CHAPTER 1

INTRODUCTION

The operator L iterated k times and is defined by

$$L^k = \left(\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \cdots + \frac{\partial^2}{\partial x_p^2}\right)^2 + \left(\frac{\partial^2}{\partial x_{p+1}^2} + \frac{\partial^2}{\partial x_{p+2}^2} + \cdots + \frac{\partial^2}{\partial x_{p+q}^2}\right)^2\right)^k \quad (1.1)$$

where $p + q = n$ is the dimension of the space $\mathbb{R}^n$, $x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$, $i = \sqrt{-1}$ and k is a nonnegative integer.

Actually the operator L^k is an extension of the operator L_1^k and the operator L_2^k. So the operator L^k can be expressed as the product of the operator L_1 and L_2, that is $L^k = L_1^k L_2^k = L_2^k L_1^k$ where

$$L_1^k = \left(\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \cdots + \frac{\partial^2}{\partial x_p^2}\right) + i\left(\frac{\partial^2}{\partial x_{p+1}^2} + \frac{\partial^2}{\partial x_{p+2}^2} + \cdots + \frac{\partial^2}{\partial x_{p+q}^2}\right)\right)^k \quad (1.2)$$

and

$$L_2^k = \left(\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \cdots + \frac{\partial^2}{\partial x_p^2}\right) - i\left(\frac{\partial^2}{\partial x_{p+1}^2} + \frac{\partial^2}{\partial x_{p+2}^2} + \cdots + \frac{\partial^2}{\partial x_{p+q}^2}\right)\right)^k . \quad (1.3)$$

A. Kananthai, S. Suantai, V. Longani ([3], Lemma 2.4 p223) has shown that the convolution $(-1)^k(-i)^{\frac{q}{2}}S_{2k}(w) * (-1)^k(-i)^{\frac{q}{2}}T_{2k}(z)$ is an elementory solution of the operator L^k, that is

$$L^k\left((-1)^k(-i)^{\frac{q}{2}}S_{2k}(w) * (-1)^k(-i)^{\frac{q}{2}}T_{2k}(z)\right) = \delta \quad (1.4)$$

where δ is the Dirac-delta distribution and the functions $S_{2k}(x)$ and $T_{2k}(x)$ are defined by (2.5) and (2.1) respectively with $\gamma = \nu = 2k$, k is nonnegative integer.

In this thesis, we study the solution of the equation

$$L^k u(x) = f(x). \quad (1.5)$$

Let $K_{\gamma,\nu}(x)$ be a distributional family and is defined by

$$K_{\gamma,\nu}(x) = S_\gamma * T_\nu \quad (1.6)$$

where S_γ is defined by (3.59) and T_ν is defined by (3.60) and γ, ν are the complex parameters.

The family $K_{\gamma,\nu}(x)$ is well-defined and is a tempered distribution since $S_\gamma * T_\nu$ is a tempered sce ([1], Lemma 2.2) and T_ν has a compact support.

In this thesis, we can show that

$$u(x) = (-1)S_{2k}(x) * \left(T_{2(k-1)}(\nu)\right)^{(m)} + (-1)K_{2k,2k}(x) * f(x)$$

is a solution of (1.5) where $m = \frac{n-4}{2}, n \geq 4$ and n is even number and $K_{2k,2k}(x)$ is defined by (1.6) with $\gamma = \nu = 2k$. Moreover, we can show that the solution $u(x)$ relates to the solution of Laplace operator $\triangle^{2k}$ defined by (3.62).