CHAPTER 1

INTRODUCTION

Let R be a commutative ring with identity and V a unitary left R-*module*. A mapping $f : V \to V$ satisfying

(I) $f(rv) = rf(v)$ for all $r \in R$, $v \in V$, and

(II) $f(v_1 + v_2) = f(v_1) + f(v_2)$ for all $v_1, v_2 \in V$

is called an R-*endomorphism* of V, and the set of all R-*endomorphisms* is denoted by $\mathrm{End}_R(\mathrm{V})$. The set $\mathrm{End}_R(\mathrm{V})$ with the binary operations of function addition and function composite is a ring.

A mapping $f : V \to V$ is called *a homogeneous function* of V if it satisfies (I). The set of all homogeneous functions on V is denoted by $\mathrm{M}_R(\mathrm{V})$. This set with the operations of function addition and function composite is a near-ring. That is it satisfies all the ring axioms, except the left distributive law.

Always, we have $\mathrm{End}_R(\mathrm{V}) \subseteq \mathrm{M}_R(\mathrm{V})$.

In 2000, C. J. Maxson and J. H. Meyer [4] called a collection $\Im = \{W_\alpha | \alpha \in \mathcal{A}\}$ of proper submodules of V *to force linearity on V* if whenever $f \in \mathrm{M}_R(\mathrm{V})$ and f is linear on each $W_\alpha \in \Im$, then $f \in \mathrm{End}_R(\mathrm{V})$ and call the minimum number of proper submodules need to force linearity, the *forcing linearity number for V*, denote by $fln(V)$. They found that if V is a vector space over a field F then the following statements hold.

i) If $dim_F(V) = 1$, then $fln(V) = 0$.

ii) If $dim_F(V) = 2$, then $fln(V) = \infty$.

iii) If F is infinite and $dim_F(V) > 1$, then $fln(V) = \infty$.

iv) If F is finite with $|F| = q$ and $dim_F(V) > 2$, then $fln(V) = q + 2$.

Later in 2000, C. J. Maxson and J. H. Meyer [4] found that if R is a local ring, not an integral domain and $\mathrm{V} = \mathrm{R}^m$, $m \geq 2$, then $fln(V) = q + 2$ where $q = |\mathrm{R/M}|$, M is a maximal ideal of R.

The purpose of this thesis is to determine the forcing linearity numbers of free modules over a local ring and determine the forcing linearity numbers for

$V = \mathbb{Z}_{p^k}^{(\mathbb{N})}$ where $\mathbb{Z}_{p^k}^{(\mathbb{N})}$ is a free modules over $\mathbb{Z}_{p^k}$.

The thesis comprises of four chapters. Chapter I is for the introduction. In Chapter II, we list some well-known results, definitions and notations that will be used throughout this thesis. Chapter III, we determined the forcing linearity numbers of free modules over a local ring, forcing linearity numbers of $\mathbb{Z}_{p^k}^{(\mathbb{N})}$ where p is prime over $\mathbb{Z}_{p^k}$ and as example we determined the forcing linearity numbers of $\mathbb{Z}_2^{(\mathbb{N})}$ over $\mathbb{Z}_2$. Chapter IV is used for the conclusion.