c$, $b < \sqrt{x_2^2 + x_2 dcos(x_2)}$, $a < \sqrt{bc}$ and $b < c$.

We have presented two methods for controlling chaos of the perturbed L**ü** chaotic dynamical system (3.1) and (3.2). Both methods, feedback control and bounded feedback control, suppress the chaotic behavior of system (3.1) and (3.2) to one of the three steady states E_1, E_2 and E_3.

2 Controlling Chaos of Perturbed Lü System to Equilibrium Point

2.1 Feedback Control Method

Theorem 3.2.1 The equilibrium point $E_1 = (0, 0, 0)$ is asymptotically stable if $k_{11} = 0$, $k_{33} > 0$ and $k_{22} > c$.

Theorem 3.2.2 The equilibrium point $E_2 = (\beta, \beta, c)$ is asymptotically stable if $k_{11}, k_{33} > 0$ and $k_{22} > c$.

Theorem 3.2.3 The equilibrium point $E_3 = (-\beta, -\beta, c)$ is asymptotically stable if $k_{11}, k_{33} > 0$ and $k_{22} > c$.

Theorem 3.2.4 The equilibrium point $E_1 = (0, 0, 0)$ is asymptotically stable if $k_{11} = 0$, $k_{33} > 0$ and $k_{22} > c$.

Theorem 3.2.5 The equilibrium point $E_2 = (x_1, x_1, c)$ is asymptotically stable if $k_{11}, k_{33} > 0$ and $k_{22} > c$.

Theorem 3.2.6 The equilibrium point $E_3 = (x_2, x_2, c)$ is asymptotically stable if $k_{11}, k_{33} > 0$ and $k_{22} > c$.

2.2 Bounded Feedback Control Method

Theorem 3.2.7 The equilibrium point $E_1 = (0, 0, 0)$ is asymptotically stable if $k > \frac{c}{a}$.

Theorem 3.2.8 The equilibrium point $E_2 = (\beta, \beta, c)$ is asymptotically stable if $k > 2$.

Theorem 3.2.9 The equilibrium point $E_3 = (-\beta, -\beta, c)$ is asymptotically stable if $k > 2$.

Theorem 3.2.10 The equilibrium point $E_1 = (0, 0, 0)$ is asymptotically stable if $k > \frac{c}{a}$.

Theorem 3.2.11 The equilibrium point $E_2 = (x_1, x_1, c)$ is asymptotically stable if $k > \frac{2x_1 + dcos(x_1)}{x_1}$.

Theorem 3.2.12 The equilibrium point $E_3 = (x_2, x_2, c)$ is asymptotically stable if $k > \frac{2x_2 + dcos(x_2)}{x_2}$.

3. Synchronization of Perturbed Lü Chaotic Dynamical System.

3.1 Adaptive Synchronization of Perturbed Lü Chaotic Dynamical System.

Theorem 4.1.1 Suppose that $|x| \leq S < +\infty$, $|y| \leq S < +\infty$,

$|z| \leq S < +\infty$, γ, θ, β, α are positive constants. When k_1, k_2 and $k_3 \geq 0$ are properly chosen so that the following matrix inequality holds,

$$P = \begin{bmatrix} \beta(k_1 + a) & -\frac{1}{2}(\beta a - S) & -S \\ -\frac{1}{2}(\beta a - S) & k_2 & 0 \\ -S & 0 & k_3 + b \end{bmatrix} > 0$$

or k_1, k_2 and k_3 can be chosen so that the following inequalities hold:

$$\begin{aligned} (i) \;\; A &= \beta(k_1 + a)k_2 - \frac{1}{4}(\beta a - S)^2 > 0 \\ (ii) \;\; B &= A(k_3 + b) - k_2 S^2 > 0, \end{aligned}$$

then the two perturbed Lü systems (4.3) and (4.4) can be synchronized under the adaptive control of (4.5) and (4.6)

3.2 Synchronization via Pecora and Carroll method.

The synchronization errors by

$e_x = x_2 - x_1$ and $e_z = z_2 - z_1$.

Using this notation and subtracting system (4.11) from system (4.12), we obtain the error system

$$\begin{aligned} \dot{e}_x &= -ae_x, \\ \dot{e}_z &= (y_1 + \frac{f(x_2) - f(x_1)}{x_2 - x_1})e_x - be_z. \end{aligned}$$

The linear system of synchronization error has two negative eigenvalues $\lambda_1 = -a$ and $\lambda_2 = -b$ and this implies that the zero solution of system satisfies

$\|e_x\| \to 0$ as $t \to +\infty$ and $\|e_z\| \to 0$ as $t \to +\infty$ Hence the zero solution of the error system is asymptotically stable and then the response system with $y-$drive configuration achieves synchonization.

Remark 1. Only one choice induces the appearance of chaos synchronization, namely (x, z) driven by y. For the other possible choices (x, y) driven by z or (y, z) driven by x, the Pecora and Carroll scheme did not succeed in achieving synchronization.

3.3 Synchronization via one-way coupling.

Theorem 4.3.1 The two Perturbed Lü system (3.27)and (3.30) are synchronized if the feedback control parameter k is chosen so that $k \geq max(k_1, k_2)$ where,

$$k_1 = min_\mu \left(c + \frac{(\mu a + S)^2}{4a\mu} \right), \quad k_2 = min_\mu \left(c + \frac{b(\mu a + S)^2}{4ab\mu - (S + \left| \left| \frac{f(x_2)-f(x_1)}{x_2-x_1} \right| \right|_S)^2} \right)$$

and

$$\mu > \frac{(S + \left| \left| \frac{f(x_2)-f(x_1)}{x_2-x_1} \right| \right|_S)^2}{4ab}$$

where $f : \mathbb{R} \to \mathbb{R}$ is satisfied Lipschitz condition, $|f(x_2) - f(x_1)| \leq c|x_2 - x_1|$, $\forall x_1, x_2 \in \mathbb{R}$, c is positive constant, or differentiable.