CHAPTER 1

INTRODUCTION

Robust control problems for uncertain systems have been extensively studied for the past two decades. Different types of parametric uncertainty descriptions including norm bounded uncertainty, positive real uncertainty and polytopic uncertainty can be found in literature, [3], [7], [9], [10], [11] and [13].

In 1999, M. C. de Olivelra, J. Bernussou and J. C Geromel [9] studied the robust stability of linear discrete-time uncertain system described by

$$x_{k+1} = A(\alpha)x_k \tag{1.1}$$

where the dynamic matrix $A(\alpha)$ belongs to a convex polytopic set defined as

$$\mathcal{A} := \left\{ A(\alpha) : A(\alpha) = \sum_{i=1}^{n} \alpha_i A_i, \sum_{i=1}^{n} \alpha_i = 1, \alpha_i \geq 0 \right\}. \tag{1.2}$$

The following result are obtained.

Theorem 1.1 [9] *The following conditions are equivalent :*

(i) There exists a symmetric matrix $P > 0$ such that

$$A_i^T P A_i - P < 0.$$

(ii) There exist a symmetric matrix P and a matrix G such that

$$\begin{bmatrix} P & A^T G^T \\ GA & G + G^T - P \end{bmatrix} > 0.$$

Theorem 1.2 [9] *Uncertain system* (1.1) *is robustly stable in uncertainty domain* (1.2) *if there exist symmetric matrices P_i and G such that*

$$\begin{bmatrix} P_i & A_i^T G^T \\ GA_i & G + G^T - P_i \end{bmatrix} > 0 \quad \textit{for all} \ \ i = 1, 2, \ldots, N.$$

In 2001, D. C. W. Ramos and P. L. D. Peres [10] studied the robust stability of linear discrete-time uncertain system given by

$$x(t+1) = Ax(t) \tag{1.3}$$

where $x \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$ is not precisely known, but belongs to a convex bounded (polytopic type) uncertain domain $\mathcal{D}$

$$\mathcal{D} = \left\{ A(\alpha) : A(\alpha) = \sum_{i=1}^{n} \alpha_i A_i, \sum_{i=1}^{n} \alpha_i = 1, \alpha_i \geq 0 \right\}, \tag{1.4}$$

such that

$$A_i^T P A_i - P < -I, \quad i = 1, 2, \ldots, N.$$

The following lemma is the main result in their studied

Lemma 1.1 [10] *If there exist positive definite Lyapunov matrices* P_i , $i = 1, 2, \ldots, N$ *such that*

$$A_i^T P_i A_i - P_i < -I, \quad i = 1, 2, \ldots, N$$

$$A_i^T P_i A_j + A_j^T P_i A_i + A_i^T P_j A_i - 2P_i - P_j < \frac{1}{(N-1)^2} I, \quad i = 1, 2, \ldots, N, \quad i \neq j \ , \ j = 1, 2, \ldots, N$$

$$A_j^T P_i A_i + A_k^T P_i A_j + A_i^T P_j A_k + A_k^T P_j A_i + A_i^T P_k A_j + A_j^T P_k A_i - 2(P_i + P_j + P_k) < \frac{6}{(N+1)^2}, \quad i = 1, 2, \ldots, N-2, \quad j = i+1, 2, \ldots, N-1, \quad k = j+1, 2, \ldots, N$$

then the uncertain system (1.3) *with* $A(\alpha) \in \mathcal{D}$ *is robustly stable.*

In 2001, J. Daafouz and J. Bernussou [3] studied the robust stability of discrete-time systems with time varying parameteric uncertainties given by

$$x(k+1) = \mathcal{A}(\xi(k))x(k), \tag{1.5}$$

where $x \in \mathbb{R}^n$ is the state vector, ξ is unknown but bounded time-varying parameter. The structure of the dynamical matrix $\mathcal{A}$ is assumed to be the form

$$\mathcal{A}(\xi(k)) = \sum_{i=1}^{N} \xi_i(k) A_i, \ \xi(k) \geq 0, \ \sum_{i=1}^{N} \xi_i(k) = 1 \tag{1.6}$$

The following result are obtained.

Theorem 1.3 [3] *System* (1.5) *is poly-quadratically stable if and only if there exist symmetric positive definite matrices S_i, S_j and matrices G_i of appropriate dimensions such that*

$$\begin{bmatrix} G_i + G_i^T - S_i & G_i^T A_i^T \\ A_i G_i & S_j \end{bmatrix} > 0,$$

for all $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n$.

In recent years, stability of different classes of neural networks with time delay, such as additive neural networks, cellular neural networks, bidirectional associative neural networks, Lotka-Volterra neural networks has been extensively studied and various stability conditions have been obtained for these models of neural networks. A neural network is computing paradigm that is looesly modeled after cortical structures of the brain. It consists of interconnected processing elements called neurons that work together to produce an output function. The output of a neural network relies on the cooperation of the individual neurons within the network to operate. Processing of information by neural networks is often done in parallel rather than on series (or sequentially). Since it relies on its member neurons collectively to perform its function. There are many researcher studying on robust stability problems for continuous-time neural networks, [7], [11] and [13].

In 2005, Y. He, Q. G. Wang and W. X. Zang [7] studied the global robust stability for delayed neural networks with polytopic type uncertainties system described by

$$\dot{z}(t) = -Az(t) + W_0 f(z(t)) + W_1 f(z(t - \tau(t))) \tag{1.7}$$

where $z(\cdot) = [z_1(\cdot), z_2(\cdot), \ldots, z_n(\cdot)]^T$ is the neuron state vector, $f(z(\cdot)) = [f_1(z_1(\cdot)), f_2(z_2(\cdot)), \ldots, f_n(z_n(\cdot))]^T$ is the neuron activation function.

$A = \ diag\{a_1, a_2, \ldots, a_n\}$ is a diagonal matrix with positive entries, W_0 and W_1 are the connection weight matrix and the delayed connection weight matrix. The matrices A, W_0, W_1 are subject to uncertainties and satisfy real convex polytopic model :

$$\begin{bmatrix} A & W_0 & W_1 \end{bmatrix} \in \Omega,$$

$$\Omega := \left\{ \begin{bmatrix} A(\xi) & W_0(\xi) & W_1(\xi) \end{bmatrix} = \sum_{k=1}^{p} \xi_k \begin{bmatrix} A_k & W_{0k} & W_{1k} \end{bmatrix}, \sum_{k=1}^{p} \xi_k = 1,\ \xi_k \geq 0 \right\} \tag{1.8}$$

where $A_k = diag\{a_{1k}, a_{2k}, \ldots, a_{nk}\}$ are diagonal matrices with positive entries, $W_{0k}, W_{1k},\ k = 1, 2, \ldots, p$, are constant matrices of compatible dimensions, and ξ_k are time-invariant uncertainties. Note that Eq. (1.8) represents polytopic type uncertainties for system (1.7). The delay $\tau(t)$ is a time-varying differentiable function and satisfies

$$\dot{\tau}(t) \leq d < 1 \tag{1.9}$$

where d is a constant. The functions $f_j(\cdot)$ satisfies the following condition:

$$0 \leq \frac{f_j(z_j)}{z_j} \leq \mu_j,\ f_j(0) = 0,\ \forall z_j \neq 0,\ j = 1, 2, \ldots, n, \tag{1.10}$$

which is equivalent to the following one:

$$f_j(z_j)[f_j(z_j) - \mu_j z_j] \leq 0\ ,\ f_j(0) = 0,\ j = 1, 2, \ldots, n.$$

A criterion of global robust stability for delayed neuron networks with polytopic type uncertainties is derived as follows :

Theorem 1.4 [7] *The origin of system* (1.7) *with fixed matrices* A, W_0 *and* W_1 *and subject to conditions* (1.9) *and* (1.10) *is globally asymptotically stable if there exist* $P = P^T > 0,\ R = R^T > 0,\ Q = Q^T > 0,\ \Lambda = diag\{\lambda_1, \lambda_2, \ldots, \lambda_n\} \geq 0$, $T = diag\{t_1, t_2, \ldots, t_n\} \geq 0,\ S = diag\{s_1, s_2, \ldots, s_n\} \geq 0$ *and any appropriate dimensional matrices* $H_i,\ i = 1, 2$ *such that the following LMI* (1.11) *is feasible,*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & \Phi_{14} & -H_1 W_1 \\ \Phi_{12}^T & \Phi_{22} & 0 & \Phi_{24} & -H_2 W_1 \\ 0 & 0 & -(1-d)R & 0 & \Theta S \\ \Phi_{14}^T & \Phi_{24}^T & 0 & Q - 2T & 0 \\ -W_1^T H_1^T & -W_1^T H_2^T & S\Theta & 0 & \Phi_{55} \end{bmatrix} < 0 \tag{1.11}$$

where

$\Phi_{11} = H_1 A + A^T H_1^T + R,$

$\Phi_{12} = P + H_1 + A^T H_2^T,$

$\Phi_{14} = -H_1 W_0 + \Theta T,$

$\Phi_{22} = H_2 + H_2^T,$

$\Phi_{24} = \Lambda - H_2 W_0,$

$\Phi_{55} = -(1-d)Q - 2S,$

$\Theta = diag\{\mu_1, \mu_2, \ldots, \mu_n\}.$

Theorem 1.5 [7] *The origin of system* (1.7) *with polytopic type uncertainties* (1.8) *and subject to conditions* (1.9) *and* (1.10) *is globally robustly stable if there exist* $P_k = P_k^T > 0,\ R_k = R_k^T > 0,\ Q_k = Q_k^T > 0,\ \Lambda_k = diag\{\lambda_{1k}, \lambda_{2k}, \ldots, \lambda_{nk}\} \geq 0,$ $T_k = diag\{t_{1k}, t_{2k}, \ldots, t_{nk}\} \geq 0,\ S_k = diag\{s_{1k}, s_{2k}, \ldots, s_{nk}\} \geq 0,\ k = 1, \ldots, p$ *and any appropriate dimensional matrices* $H_i,\ i = 1, 2$ *such that the following LMIs* (1.12) *is feasible for* $k = 1, \ldots, p,$

$$\Phi^{(k)} = \begin{bmatrix} \Phi_{11}^{(k)} & \Phi_{12}^{(k)} & 0 & \Phi_{14}^{(k)} & -H_1 W_{1k} \\ [\Phi_{12}^{(k)}]^T & \Phi_{22}^{(k)} & 0 & \Phi_{24}^{(k)} & -H_2 W_{1k} \\ 0 & 0 & -(1-d)R_k & 0 & \Theta S_k \\ [\Phi_{14}^{(k)}]^T & [\Phi_{24}^{(k)}]^T & 0 & Q_k - 2T_k & 0 \\ -W_{1k}^T H_1^T & -W_{1k}^T H_2^T & S_k \Theta & 0 & \Phi_{55}^{(k)} \end{bmatrix} < 0 \quad (1.12)$$

where

$\Phi_{11}^{(k)} = H_1 A_k + A_k^T H_1^T + R_k,$

$\Phi_{12}^{(k)} = P_k + H_1 + A_k^T H_2^T,$

$\Phi_{14}^{(k)} = -H_1 W_{0k} + \Theta T_k,$

$\Phi_{22}^{(k)} = H_2 + H_2^T,$

$\Phi_{24}^{(k)} = \Lambda_k - H_2 W_0,$

$\Phi_{55}^{(k)} = -(1-d)Q_k - 2S_k,$

$\Theta = diag\{\mu_1, \mu_2, \ldots, \mu_n\}.$

In 2005, H. Zhang and X. Liao [13] studied the LMI-based robust stability analysis of neural networks with time-varying delay given by

$$\dot{x}(t) = -(A + \Delta A)x(t) + (W + \Delta W)g(x(t)) + (W_1 + \Delta W_1)g(x(t - \tau(t))) \quad (1.13)$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$ is the neuron state vector, $A = diag\{a_1, a_2, \ldots, a_n\}$ is a positive diagonal matrix, W and W_1 are interconnection weight matrices, and $0 \leq \tau(t) < \tau_0$ is differentiable time delay, and it assumed that $\dot{\tau}(t) \leq d < 1$, ΔA, ΔW, ΔW_1 are parametric uncertainties and $g(x(\cdot)) = [g_1(x_1(\cdot)), g_2(x_2(\cdot)) \ldots, g_n(x_n(\cdot))]$ is the neuron activation function which satisfies condition

$$0 \leq \frac{g_j(x_j)}{x_j} \leq k \ , \ g_j(0) = 0 \ , \ \forall x_j \neq 0, \ \ j = 1, 2, \ldots, n,$$

which is equivalent to

$$g_j(x_j)[f_j(x_j) - kx_j] \leq 0 \ , \ f_j(0) = 0 \ , \ j = 1, 2, \ldots, n \ , \ k \in \mathbb{R}.$$

The uncertainties ΔA, ΔW and ΔW_1 are defined by

$$\Delta A = H_0F_0E_0, \ \Delta W = HFE \text{ and } \Delta W_1 = H_1F_1E_1$$

where H_0, H, H_1, E_0, E and E_1 are known constant matrices of appropriate dimensions, F_0, F and F_1 are unknown matrices representing the parameter uncertainty, which satisfy

$$F_0^TF_0 \leq I, \ F^TF \leq I \text{ and } F_1^TF_1 \leq I$$

where I is the identity matrix of appropriate dimensions. The main result obtained in [13] is the following :

Theorem 1.6 [13] *If there exist a symmetric positive definite matrix P, a positive diagonal definite matrix $D = diag(d_1, d_2, \ldots, d_n)$, and scalars $e_0 > 0$, $e > 0$, $e_1 > 0$ such that the following LMI holds*

$$M = \begin{bmatrix} (1,1) & PW & PW_1 & -PH_0 & PH & PH_1 \\ W^TP & (2,2) & DW_1 & -DH_0 & DH & DH_1 \\ W_1^TP & W^TD & -(1-d)Q + e_1E_1^TE_1 & 0 & 0 & 0 \\ -H_0^TP & -H_0^TD & 0 & -e_0I & 0 & 0 \\ H^TP & H^TD & 0 & 0 & -eI & 0 \\ H_1^TP & H_1^TD & 0 & 0 & 0 & -e_1I \end{bmatrix} < 0$$

where $(1,1) = -(PA + A^TP) + e_0E_0^TE_0$ *and* $(2,2) = -(2/k)DA + Q + DW + W^TD + eE^TE$ *then the origin of system* (1.13) *is robustly stable for all time delay* $0 \leq \tau(t) \leq \tau_0$ *and* $\dot{\tau}(t) \leq d < 1$.

In 2006, V. Singh [11] studied the global robust stability for delayed cellular neural networks based on norm-bounded uncertainties defined by the following state equations :

$$\dot{x}(t) = -x(t) + (A + \Delta A)f(x(t)) + (B + \Delta B)f(x(t-\tau)) \tag{1.14}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$ is the neuron state vector, $A = \{a_{ij}\} \in \mathbb{R}^{n\times n}$, $B = \{b_{ij}\} \in \mathbb{R}^{n\times n}$ are the weight coefficients of the neurons, ΔA and ΔB are parametric uncertainties in A and B, $f(x(\cdot)) = [f_1(x_1(\cdot)), f_2(x_2(\cdot))\ldots, f_n(x_n(\cdot))]$ is the neuron activation function which satisfies

$$0 \leq \frac{f_i(x_i)}{x_i} \leq 1,\ g_i(0) = 0,\ \forall x_i \neq 0,\quad i = 1, 2, \ldots, n.$$

Definition 1.1 [11] The system (1.14) is globally robustly stable if there is a unique equilibrium point $x^* = [x_1^*, x_2^*, \ldots, x_n^*]$ of system, which is globally asymptotically stable in the presence of the parametric uncertainties ΔA and ΔB.

The uncertainties ΔA and ΔB are assumed to satisfies

$$\Delta A = HFE\ \ ,\ \ \Delta B = H_1F_1E_1$$

where H, H_1, E and E_1 are known constant matrices of appropriate dimensions, F, F_1 are unknown matrices representing the parameter uncertainty, which satisfy

$$F^TF \leq I\ ,\ F_1^TF_1 \leq I$$

where I is the identity matrix of appropriate dimensions. In order establish the main result, V. Singh used the following well-known lemma

Lemma 1.4 [11] *Let* $U,\ V,\ W,$ *and* M *be real matrices of appropriate dimensions with* M *satisfying* $M = M^T$, *then*

$$M + UVW + W^TV^TU^T < 0$$

for all $V^TV \leq I$, if and only if there exists a scalar $\epsilon > 0$ such that

$$M + \epsilon^{-1}UU^T + \epsilon W^TW < 0.$$

The main result [11] is stated in the following.

Theorem 1.6 [11] *The system* (1.14) *is globally robustly stable if there is a positive definite matrix $P = P^T = \{p_{ij}\} \in \mathbb{R}^{n\times n}$, positive definite diagonal matrices $D = diag\{d_i > 0\} \in \mathbb{R}^{n\times n}$, $C = diag\{c_i > 0\} \in \mathbb{R}^{n\times n}$ and $S = diag\{s_i > 0\} \in \mathbb{R}^{n\times n}$, positive semidefinite diagonal matrices $K = diag\{k_i \leq 0\} \in \mathbb{R}^{n\times n}$ and $L = diag\{l_i \leq 0\} \in \mathbb{R}^{n\times n}$ and scalars $\epsilon_1 > 0$ and $\epsilon_2 > 0$ satisfying the following LMI :*

$$\begin{bmatrix} 2P - S & -PA - K & -PB & 0 & -PH & -PH_1 \\ -A^TP - K & \delta - \epsilon_1 E_1^T E_1 & -DB & 0 & -DH & -DH_1 \\ -B^TP & -B^TD & C + 2L - \epsilon_2 E^T E & -L & 0 & 0 \\ 0 & 0 & -L & S & 0 & 0 \\ -H^TP & -H^TD & 0 & 0 & \epsilon_2 I & 0 \\ -H_1^TP & -H_1^TD & 0 & 0 & 0 & \epsilon_1 I \end{bmatrix} > 0$$

where $\delta = 2D + 2K - DA - A^TD - C$.

In summary, from [7], [11] and [13] authors gave sufficient condition for robust stability of continuous time neural networks with uncertainties. There has been far less research studying an interesting problem of robust stability for discrete-time neural networks. In this thesis, we proposed to study robust stability problem for discrete-time neural networks. In Chapter 3 we give sufficient conditions for robust stability of zero solution of discrete-time linear parameter dependent cellular neural networks (CNNs) with time delay. Numerical simulations are also given to illustrate the efficiency of our theoretical results. Conclusion is provided in Chapter 4.