CHAPTER 4

CONCLUSION

In this work, we study robust stability of zero solution of discrete-time cellular neural networks with time delay. We give sufficient conditions for robust stability of system and robust stability criterion for discrete-time cellular neural networks with time delay system with polytopic type uncertainties and discrete-time cellular neural networks with time delay system with time-varying polytopic type uncertainties. The main results are summarized as follows:

4.1 Robust Stability of Discrete-Time Cellular Neural Networks with Time Delay Systems

Theorem 3.1.1 *The zero solution of system* (3.5) *is robustly stable if there exist* $P = P^T > 0$, $G = G^T \geq 0$, $Q = Q^T > 0$ *and* $L = diag\{l_1, l_2, \ldots, l_n\} > 0$ *with* $\tau \geq 0$ *such that the following LMI holds*

$$M = \begin{bmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{bmatrix} < 0$$

where

$$\begin{aligned}
(1,1) = {} & A^TPA - P + \epsilon A^TPWW^TPA + \epsilon A^TPH_0E_0E_0^TH_0^TPA \\
& + \epsilon A^TPHEE^TH^TPA + \epsilon E_0^TH_0^TPWW^TPH_0E_0 \\
& + \epsilon E_0^TH_0^TPHEE^TH^TPH_0E_0 + \epsilon A^TPH_1E_1E_1^TH_1^TPA \\
& + \epsilon A^TPW_1W_1^TPA + \epsilon E_0^TH_0^TPW_1W_1^TPH_0E_0 + \tau G + Q \\
& + \epsilon LW^TPW_1W_1^TPWL + \epsilon E_0^TH_0^TPH_1E_1E_1^TH_1^TPH_0E_0 \\
& + \epsilon LW^TPHEE^TH^TPWL + \epsilon LW^TPH_1E_1E_1^TH_1^TPWL \\
& + E_0^TH_0^TPH_0E_0 + \epsilon LE^TH^TPW_1W_1^TPHEL + LW^TPWL \\
& + LE^TH^TPHEL + \epsilon^{-1}I + 6\epsilon^{-1}L^2,
\end{aligned}$$

$$(2,2) = \epsilon L E_1^T H_1^T P H E E^T H^T P H_1 E_1 L + \epsilon L W_1^T H_1 E_1 E_1^T H_1^T P W_1 L$$
$$+ L E_1^T H_1^T P H_1 E_1 L + L W_1^T P W_1 L + 8\epsilon^{-1} L^2 - Q,$$
$$(3,3) = -\tau G.$$

Theorem 3.1.2 *The zero solution of system* (3.5) *is robustly stable if there exist* $P = P^T > 0,\ Q = Q^T > 0, T = \ diag\{t_1, t_2, \ldots, t_n\} \geq 0,\ S = \ diag\{s_1, s_2, \ldots, s_n\}$ ≥ 0 *and scalars* $e_0 > 0,\ e > 0$ *and* $e_1 > 0$ *such that the following LMI holds*

$$M = \begin{bmatrix} \prod_{11} & 0 & \prod_{13} & -A^T P W_1 & A^T P H_0 & -A^T P H & -A^T P H_1 \\ * & \prod_{22} & 0 & LS & 0 & 0 & 0 \\ * & * & \prod_{33} & W^T P W_1 & -W^T P H_0 & W^T P H & W^T P H_1 \\ * & * & * & \prod_{44} & -W_1^T P H_0 & W_1^T P H & W_1^T P H_1 \\ * & * & * & * & \prod_{55} & -H_0^T P H & -H_0^T P H_1 \\ * & * & * & * & * & \prod_{66} & H^T P H_1 \\ * & * & * & * & * & * & \prod_{77} \end{bmatrix} < 0$$

where

$\prod_{11} = A^T P A - P + Q + e_0 E_0^T E_0,$

$\prod_{13} = -A^T P W - LT,$

$\prod_{22} = -Q,$

$\prod_{33} = W^T P W - 2T + e E^T E,$

$\prod_{44} = W_1^T P W_1 - 2S + e_1 E_1^T E_1,$

$\prod_{55} = H_0^T P H_0 - e_0 I,$

$\prod_{66} = H^T P H - e I,$

$\prod_{77} = H_1^T P H_1 - e_1 I.$

Theorem 3.1.3 *The zero solution of system* (3.5) *is robustly stable if there exist* $P = P^T > 0$, $Q = Q^T > 0$, $T = diag\{t_1, t_2, \ldots, t_n\} \geq 0$, $S = diag\{s_1, s_2, \ldots, s_n\} \geq 0$ *and scalars* $\epsilon > 0$, $e_0 > 0$, $e > 0$ *and* $e_1 > 0$ *such that the following LMI holds*

$$M = \begin{bmatrix} \prod_{11} & 0 & \prod_{13} & -A^TPW_1 & A^TPH_0 & -A^TPH & -A^TPH_1 \\ * & \prod_{22} & 0 & LS & 0 & 0 & 0 \\ * & * & \prod_{33} & W^TPW_1 & -W^TPH_0 & W^TPH & W^TPH_1 \\ * & * & * & \prod_{44} & -W_1^TPH_0 & W_1^TPH & W_1^TPH_1 \\ * & * & * & * & \prod_{55} & -H_0^TPH & -H_0^TPH_1 \\ * & * & * & * & * & \prod_{66} & H^TPH_1 \\ * & * & * & * & * & * & \prod_{77} \end{bmatrix} < 0$$

where

$\prod_{11} = A^T(P^{-1} - \epsilon^{-1}H_0H_0^T)A + \epsilon E_0^TE_0 - P + Q + e_0E_0^TE_0,$

$\prod_{13} = -A^TPW - LT,$

$\prod_{22} = -Q,$

$\prod_{33} = W^T(P^{-1} - \epsilon^{-1}HH^T)W + \epsilon E^TE - 2T + eE^TE,$

$\prod_{44} = W_1^T(P^{-1} - \epsilon^{-1}H_1H_1^T)W_1^T + \epsilon E_1^TE_1 - 2S + e_1E_1^TE_1,$

$\prod_{55} = -e_0I$, $\prod_{66} = -eI$, $\prod_{77} = -e_1I.$

4.2 Robust Stability of Discrete-Time Cellular Neural Networks with Time Delay Systems with Polytopic Type Uncertainties

Theorem 3.2.1 *The zero solution of system* (3.5) *with polytopic type uncertainties* (3.2) *is robustly stable if there exist* $P_i = P_i^T > 0$, $Q_i = Q_i^T > 0$, $T_i = diag\{t_{1i}, t_{2i}, \ldots, t_{ni}\} \geq 0$, $S_i = diag\{s_{1i}, s_{2i}, \ldots, s_{ni}\} \geq 0$ *and scalars* $e_{0i} > 0$, $e_i >$

0 *and* $e_{1i} > 0$, $i = 1, 2, \ldots, N$ *satisfy this condition*

$$(i)\ M_{i,i,i} + N_i < -I,\ i = 1, 2, \ldots, N$$

$$(ii)\ M_{i,i,j} + M_{j,i,i} + M_{i,j,i} + 2N_i + N_j < \frac{1}{(N-1)^2} I,$$
$$i = 1, 2, \ldots, N,\ i \neq j,\ j = 1, 2, \ldots, N$$

$$(iii)\ M_{i,j,l} + M_{i,l,j} + M_{j,i,l} + M_{j,l,i} + M_{l,i,j} + M_{l,j,i}$$
$$+ 2N_i + 2N_j + 2N_l < \frac{6}{(N-1)^2} I,$$
$$i = 1, 2, \ldots, N-2,\ j = i+1, 2, \ldots, N-1,\ l = 1, 2, \ldots, N,$$

where

$$M_{i,j,l} =$$
$$\begin{bmatrix} A_i^T P_j A_l & 0 & -A_i^T P_j W_l & -A_i^T P_j W_{1l} & A_i^T P_j H_0 & -A_i^T P_j H & -A_i^T P_j H_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -W_l^T P_j A_i & 0 & W_i^T P_j W_l & W_i^T P_j W_{1l} & -W_i^T P_j H_0 & W_i^T P_j H & W_i^T P_j H_1 \\ -W_{1l}^T P_j A_i & 0 & W_{1l}^T P_j W_i & W_{1i}^T P_j W_{1l} & -W_{1i}^T P_j H_0 & W_{1i}^T P_j H & W_{1i}^T P_j H_1 \\ H_0^T P_j A_i & 0 & -H_0^T P_j W_i & -H_0^T P_j W_{1i} & 0 & 0 & 0 \\ -H^T P_j A_i & 0 & H^T P_j W_i & H^T P_j W_{1i} & 0 & 0 & 0 \\ -H_1^T P_j A_i & 0 & H_1^T P_j W_i & H_1^T P_j W_{1i} & 0 & 0 & 0 \end{bmatrix},$$

$$N_i = \begin{bmatrix} \prod_{11}(i) & 0 & -LT_i & 0 & 0 & 0 & 0 \\ 0 & -Q_i & 0 & LS_i & 0 & 0 & 0 \\ -LT_i & 0 & \prod_{33}(i) & 0 & 0 & 0 & 0 \\ 0 & LS_i & 0 & \prod_{44}(i) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \prod_{55}(i) & -H_0^T P_i H & -H_0^T P_i H_1 \\ 0 & 0 & 0 & 0 & -H^T P_i H_0 & \prod_{66}(i) & H^T P_i H_1 \\ 0 & 0 & 0 & 0 & -H_1^T P_i H_0 & H_1^T P_i H & \prod_{77}(i) \end{bmatrix},$$

and $\prod_{11}(i) = e_{0i} E_0^T E_0 - P_i + Q_i$, $\prod_{33}(i) = -2T_i + e_i E^T E$,

$\prod_{44}(i) = -2S_i + e_{1i} E_1^T E_1$, $\prod_{55}(i) = H_0^T P_i H_0 - e_{0i} I$, $\prod_{66}(i) = H^T P_i H - e_i I$,

$\prod_{77}(i) = H_1^T P_i H_1 - e_{1i} I.$

Theorem 3.2.2 *The zero solution of system* (3.5) *with polytopic type uncertainties* (3.2) *is robustly stable if there exist* $P_i = P_i^T > 0$, $Q_i = Q_i^T > 0$, $T_i = diag\{t_{1i}, t_{2i}, \dots, t_{ni}\} \geq 0$, $S_i = diag\{s_{1i}, s_{2i}, \dots, s_{ni}\} \geq 0$ *and scalars* $\epsilon > 0$, $e_{0i} > 0$, $e_i > 0$ *and* $e_{1i} > 0$, $i = 1, 2, \dots, N$ *satisfy this condition where*

$$(i)\ M_{i,i,i} + N_i < -I,\ i = 1, 2, \dots, N$$

$$(ii)\ M_{i,i,j} + M_{j,i,i} + M_{i,j,i} + 2N_i + N_j < \frac{1}{(N-1)^2}I,$$

$$i = 1, 2, \dots, N,\ i \neq j,\ j = 1, 2, \dots, N$$

$$(iii)\ M_{i,j,l} + M_{i,l,j} + M_{j,i,l} + M_{j,l,i} + M_{l,i,j} + M_{l,j,i}$$

$$+ 2N_i + 2N_j + 2N_l < \frac{6}{(N-1)^2}I,$$

$$i = 1, 2, \dots, N-2,\ j = i+1, 2, \dots, N-1,\ l = 1, 2, \dots, N,$$

where

$$M_{i,j,l} =$$

$$\begin{bmatrix} \theta_{11}(i) & 0 & -A_i^T P_j W_l & -A_i^T P_j W_{1l} & A_i^T P_j H_0 & -A_i^T P_j H & -A_i^T P_j H_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -W_l^T P_j A_i & 0 & \theta_{33}(i) & W_i^T P_j W_{1l} & -W_i^T P_j H_0 & W_i^T P_j H & W_i^T P_j H_1 \\ -W_{1l}^T P_j A_i & 0 & W_{1l}^T P_j W_i & \theta_{44}(i) & -W_{1i}^T P_j H_0 & W_{1i}^T P_j H & W_{1i}^T P_j H_1 \\ H_0^T P_j A_i & 0 & -H_0^T P_j W_i & -H_0^T P_j W_{1i} & 0 & 0 & 0 \\ -H^T P_j A_i & 0 & H^T P_j W_i & H^T P_j W_{1i} & 0 & 0 & 0 \\ -H_1^T P_j A_i & 0 & H_1^T P_j W_i & H_1^T P_j W_{1i} & 0 & 0 & 0 \end{bmatrix},$$

$$\theta_{11}(i) = A_i^T(P_j^{-1} - \epsilon^{-1} H_0 H_0^T)A_l,\ \theta_{33}(i) = W_i^T(P_j^{-1} - \epsilon^{-1} H H^T)W_l$$

$$\theta_{44}(i) = W_{1i}^T(P_j^{-1} - \epsilon^{-1} H_1 H_1^T)W_{1l}^T$$

$$N_i = \begin{bmatrix} \prod_{11}(i) & 0 & -LT_i & 0 & 0 & 0 & 0 \\ 0 & -Q_i & 0 & LS_i & 0 & 0 & 0 \\ -LT_i & 0 & \prod_{33}(i) & 0 & 0 & 0 & 0 \\ 0 & LS_i & 0 & \prod_{44}(i) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \prod_{55}(i) & -H_0^T P_i H & -H_0^T P_i H_1 \\ 0 & 0 & 0 & 0 & -H^T P_i H_0 & \prod_{66}(i) & H^T P_i H_1 \\ 0 & 0 & 0 & 0 & -H_1^T P_i H_0 & H_1^T P_i H & \prod_{77}(i) \end{bmatrix},$$

and $\prod_{11}(i) = \epsilon E_0^T E_0 + e_{0i} E_0^T E_0 - P_i + Q_i$, $\prod_{33}(i) = \epsilon E^T E - 2T_i + e_i E^T E$, $\prod_{44}(i) = \epsilon E_1^T E_1 - 2S_i + e_{1i} E_1^T E_1$, $\prod_{55}(i) = -e_{0i} I$, $\prod_{66}(i) = -e_i I$, $\prod_{77}(i) = -e_{1i} I$.

For future investigations, we propose study time-varying system describe by

$$x(k+1) = -[A(\xi(k)) + \Delta A]x(k) + [W(\xi(k)) + \Delta W]f(x(k)) + [W_1(\xi(k)) + \Delta W_1]f(x(k-\tau(k))) + b$$

where $x(k) = [x_1(k), \dots, x_n(k)]^T \in \mathbb{R}^n$ is the neuron state vector, $f(x(\cdot)) = [f_1(x_1(\cdot)), \dots, f_n(x_n(\cdot))]^T$ is the activation function, $b = [b_1, \dots, b_n]^T$ is constant input vector, $A(\xi(k))$ is positive diagonal matrix, $W(\xi(k))$ and $W_1(\xi(k))$ are the interconnection matrices of polytopic type where

$$\begin{bmatrix} A(\xi(k)) & W(\xi(k)) & W_1(\xi(k)) \end{bmatrix} \in \Omega,$$

$$\Omega = \Big\{ \big[A(\xi(k)) \; W(\xi(k)) \; W_1(\xi(k)) \big] = \sum_{i=1}^{N} \xi_i(k) \begin{bmatrix} A_i & W_i & W_{1i} \end{bmatrix}, \sum_{i=1}^{N} \xi_i(k) = 1, \; \xi_i(k) \geq 0 \Big\},$$

where A_i, W_i and W_{1i} are known constant matrices and ΔA, ΔW and ΔW_1 are uncertainty matrices which are of the form

$$\Delta A = H_0 \sum_{i=1}^{N} \xi_i(k) F_{0i} E_0, \; \Delta W = H \sum_{i=1}^{N} \xi_i(k) F_i E \; \text{ and } \Delta W_1 = H_1 \sum_{i=1}^{N} \xi_i(k) F_{1i} E_1$$

where H_0, H, H_1, E_0, E and E_1 are known constant matrices F_0, F and F_1 are unknown matrices

$$\begin{bmatrix} F_0(\xi(k)) & F(\xi(k)) & F_1(\xi(k)) \end{bmatrix} \in \Omega,$$

$$\Omega = \Big\{ \big[F_0(\xi(k)) \; F(\xi(k)) \; F_1(\xi(k)) \big] = \sum_{i=1}^{N} \xi_i(k) \begin{bmatrix} F_{0i} & F_i & F_{1i} \end{bmatrix}, \sum_{i=1}^{N} \xi_i(k) = 1, \; \xi_i(k) \geq 0 \Big\},$$

which satisfy

$$\sum_{i=1}^{N}\xi_i(k)F_{0i}^T\sum_{i=1}^{N}\xi_i(k)F_{0i}\leq I,\ \ \sum_{i=1}^{N}\xi_i(k)F_i^T\sum_{i=1}^{N}\xi_i(k)F_i\leq I$$
$$\text{and}\ \sum_{i=1}^{N}\xi_i(k)F_{1i}^T\sum_{i=1}^{N}\xi_i(k)F_{1i}\leq I$$

where I is the identity matrix of appropriate dimension , $\tau(k)$ is a positive integer denotes the time-varying delay satisfying

$$\tau_1\leq\tau(k)\leq\tau_2.$$