CHAPTER 2

PRINCIPLE AND THEORY

Much research has been conducted on using the electrical resistivity method with various electrode configurations in the field, but relatively very little effort has been expended on making resistivity measurements of soil sample in the laboratory. Laboratory measurements are necessary to enhance the effectiveness and credibility of using this method.

In an attempt to interpret the complexities of field-based resistivity investigations, a laboratory-based electrical resistivity measurement of clay rich contaminated sediment was made. The important principles and theory concerning both field resistivity investigations and laboratory electrical measurements will be shown in this chapter.

2.1 Archie's equation

Resistivity is one of the most variable of the physical properties of geological materials. Most rock forming minerals are insulators, and electrical current is carried through a rock mainly via ions in pore waters. Thus most rocks conduct electricity by electrolyte rather than electronic processes. It follows that porosity is the major control of resistivity in rocks and soils.

Conduction of electricity through porous media occurs by two mechanisms. The primary mode of conduction is by movement of ions through the bulk saturating electrolyte. Conduction also takes place as dissolved ions move along the surfaces of pores and cracks. Thus, conductivity of a porous medium is related to microstructural properties such as porosity, pore geometry, and surface morphology of the mineral grains lining the pores, as well as the dielectric properties of the mineral grains and pore fluid. Previous theoretical and experimental works have investigated these relationships.

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A pioneering work in electrical methods was carried out by Archie (1942) who derived the first general relationship between the electrical resistivity of a saturated soil and the resistivity of its pore fluid. Archie's equation assumes that bulk resistivity is directly related to pore fluid resistivity and the pore structures themselves. An equation commonly used to related bulk resistivity to pore fluid resistivity

$$\rho_{bulk} = \rho_{fluid}\, a\, \varphi^{-m} S_w^{-n} \qquad (2.1)$$

where ρ_{bulk} is the bulk resistivity of the rock (ohm-m)

S_w is the saturation index

ρ_w is the pore liquid resistivity (ohm-m)

φ is the fractional porosity

a is the pore geometry factor

m is the cementation factor

n is the saturation exponent

a, m and n are constants for the type of rock or sediment being considered (Figure 2.1). The empirical values for these parameters were derived by Archie (1942). A value of the saturation exponent n is approximately 2 with the cementation factor m ranging between 1.3 and 1.4 proposed for clean, unconsolidated sandstones.

Considering other kinds of rocks, other researchers measured values where a ranges from 0.55 to 2.26, n varies between 1.7 and 2.2 (Pearson *et al.*,1983), and m can have values between 1 and 3 (Labo, 1987).

The coefficients a and m are determined by plotting the bulk resistivity versus the measured porosity and fitting an exponential curve to these data.
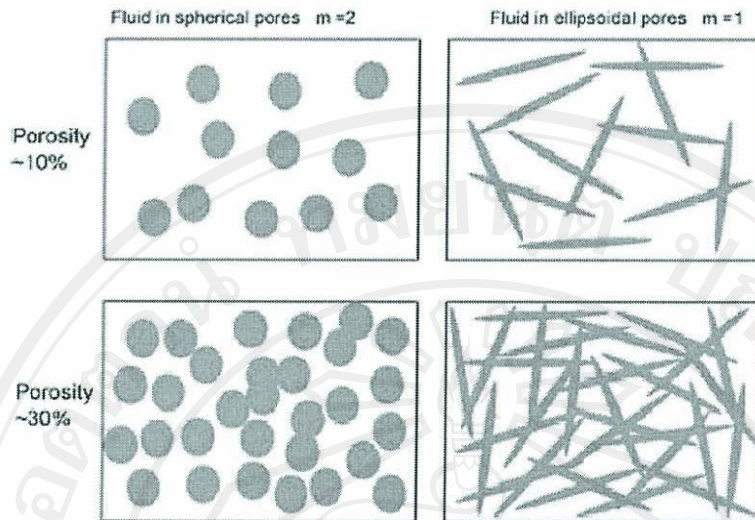
Figure 2.1 Physical explanation of the cementation factor, *m* (Glover, *et al.*, 2000).
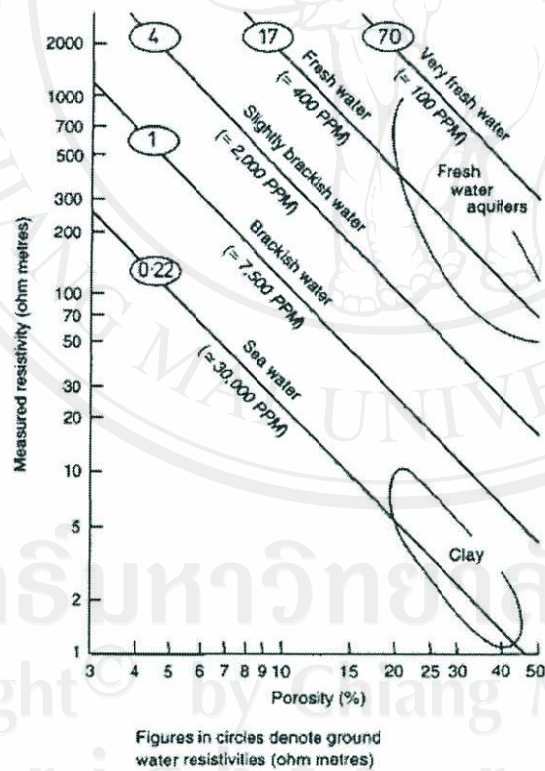

Figure 2.2 Effects of groundwater salinity and porosity on the measured resistivity. These graphs show theoretical results when ρ_w are in circle (in ohm-m). (modified from Schlumberger, 1977).

However, these relationships   are valid only for clean formations (clay-free) under high salinity conditions. The simple relationships given by Archie's equation explain only the primary conduction by the flow of ions through the electrolyte in the pore. If a rock contains clay minerals, then an extra conduction pathway is possible via the electrical double-layer that forms at the interface of the clay mineral and the water. This effectively allows ions to move through the system with a lower effective viscosity than in the liquid phase. This layer carried extra charges adjacent to the mineral surfaces that are able to conduct current when an electrical potential is applied, and gives rise to excess conductivity. The negative charge on the surfaces of the clay minerals is caused by broken bonds and lattice substitutions and the magnitude from these kind of surface effects is much larger than with other rock forming mineral (Figure 2.3). The size of the surface charge in clay is dependent on clay structure and degree of crystallization (Taylor and Baker, 2006).

In order to improvement Archie's equation by accounting for the presence of access conductivity in rocks, Waxman and Smits (1968) modified the Archie's equation to include a surface – conduction term. Their model is based on the assumption that rock or soil can be represented as two parallel resistors. One represents the resistance due to the free electrolyte in the pore space of the rock and the other resistance due to clay minerals within the rock. The second term representing the resistance of clay minerals is expressed using physical properties such as CEC per unit volume and the equivalent ionic conductance of clay exchange ions.
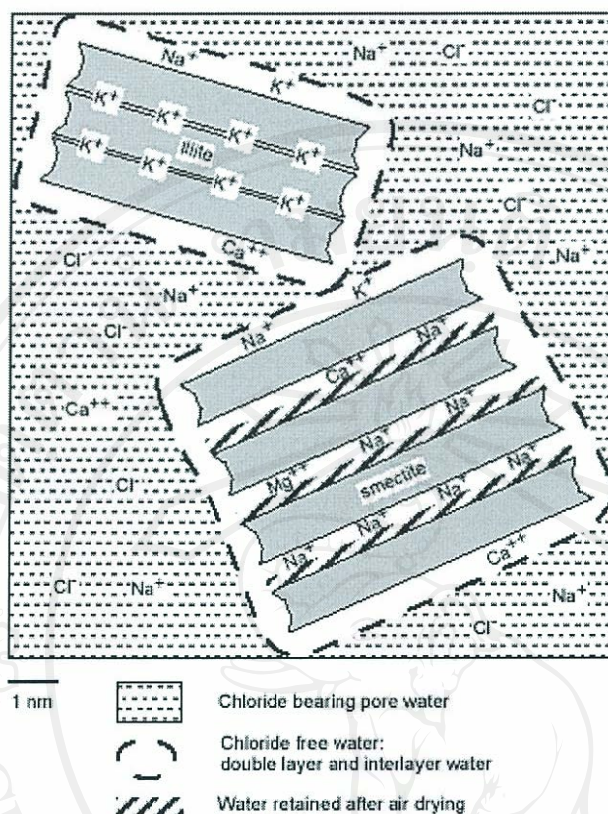
Figure 2.3 Schematic representation of the water and ion distribution in a clay sample. The 1-nm scale applies to the thickness of the clay layers, the interlayers, and the layer of chloride exclusion around the particles (double layer). In illites, all the exchangeable cations are adsorbed on the external surfaces of the particle. In smectites, only a small fraction of the exchangeable cations are in the double layers, most are in the hydrated interlayers (modified from Henry, 1997).

The Waxman-Smits model have proved to be appropriate for describing materials saturated both fully or partially with low salinity electrolytes and where the matrix material is conductive (Taylor and Baker, 2006).

In order to apply Archie's theory, all of the parameters that are needed in the equation must be measured before the empirical data can be fitted to the theoretical curve. The first necessary parameter is the fractional porosity (φ) of sediment; this parameter must be controlled by making sure that the sediment samples are well packed in the resistivity box so that the same porosity value is measured each time.

The bulk resistivity of the sediments (ρ_{bulk}) can be measure by varying the pore liquid resistivity (ρ_w) and the saturation degree (S_w). The explanations of each the parameter needed in Archie's eqaution will be discussed below.

2.2 Ohm's law

Resistivity measurements obey Ohm's law. In any electrical system, there are three equally important fundamental quantities: voltage, current and resistance. In any electrical system the pressure is the applied voltage, and the result is the flow of charge or current. The resulting level of current is controlled by the resistance of the system. The greater the resistance, the less the current, and vice versa. This relationship is explained by the most fundamental law of electrical circuits: Ohm's law. If the resistance is constant over a considerable range of voltage, then Ohm's law

$$I = \frac{V}{R} \tag{2.2}$$

where   I   is the current (ampere, A)
            V   is the voltage (volt, V)
            R   is the resistance (ohm, Ω)

This Ohm's law can be used to predict the behavior of the material. Although the definition above involves DC current and voltage, the same equation holds for the active currents.

Whether or not a material obeys Ohm's law, its resistance can be described in terms of its bulk resistivity. The resistivity, and thus the resistance, is temperature dependent. At any given temperature, the resistivity can be calculated by the resistance of a material and the materials geometry.
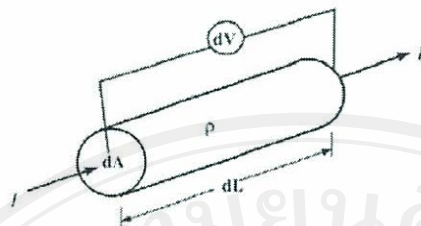
Figure 2.4 The parameters used in defining resistivity (modified from Kearey and Brooks, 1991).

At a temperature of 20°C, the resistivity of a material is defined as the resistance in ohm between the opposite faces of a unit cube of the material. For the conducting cylinder with resistance R, length dL and cross-sectional area dA (Figure 2.4) the resistivity ρ is given by the following Equation (2.3):

$$\rho = \frac{RdA}{dL} \qquad (2.3)$$

The SI units of resistivity are ohm-meters (Ω m) and the reciprocal of the resistivity is the conductivity (units: siemens(S) per meter; $1Sm^{-1} = 1\Omega^{-1}m^{-1}$). The dimension of length and area can be considering as a "geometry factor".

In this experiment the resistance of the material in the resistivity box was measure by applying Ohm's law. The resistivity was then calculated by applying the "geometry factor" of the box. Because the box is cylindrical in shape and the electrode is placed on the curved part of the cylinder, the geometry factor is not easy to calculate directly from the dimensions of the box. Instead, the geometry factor was measured by calibrating the box with the known resistivity of a standard solution. The details of the resistivity box calibration are included below in Chapter 3.

2.3 The resistivity measurement

Using an experimental configuration where the two current electrodes are placed relatively close to one another and using two potential electrodes place between the two current electrodes, an estimate of the resistivity of our homogeneous material can be made. The configuration of the four electrodes and its signal contribution is shown below in Figure 2.5. Let the distances between the four electrodes be given by $R_1, R_2, R_3,$ and R_4 as shown in the Figure 2.6.
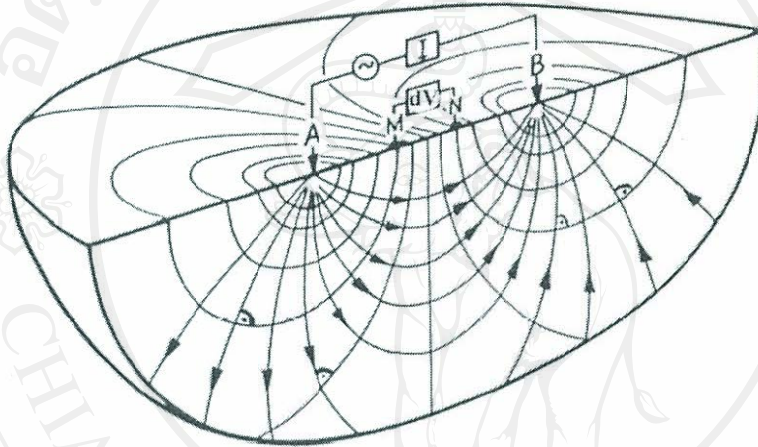

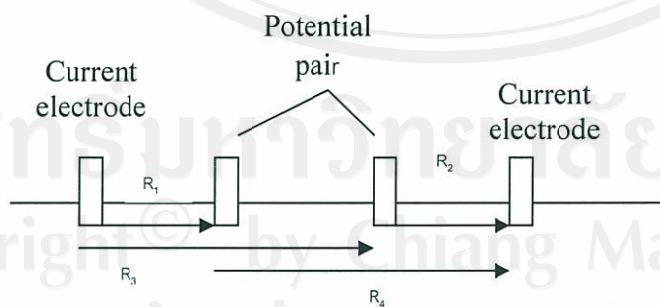Figure 2.5 The common four electrode array and its signal contribution (modified from Dobrin, 1960).

Figure 2.6 Distances between the four electrodes

Knowing the locations of the four electrodes, and by measuring the amount of current input into the ground, I, and the voltage difference between the two

potential electrodes, V, we can compute the resistivity of the medium using the following equation.

$$\rho = 2\pi \frac{V}{I} \left[\frac{1}{\dfrac{1}{r_1} - \dfrac{1}{r_4} - \dfrac{1}{r_3} + \dfrac{1}{r_2}} \right] \tag{2.4}$$

The resistivity computed using the equation given above is referred to as the *apparent resistivity* which will be exactly equal to the resistivity of the medium if the earth has a constant resistivity. If, however, the earth does not have a constant resistivity, the resistivity computed by the above equation may not represent the true resistivity of the earth.

Equation (2.4) can be derived by considering a single current electrode on the surface of the medium (Figure 2.7). The circuit is completed by a continuous current flowing radially from the electrode through an isotropic homogenous medium with resistivity ρ. The current distribution is uniform over hemispherical shells centered on the source. The displacement current (I) is passed through a surface (A) causing a potential drop $-\delta V$ between the ends of surface.

Invoking Ohm's law such that V= IR,

so
$$- dV = dRI \tag{2.5}$$

and from Equation (2.3)
$$dR = - \frac{\rho dL}{dA} \tag{2.6}$$

Replace (2.7) in (2.6)
$$dV = - \frac{\rho dLI}{dA} \tag{2.7}$$

so that
$$\frac{dV}{dL} = - \frac{\rho I}{dA} \tag{2.8}$$

Equation (2.8) represents the potential gradient through the surface element, so with a surface area of $2\pi r^2$, the potential gradient at a distance r is

$$\frac{dV}{dr} = -\frac{\rho I}{2\pi r^2} \qquad (2.9)$$

The potential V_r at distance r is then obtained by integration

$$V_r = \int dV = -\int \frac{\rho I dr}{2\pi r^2} = \frac{\rho I}{2\pi r} \qquad (2.10)$$

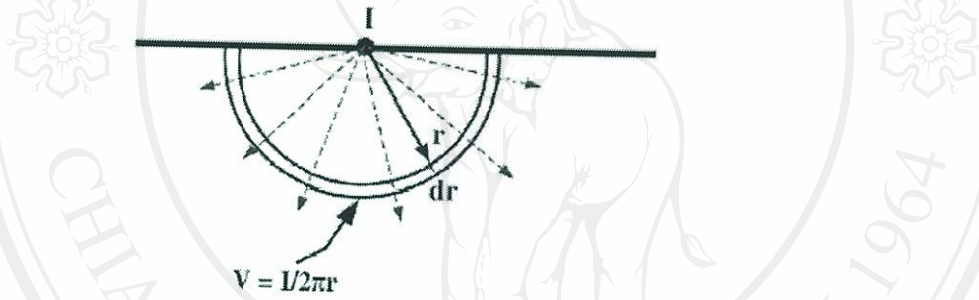The constant of integration is zero since $V_r = 0$ when $r = \infty$,

Figure 2.7 Current flow from a single source electrode (modified from Kearey and Brooks, 1991).

Now consider the case when the distance between the electrodes is finite with four electrodes in the configuration (Figure 2.6). The potential at a nearby surface point will be affected by both current electrodes. Finally, the difference in potential electrodes will be

$$\Delta V = \frac{I\rho}{2\pi}\left\{\left(\frac{1}{r_1}-\frac{1}{r_4}\right)-\left(\frac{1}{r_3}-\frac{1}{r_2}\right)\right\} \qquad (2.11)$$

Thus by Ohm's law, the Equation (2.11) can be rewritten as Equation (2.4).

In this four electrodes configuration, the voltage recorded by the voltmeter (V) is relatively small. The voltage will increase by moving the two potential

electrodes outward, closer to the two current electrodes. For a variety of reasons, some related to the reduction of noise and some related to maximizing the depth over which the measurements are sensitive, it is better not to move the potential and current electrodes closer together. Thus, a very sensitive voltmeter must be used that is able to record voltage differences down to mV.

2.4 Porosity and degree of saturation

Archie (1942) showed that porosity (φ) has a strong influence on the process of electric-current conduction through saturated porous media.

Porosity is measure of the density of a soil or in other words, the pore space ratio to the whole volume. Porosity is the sum of the aeration and water-filled porosity of soil. Usually individual grains of soil are linked into clusters and aggregated an intricate system of pore space.

Characterizations of the pore system are important in investigations of the storage and movement of water and gases and in investigation of soil strength. The geometry of the pore system of soil is very complex. Just as soil particles vary in size, shape, regularity, and swelling tendency, so the pores differ greatly from one another in size, shape, lateral dimensions, length, continuity, and other characteristics.

The porosity is defined as the ratio of void volume to total volume in the soil mass. Soil is a three phase system that includes solid particles, pore water, and air, so that the definition of porosity becomes:

$$\text{Porosity } \varphi = (V_l + V_g)/V_t \tag{2.12}$$

Where V_l, V_g, V_t is the volume of liquid, gas and the total or bulk volume of a quantity of soil, respectively. Note that $V_t = V_l + V_g + V_s$, where V_s is the volume of solid particle of soil.

The porosity changes depend on the degree of a sediment's compaction. Soil compaction occurs when the soil particles are pressed together, reducing the pore space between them. This increases the weight of solids per unit volume of soil (bulk density). Soil compaction may occur in response to pressure (weight per unit area) exerted by human or animal activity, but the risk for soil compaction is greatest when the soil is wet.

In order to measure the porosity of soil all three quantities in Equation (2.12) must be known.

Density of solid particles, $\quad \sigma_s = m_s / V_s$ $\hspace{2cm}$ (2.13)

Bulk density of soil, $\quad\quad\quad \sigma_b = m_s / V_t$ $\hspace{2cm}$ (2.14)

By combining the quantities above, Equation (2.12) can be written as

$$\varphi = (\sigma_s - \sigma_b) / \sigma_s \hspace{2cm} (2.15)$$

Soil particle density is a measure of the mass per unit volume of the soil solids only. Texture and structure do not affect particle density. However, organic matter, which is a soil solid, readily influences particle density. Organic matter weighs much less per unit volume than soil minerals. Soils high in organic matter have lower particle densities than soils similar in texture that are low in organic matter.

Particle density varies with the type of soil minerals present as well as the amount of organic matter. The particle density of most mineral soils is in the range of 2.60 to 2.75 g/cm^3. Particle density is used in the calculation of pore space and bulk density on a coarse fragment free basis. When unknown, particle density of mineral soils is assumed to be 2.65 g/cm^3. Generally quartz, feldspars, and colloidal silicates dominate the mineral fraction of soils. The particle density of these minerals averages about 2.65 g/cm^3. When large amounts of heavy minerals, such as hornblende or magnetite, are present, the soil particle density is greater than 2.65 g/cm^3 (Marshall and Holmes, 1981).

Methods for measuring particle density and bulk density are described by McIntyre and Loveday (1974) and Blake (1965). The most usual method for measuring bulk density of a soil is to take cylindrical core of known volume and find the mass of the dried soil.

The degree of saturation, S_W, is the ratio of the volume of pore water in the soil to the total volume of pore space.

$$S_W = \frac{V_l}{V_l + V_g} \hspace{2cm} (2.16)$$