CHAPTER 4

CONCLUSION

In This thesis, we study oscillation criteria for second order nonlinear neutral differential equations of the form (E). We provide sufficient conditions for oscillation of Eq.(E). The results are summarized as follows.

1. Some new oscillation criteria

Theorem 3.1.1 *Let $(H_1)-(H_5)$ and (S_1) be satisfied. Suppose that there exists a constant $\bar{p}$ such that $0 \le p(t) \le \bar{p} < 1$. If*

$$\int^{\infty}\Big\{R^{\alpha}(\sigma(t))q(t) - M\Big(\frac{\alpha}{\gamma p^*}\Big)^{\alpha}\Big(\frac{\alpha}{\alpha+1}\Big)^{\alpha+1}\frac{\sigma'(t)}{R(\sigma(t))r^{1/\alpha}(\sigma(t))}\Big\}dt = \infty,$$

where $p^* = 1-\bar{p}$, then Eq.(E) is oscillatory.

Theorem 3.1.2 *Let $(H_1)-(H_5)$ and (S_2) be satisfied. If*

$$\int^{\infty}\Big(\beta R^{\alpha}(\sigma(t))Q(t) - M\Big(\frac{\alpha}{\alpha+1}\Big)^{\alpha+1}\frac{\sigma'(t)}{R(\sigma(t))r^{1/\alpha}(\sigma(t))}\Big)dt = \infty,$$

where $Q(t) = q(t)\big(1-p(\sigma(t))\big)^{\alpha}$, then Eq.$(E)$ is oscillatory.

2. Philos's type oscillation criteria

Theorem 3.2.1 *Let $(H_1)-(H_5)$ and (S_1) be satisfied. Suppose that there exists a constant $\bar{p}$ such that $0 \le p(t) \le \bar{p} < 1$. If there exist function $H \in \mathcal{P}$ and positive function $\rho \in C^1\big([t_0,\infty);\mathbb{R}^+\big)$ such that*

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\Big\{H(t,s)\rho(s)q(s) - M\Big(\frac{1}{\alpha+1}\Big)^{\alpha+1}\Big(\frac{\alpha}{\gamma p^*}\Big)^{\alpha}G(t,s)\Big\}ds = \infty,$$

then Eq.(E) is oscillatory, where $p^* = 1-\bar{p}$ and

$$G(t,s) = \frac{\rho(s)r(\sigma(s))}{(\sigma'(s))^{\alpha}\big(\sqrt{H(t,s)}\big)^{\alpha-1}}\Big(h(t,s) + \frac{\rho'(s)}{\rho(s)}\sqrt{H(t,s)}\Big)^{\alpha+1}.$$

Theorem 3.2.2 *Let $(H_1)-(H_5)$ and (S_1) be satisfied. Suppose that there exists a constant $\bar{p}$ such that $0 \le p(t) \le \bar{p} < 1$ and there exists function $H \in \mathcal{P}$ such that*

$$0 < \inf_{s \ge t_0}\left[\liminf_{t\to\infty}\frac{H(t,s)}{H(t,t_0)}\right] \le \infty$$

and there exists a positive function $\rho \in C^1([t_0,\infty);\mathbb{R}^+)$ such that

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t} M\Big(\frac{1}{\alpha+1}\Big)^{\alpha+1}\Big(\frac{\alpha}{\gamma p^*}\Big)^{\alpha} G(t,s)ds < \infty.$$

If there exists a function $\phi \in C([t_0,\infty))$ such that for every $T \ge t_0$

$$\limsup_{t\to\infty}\frac{1}{H(t,T)}\int_{T}^{t}\Big\{H(t,s)\rho(s)q(s) - M\Big(\frac{1}{\alpha+1}\Big)^{\alpha+1}\Big(\frac{\alpha}{\gamma p^*}\Big)^{\alpha} G(t,s)\Big\}ds \ge \phi(T),$$

$$\int_{t_0}^{\infty}\frac{\sigma'(s)}{\big(\rho(s)r(\sigma(s))\big)^{\frac{1}{\alpha}}}\,(\phi_+(s))^{\frac{\alpha+1}{\alpha}}ds = \infty,$$

then Eq.(E) is oscillatory, where $p^* = 1-\bar{p}$, $\phi_+(s) = \max\{\phi(s),0\}$ and

$$G(t,s) = \frac{\rho(s)r(\sigma(s))}{(\sigma'(s))^{\alpha}\big(\sqrt{H(t,s)}\big)^{\alpha-1}}\Big(h(t,s) + \frac{\rho'(s)}{\rho(s)}\sqrt{H(t,s)}\Big)^{\alpha+1}.$$

Theorem 3.2.3 *Let $(H_1)-(H_5)$ and (S_1) be satisfied. Suppose that there exists a constant $\bar{p}$ such that $0 \le p(t) \le \bar{p} < 1$ and there exists a function $H \in \mathcal{P}$ such that (3.27) holds and there exists a positive function $\rho \in C^1([t_0,\infty);\mathbb{R}^+)$ such that*

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t} H(t,s)\rho(s)q(s)ds < \infty.$$

If there exists a function $\phi \in C([t_0,\infty))$ such that for every $T \ge t_0$

$$\liminf_{t\to\infty}\frac{1}{H(t,T)}\int_{T}^{t}\Big\{H(t,s)\rho(s)q(s) - M\Big(\frac{1}{\alpha+1}\Big)^{\alpha+1}\Big(\frac{\alpha}{\gamma p^*}\Big)^{\alpha} G(t,s)\Big\}ds \ge \phi(T)$$

and (3.30) hold, then Eq.(E) is oscillatory, where $p^* = 1-\bar{p}$ and

$$G(t,s) = \frac{\rho(s)r(\sigma(s))}{(\sigma'(s))^{\alpha}\big(\sqrt{H(t,s)}\big)^{\alpha-1}}\Big(h(t,s) + \frac{\rho'(s)}{\rho(s)}\sqrt{H(t,s)}\Big)^{\alpha+1}.$$

Theorem 3.2.4 *Let $(H_1)-(H_5)$ and (S_2) be satisfied. If there exist function $H \in \mathcal{P}$ and positive function $\rho \in C^1([t_0,\infty);\mathbb{R}^+)$ such that*

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\Big\{\beta H(t,s)\rho(s)Q(s) - M\Big(\frac{1}{\alpha+1}\Big)^{\alpha+1} G(t,s)\Big\}ds = \infty,$$

then Eq.(E) is oscillatory, where $Q(t) = q(t)\big(1 - p(\sigma(t))\big)^{\alpha}$ *and*

$$G(t,s) = \frac{\rho(s) r(\sigma(s))}{(\sigma'(s))^{\alpha}\big(\sqrt{H(t,s)}\big)^{\alpha-1}}\Big(h(t,s) + \frac{\rho'(s)}{\rho(s)}\sqrt{H(t,s)}\Big)^{\alpha+1}.$$

Theorem 3.2.5 *Let* $(H_1) - (H_5)$ *and* (S_2) *be satisfied. Suppose that there exists a function* $H \in \mathcal{P}$ *such that* (3.27) *holds and there exists a positive function* $\rho \in C^1\big([t_0, \infty); \mathbb{R}^+\big)$ *such that*

$$\limsup_{t\to\infty} \frac{1}{H(t,t_0)} \int_{t_0}^{t} M\Big(\frac{1}{\alpha+1}\Big)^{\alpha+1} G(t,s)\,ds < \infty.$$

If there exists a function $\phi \in C\big([t_0, \infty)\big)$ *such that for every* $T \geq t_0$

$$\limsup_{t\to\infty} \frac{1}{H(t,T)} \int_{T}^{t} \Big\{\beta H(t,s)\rho(s)Q(s) - M\Big(\frac{1}{\alpha+1}\Big)^{\alpha+1} G(t,s)\Big\}ds \geq \phi(T)$$

and (3.30) *holds, then Eq.(E) is oscillatory, where* $Q(t) = q(t)\big(1 - p(\sigma(t))\big)^{\alpha}$ *and*

$$G(t,s) = \frac{\rho(s) r(\sigma(s))}{(\sigma'(s))^{\alpha}\big(\sqrt{H(t,s)}\big)^{\alpha-1}}\Big(h(t,s) + \frac{\rho'(s)}{\rho(s)}\sqrt{H(t,s)}\Big)^{\alpha+1}.$$

Theorem 3.2.6 *Let* $(H_1) - (H_5)$ *and* (S_2) *be satisfied. Suppose that there exists a function* $H \in \mathcal{P}$ *such that* (3.27) *holds and there exists a positive function* $\rho \in C^1\big([t_0, \infty); \mathbb{R}^+\big)$ *such that*

$$\limsup_{t\to\infty} \frac{1}{H(t,t_0)} \int_{t_0}^{t} \beta H(t,s)\rho(s)Q(s)\,ds < \infty.$$

If there exists a function $\phi \in C\big([t_0, \infty)\big)$ *such that for every* $T \geq t_0$

$$\liminf_{t\to\infty} \frac{1}{H(t,T)} \int_{T}^{t} \Big\{\beta H(t,s)\rho(s)Q(s) - M\Big(\frac{1}{\alpha+1}\Big)^{\alpha+1} G(t,s)\Big\}ds \geq \phi(T)$$

and (3.30) *hold, then Eq.(E) is oscillatory, where* $Q(t) = q(t)\big(1 - p(\sigma(t))\big)^{\alpha}$ *and*

$$G(t,s) = \frac{\rho(s) r(\sigma(s))}{(\sigma'(s))^{\alpha}\big(\sqrt{H(t,s)}\big)^{\alpha-1}}\Big(h(t,s) + \frac{\rho'(s)}{\rho(s)}\sqrt{H(t,s)}\Big)^{\alpha+1}.$$