Chapter 1

Introduction

The presence or absence of a fixed point is an intrinsic property of a map. However, many necessary or sufficient conditions for the existence of such points involve a mixture of algebraic, order theoretic, or topological properties of the mapping or its domain.

The origins of the theory, which date to the latter part of the nineteenth century, rest in the use of successive approximations to establish the existence and uniqueness of solutions, particularly to differential equations. This method is associated with the names of such celebrated mathematicians as Cauchy, Liouville, Lipschitz, Peano, Fredholm and, especially, Picard. In fact the precursors of a fixed point theoretic approach are explicit in the work of Picard. However, it is the Polish mathematician Stefan Banach who is credited with placing the underlying ideas into an abstract framework suitable for broad applications well beyond the scope of elementary differential and integral equations. Around 1922, Banach recognized the fundamental role of metric completeness; a property shared by all of the spaces commonly exploited in analysis. For many years, activity in metric fixed point theory was limited to minor extensions of Banach's contraction mapping principal and its manifold applications. The theory gained new impetus largely as a result of pioneering work of Felix Browder in the mid-nineteen sixties and the development of nonlinear functional analysis as an active and vital branch of mathematics. Pivotal in this development were the 1965 existence theorems of Browder, Göhde, and Kirk and the eary metric results of Edelstein. By the end of the decade, a rich fixed point theory for nonexpansive mapping was clearly emerging and it was equally clear that such mappings played a fundamental role in many aspects of nonlinear functional analysis with links to variational inequalities and the theory of monotone and accretive operators.

Nonexpansive mappings represent the limiting case in the theory of contractions, where the Lipschitz constant is allows to become one, and it was clear from the outset that the study of such mappings required techniques going far beyond purely metric arguments. The theory of nonexpansive mappings has involved an intertwining of geometrical and topological arguments. The original theorems of Browder and Göhde exploited special convexity properties of the norm in certain Banach spaces, while Kirk identified the underlying property of normal structure and the role played by weak compactness. The early phases of the development centred around the identification of spaces whose bounded convex sets possessed normal structure, and it was soon discovered that certain weakenings and variants of normal structure also sufficed. By the mid-nineteen seventies it was apparent that normal structure was a substantially stronger condition than necessary. And, armed with the then newly discovered Goebel Karlovitz lemma the quest turned toward classifying those Banach spaces in which all nonexpansive self-mappings of a nonempty weakly compact convex subset have a fixed point. This has yielded many elegant results and led to numerous discoveries in Banach space geometry, although the question itself remains open. Asymptotic regularity of the averaged map was an important contribution of the late seventies, that has been exploited in many subsequent arguments.

Fixed-point iteration processes for approximating fixed point of nonexpansive mapping in Banach spaces have been studied by various mathematicians using the Mann iteration process (see [19]) or the Ishikawa iteration process (see [11, 12, 39, 48]). In 2000, Noor [22] introduced a three-step iterative scheme and study the approximate solutions of variational inclusion in Hilbert spaces . In 1998, Takahashi and Kim [37] proved strong convergence of approximants to fixed points of nonexpansive nonself-mappings in reflexive Banach spaces with uniformly Gâteaux differentiable norm. In the same year, Jung and Kim [13] proved the existence of a fixed point for nonexpansive nonself-mapping in a uniformly convex Banach space with a uniformly Gâteaux differentiable norm. In [39], Tan and Xu introduced a modified Ishikawa process to approximate fixed points of nonexpansive self-mappings defined on nonempty closed convex bounded subsets of a uniformly convex Banach space. In [49], H. Zhou, R.P. Agarwal, Y.J. Cho and Y.S. Kim gave criteria for weak convergence theorems of the Ishikawa iterative scheme for nonexpansive self-mapping in a uniformly convex Banach space which satisfies Opial's condition, and for strong convergence theorems for nonexpansive self-mapping in a uniformly convex Banach space which satisfies the *condition*(A). Recently, Shahzad [33] extended Tan and Xu results([39],Theorem 1, p.305) to the case of nonexpansive nonself-mapping in a uniformly convex Banach space.

The convergence problems of an implicit iteration process for a finite family of nonexpansive mappings have been studied by Browder [2, 3], Xu and Yin [47], Takahashi and Kim [37], and Jung and Kim [13] respectively. In [50], Zhou and Chang studied the weak and strong convergence of this implicit process to a common fixed point for a finite family of nonexpansive mappings in a uniformly convex Banach space. Recently, Chidume and Shahzad [8] proved that Xu and Ori's iteration process converges strongly to a common fixed point for a finite family of nonexpansive mappings if one of the mappings is semi-compact.

The concept of asymptotically nonexpansiveness was introduced by Goebel and Kirk [10] in 1972. The iterative approximation problems for nonexpansive mapping, asymptotically nonexpansive mapping and asymptotically quasi-nonexpansive mapping were studied extensively by Browder [2, 3], Goebel and Kirk [10], Liu [18], Wittmann [41], Reich [28], Shoji and Takahashi [34], Chang et al. [6] in the settings of Hilbert spaces and uniformly convex Banach spaces.

In 2004, Cho, Zhou and Guo [9] defined and studied a new three-step iterations with errors for asymptotically nonexpansive mappings in a uniformly convex Banach space. Suantai [35] defined a new three-step iterations which is an extension of Noor iterations and gave some weak and strong convergence theorems of such iterations for asymptotically nonexpansive mappings in uniformly convex Banach space.

In 2003, Chidume, Ofoedu and Zegeye [7] introduce the class of asymptotically nonexpansive nonself-maps and prove demiclosed principle for such maps. Then, an iteration scheme for approximating a fixed point of any map belonging to this class (where such a fixed point exists) is constructed; and strong and weak convergence theorems are proved.

Let C be a nonempty set and $T : C \to C$. The fixed point set of is defined by

$$Fix(T) = \{x \in C : Tx = x\}.$$

(Banach Contraction Mapping Principle [16]) Let (X, d) be a complete metric space and let $T : X \to X$ be a contraction. Then T has a unique fixed point x_0. Moreover, for each $x \in X$,

$$\lim_{n\to\infty} T^n(x) = x_0$$

and in fact for each $x \in X$,

$$d(T^n(x), x_0) \leq \frac{k^n}{1-k} d(x, T(x)), \;\; n = 1, 2, 3, \ldots.$$

T : $X \to X$ is said to be contraction if $\exists k$, $0 \leq k < 1$ such that

$$d(T(x), T(y)) \leq kd(x, y),$$

for all $x, y \in X$.

Let X be normed space and C be a nonempty subset of X. A mapping $T : C \to C$ is said to be nonexpansive on C if

$$\|Tx - Ty\| \leq \|x - y\|,$$

for all $x, y \in C$ and each $n \geq 1$.

In 1965, Browder [1] proved that, if C is a bounded closed convex subset of a Hilbert space E and T is a nonexpansive mapping of C into itself, then T has a fixed point in C.

Furthermore, in [2] he proved the same is true if X is a uniformly convex Banach space.

In the same year, Kirk [15] proved the following theorem: Let X be a reflexive Banach space, and C a closed convex bounded and nonempty subset of X with normal structure. Let $T : C \to C$ be a nonexpansive mapping. Then T has a fixed point in C.

In 1953, Mann [19] introduced an iteration procedure for approximating fixed points of a mapping T in a Hilbert space as follows: let $x_1 \in C$ and

$$x_{n+1} = \alpha_n T x_n + (1 - \alpha_n) x_n \quad \text{for} \quad n \geq 1. \tag{1.1}$$

In 1976, Ishikawa [12] proved the following: Let C be a compact convex subset of a Banach space X and T a nonexpansive mapping of C into itself. Let $x_1 \in C$ and define a sequence $\{x_n\}$ in C by

$$x_{n+1} = \alpha_n T x_n + (1 - \alpha_n) x_n,$$

where $\{\alpha_n\}$ is a sequence in $[0, 1]$ such that

$$\sum_{n=1}^{\infty} \alpha_n = \infty, \quad \limsup_{n \to \infty} \alpha_n < 1.$$

Then $\{x_n\}$ converges strongly to a fixed point of T.

Let C be a closed convex subset of a Hilbert space X and T a nonexpansive mapping of C into itself. Let $x, y_0 \in C$ and define a sequence $\{y_n\}$ in C by

$$y_{n+1} = \alpha_{n+1} x + (1 - \alpha_{n+1}) T y_n, \; n \geq 0, \tag{1.2}$$

where $\{\alpha_n\}$ is a sequence in $[0, 1]$ such that $\alpha_n \to 0$.

In 1992, Wittmann [41] proved the following: Let C be a closed convex subset of a Hilbert space and T a nonexpansive mapping of C into itself such that $Fix(T) \neq \phi$ and let P be a metric projection from C onto $Fix(T)$ Let $x \in C$ and let $\{\alpha_n\}$ be a real sequence which satisfies

$$0 \leq \alpha_n \leq 1, \lim_{n\to\infty} \alpha_n = 0, \sum_{n=0}^{\infty} \alpha_n = \infty, \sum_{n=0}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty.$$

Then the sequence $\{y_n\}$ defined by (1.2) converges strongly to Px.

In 1979, Reich [27] proved the following: Let C be a bounded closed convex subset of a uniformly convex Banach space X whose norm is Frechet differentiable and T a nonexpansive mapping of C into itself. Then the sequence $\{x_n\}$ defined by (1.1) converges weakly to a fixed point of T if $\sum_{n=1}^{\infty} \alpha_n(1-\alpha_n) = \infty$.

In 1993, Tan and Xu [39] proved the following: Let C be a bounded closed convex subset of a uniformly convex Banach space X which satisfies Opial's condition or whose norm is Frechet differentiable and T a nonexpansive mapping of C into itself. Let $x_1 \in C$, and let the sequence $\{x_n\}$ be defined by

$$x_{n+1} = \alpha_n T(\beta_n T x_n + (1-\beta_n)x_n) + (1-\alpha_n)x_n \quad \text{for} \quad n \geq 1,$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are chosen so that

$$\sum_{n=1}^{\infty} \alpha_n(1-\alpha_n) = \infty, \sum_{n=1}^{\infty} \beta_n(1-\beta_n) < \infty, \limsup_{n\to\infty} \beta_n < 1,$$

converges weakly to a fixed point of T.

In 1998, Takahashi and Kim [38] proved the following: Let X be a uniformly convex Banach space which satisfies Opial's condition, or whose norm is Frechet differentiable, let C be a nonempty closed convex subset of X and $T : C \to C$ a nonexpansive mapping with a fixed point. Suppose $x_1 \in C$, and let the sequence $\{x_n\}$ be defined by

$$x_{n+1} = \alpha_n T(\beta_n T x_n + (1-\beta_n)x_n) + (1-\alpha_n)x_n \quad \text{for} \quad n \geq 1,$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are chosen so that $\alpha_n \in [a, b]$ and $\beta_n \in [0, b]$ or $\alpha_n \in [a, 1]$ and $\beta_n \in [a, b]$ for some a, b with $0 < a \leq b < 1$. Then $\{x_n\}$ converges weakly to a fixed point of T.

In 2001, Xu and Ori [46] introduced the following implicit iteration process for a finite family of nonexpansive mappings $\{T_i : i \in J\}$ (here $J =$

$\{1, 2, \ldots, N\}$) with $\{\alpha_n\}$ is a real sequence in $(0, 1)$, and an initial point $x_0 \in C$:

$$\begin{aligned} x_1 &= \alpha_1 x_0 + (1 - \alpha_1) T_1 x_1, \\ x_2 &= \alpha_2 x_1 + (1 - \alpha_2) T_2 x_2, \\ &\vdots \\ x_N &= \alpha_N x_{N-1} + (1 - \alpha_N) T_N x_N, \\ x_{N+1} &= \alpha_{N+1} x_N + (1 - \alpha_{N+1}) T_{N+1} x_{N+1}, \\ &\vdots \end{aligned}$$

which can be written in the following compact form:

$$x_n = \alpha_n x_{n-1} + (1 - \alpha_n) T_n x_n, \quad n \geqslant 1 \tag{1.3}$$

where $T_n = T_{n(mod\ N)}$ (here the *mod N* function takes values in J). Xu and Ori also proved the weak convergence of this process to a common fixed point of the finite family of nonexpansive mappings in a Hilbert space.

In 2002, Zhou and Chang [50] proved the following: Let C be a uniformly convex Banach space and C a nonempty closed convex subset of X. Let $\{T_i : i \in J\}$ be N semi-compact nonexpansive self-mappings of C with $F = \cap_{i=1}^{N} F(T_i) \neq \emptyset$ (here $F(T_i)$ denotes the set of fixed points of T_i.) Suppose that $x_0 \in K$ and $\{\alpha_n\} \subset (b, c)$ for some $b, c \in (0, 1)$. Then the sequence $\{x_n\}$ defined by the implicit iteration process (1.3) converges strongly to a common fixed point in F.

A mapping $T : C \to C$ is called semi-compact(or hemicompact) if any sequence $\{x_n\}$ in C satisfying $\|x_n - Tx_n\| \to 0$ as $n \to \infty$ has a convergent subsequence.

In 2005, Chidume and Shahzad [8] proved the following: Let X be a uniformly convex Banach space and C a nonempty closed convex subset of X. Let $\{T_i : i \in J\}$ be N nonexpansive self-mappings of C with $F = \cap_{i=1}^{N} F(T_i) \neq \emptyset$. Suppose that one of the mappings in $\{T_i : i \in J\}$ is semi-compact. Let $\{\alpha_n\}_{n \geq 1} \subset [\delta, 1 - \delta]$ for some $\delta \in [0, 1]$. From arbitrary $x_0 \in C$, define the sequence $\{x_n\}$ by (1.3). Then $\{x_n\}$ converges strongly to a common fixed point of the mappings $\{T_i : i \in J\}$.

Let X be normed space and C be a nonempty subset of X. A mapping $T : C \to C$ is said to be *asymptotically nonexpansive* on C if there exists a sequence $\{k_n\}$, $k_n \geq 1$ with $\lim_{n \to \infty} k_n = 1$, such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|,$$

for all $x, y \in C$ and each $n \geq 1$.

If $k_n \equiv 1$, then T is known as a nonexpansive mapping. The mapping T is called *uniformly* $L-$ *Lipschitzian* if there exists a positive constant L such that

$$\|T^n x - T^n y\| \leq L\|x - y\|,$$

for all $x, y \in C$ and each $n \geq 1$.

It is easy to see that if T is asymptotically nonexpansive, then it is uniformly $L-$Lipschitzian with the uniform Lipschitz constant $L = \sup\{k_n : n \geq 1\}$.

In 1972, Goebel and Kirk [10] proved the following: Let C be a nonempty, closed, convex and bounded subset of a uniformly convex Banach space X, and let T be a asymptotically nonexpansive self-map of C. Then T has a fixed point.

In 1991, Schu [30] introduced the following iteration methods.

(The Modified Ishikawa Iteration Method) Let C be a nonempty convex subset of a Banach space X and $T : C \to C$ a giving mapping. For $x_1 \in C$ the modified Ishikawa iteration method $\{x_n\}$ is given by

$$\begin{aligned} y_n &= b_n T^n x_n + (1 - b_n)x_n \\ x_{n+1} &= \alpha_n T^n y_n + (1 - \alpha_n)x_n, \quad n \geq 1, \end{aligned}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are suitable sequence in $[0, 1]$.

(The Modified Mann Iteration Method)

$$x_{n+1} = \alpha_n T^n x_n + (1 - \alpha_n)x_n, \quad n \geq 1.$$

Let H be a Hilbert space, C a nonempty closed, bounded and convex subset of H. Let T be a completely continuous asymptotically nonexpansive mapping with $\{k_n\}$ satisfying $k_n \geq 1$ and $\sum_{n=1}^{\infty}(k_n^2 - 1) < \infty$. Let $\{\alpha_n\}$ be a real sequence in $[0, 1]$ satisfying the condition $\varepsilon \leq \alpha_n \leq 1 - \varepsilon$ from all $n \geq 1$. and for some $\varepsilon > 0$. Then the sequence $\{x_n\}$ generated from an arbitary $x_1 \in C$ by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n x_n, \quad n \geq 1$$

converges strongly to some fixed point of T.

In 1994, Rhoades [29] extended above to uniformly convex Banach space and to the modified Ishikawa iteration method.

In 2002, Osilike and Aniagbosor [26] proved the following: Let X be a uniformly convex Banach space satisfying Opial's condition, and let C be a nonempty closed convex subset of X. Let T be an asymptotically nonexpansive mapping with $\{k_n\}$ satisfying $k_n \geq 1$ and $\sum_{n=1}^{\infty}(k_n - 1) < \infty$. Let $\{u_n\}$ and

$\{v_n\}$ be bounded sequences in C and let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{a'_n\}$, $\{b'_n\}$ and $\{c'_n\}$ be real sequence in $[0, 1]$ satisfying the conditions: $a_n+b_n+c_n = a'_n+b'_n+c'_n = 1$ $\forall n \geq 1$. Then the sequence generated from an arbitary $x_1 \in K$ by

$$\begin{aligned} y_n &= a_n x_n + b_n T^n x_n + c_n u_n \\ x_{n+1} &= a'_n x_n + b'_n T^n y_n + c'_n v_n, n \geq 1 \end{aligned}$$

converges weakly to some fixed point of T.

In 2002, Xu and Noor [45] proved the following: Let X be a uniformly convex Banach space, and let C be a closed, bounded and convex subset of X. Let T be a completely continuous asymptotically nonexpansive self-map of C with $\{k_n\}$ satisfying $k_n \geq 1$ and $\sum_{n=1}^{\infty}(k_n - 1) < \infty$. Let $\{a_n\}, \{b_n\}, \{\alpha_n\}$ be real sequences in $[0, 1]$ satisfying

(*i*) $0 < \liminf_{n\to\infty} b_n \leq \limsup_{n\to\infty} b_n < 1$, and
(*ii*) $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty} \alpha_n < 1$,

For a given $x_1 \in C$, define

$$\begin{aligned} z_n &= a_n T^n x_n + (1 - a_n)x_n \\ y_n &= b_n T^n z_n + (1 - b_n)x_n \\ x_{n+1} &= \alpha_n T^n y_n + (1 - \alpha_n)x_n, \quad n \geq 1. \end{aligned}$$

Then $\{x_n\}, \{y_n\}$, $\{z_n\}$ converge strongly to a fixed point of T.

In 2005, Suantai [35] proved the following: Let X be a uniformly convex Banach space, and C a nonempty closed, bounded and convex subset of X. Let T be a completely continuous asymptotically nonexpansive self-map of C with $\{k_n\}$ satisfying $k_n \geq 1$ and $\sum_{n=1}^{\infty}(k_n - 1) < \infty$. Let $\{a_n\}, \{b_n\}, \{c_n\}, \{\alpha_n\}$, $\{\beta_n\}$ be sequences of real numbers in $[0, 1]$ with $b_n + c_n \in [0, 1]$ and $\alpha_n + \beta_n \in [0, 1]$ for all $n \geq 1$, and

(i) $0 < \liminf_{n\to\infty} b_n \leq \limsup_{n\to\infty}(b_n + c_n) < 1$, and
(ii) $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty}(\alpha_n + \beta_n) < 1$.

For a given $x_1 \in C$, define

$$\begin{aligned} z_n &= a_n T^n x_n + (1 - a_n)x_n \\ y_n &= b_n T^n z_n + c_n T^n x_n + (1 - b_n - c_n)x_n \\ x_{n+1} &= \alpha_n T^n y_n + \beta_n T^n z_n + (1 - \alpha_n - \beta_n)x_n, \quad n \geq 1. \end{aligned}$$

Then $\{x_n\}, \{y_n\}$ and $\{z_n\}$ converge strongly to a fixed point of T.

Let X be a uniformly convex Banach space which satisfies Opial's condition, and C a nonempty closed, bounded and convex subset of X. Let T be an asymptotically nonexpansive self-map of C with $\{k_n\}$ satisfying $k_n \geq 1$ and $\sum_{n=1}^{\infty}(k_n - 1) < \infty$. Let $\{a_n\}, \{b_n\}, \{c_n\}, \{\alpha_n\}, \{\beta_n\}$ be sequences of real numbers in $[0,1]$ with $b_n + c_n \in [0,1]$ and $\alpha_n + \beta_n \in [0,1]$ for all $n \geq 1$, and

(i) $0 < \liminf_{n\to\infty} b_n \leq \limsup_{n\to\infty}(b_n + c_n) < 1$, and
(ii) $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty}(\alpha_n + \beta_n) < 1$.

Let $\{x_n\}, \{y_n\}$ and $\{z_n\}$ be the sequences defined by

$$\begin{aligned} z_n &= a_n T^n x_n + (1 - a_n)x_n \\ y_n &= b_n T^n z_n + c_n T^n x_n + (1 - b_n - c_n)x_n \\ x_{n+1} &= \alpha_n T^n y_n + \beta_n T^n z_n + (1 - \alpha_n - \beta_n)x_n, \quad n \geq 1. \end{aligned}$$

Then $\{x_n\}$ converges weakly to a fixed point of T.

Let X be a real normed linear space, C a nonempty subset of X. Let $P : X \to C$ be the nonexpansive retraction of X onto C. A mapping $T : C \to X$ is said to be *asymptotically nonexpansive* if there exists a sequence $\{k_n\}$, $k_n \geq 1$ with $\lim_{n\to\infty} k_n = 1$, such that the following inequality holds:

$$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq k_n\|x - y\|,$$

for all $x, y \in C$ and each $n \geq 1$.

The mapping T is called *uniformly $L-$ Lipschitzian* if there exists a positive constant L such that

$$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq L\|x - y\|,$$

for all $x, y \in C$ and each $n \geq 1$.

Let C be a nonempty closed convex subset of a real uniformly convex Banach space X. The following iteration scheme is studied:

$$x_{n+1} = P((1 - \alpha_n)x_n + \alpha_n T(PT)^{n-1}x_n), \tag{1.4}$$

where $\{\alpha_n\}_{n\geq 1}$ is a sequence in $(0,1)$.

(Demiclosed principle for nonself-map) Let X be a uniformly convex Banach space, C a nonempty closed convex subset of X, and let $T : C \to X$ be an asymptotically nonexpansive mapping with a sequence $\{k_n\} \subset [1, \infty)$ and $k_n \to 1$ as $n \to \infty$. Then $I - T$ is demiclosed at zero.

In 2003, Chidume, Ofoedu and Zegeye [7] proved the following: Let X be a real uniformly convex Banach space, C closed convex nonempty subset of X. Let $T : C \to X$ be completely continuous and asymptotically nonexpansive map with sequence $\{k_n\} \subset [1, \infty)$ such that $\sum_{n \geq 1}(k_n^2 - 1) < \infty$ and $F(T) \neq \emptyset$. Let $\{\alpha_n\} \subset (0, 1)$ be such that $\epsilon \leq 1 - \alpha_n \leq 1 - \epsilon$, $\forall n \geq 1$ and some $\epsilon > 0$. From arbitrary $x_1 \in C$, define the sequence $\{x_n\}$ by (1.4). Then $\{x_n\}$ converges strongly to some fixed point of T.

Furthermore, in [7] they proved that the sequence $\{x_n\}$ defined by (1.4) converges weakly to some fixed point of T, where X be a real uniformly convex Banach space which has a Fréchet differentiable norm.

The purpose of this research is three fold. Firstly, we define a new iterative scheme for approximating fixed points of nonexpansive and asymptotically nonexpansive mappings in a uniformly convex Banach space. Secondly, we prove weak and strong convergence of the new iterative schemes to a fixed point of nonexpansive and asymptotically nonexpansive mappings and study sufficient conditions for weak and strong convergence theorems. Finally, we define and study a new iterative scheme for a finite family of nonexpansive mappings in a uniformly convex Banach space and prove weak and strong convergence theorems of the iterative scheme to a common fixed point of those finite family of nonexpansive mappings under some certain conditions.

This thesis is divided into 5 chapters. Chapter 1 is an introduction to the research problems. Chapter 2 deals with some preliminaries and give some useful results that will be used in later chapters. Chapter 3, Chapter 4 and Chapter 5 are the main results of this research. Precisely, in Section 3.1 we give sufficient conditions for weak and strong convergence to a fixed point of nonxepansive mapping. In Section 3.2, we give sufficient conditions for weak and strong convergence theorem to a common fixed point of nonxepansive mapping. In Section 4.1 we give sufficient conditions for weak and strong convergence to a fixed point of asymptotically nonxepansive mapping. In Section 4.2 we give sufficient conditions for weak and strong convergence to a common fixed point of asymptotically nonxepansive mapping. In Chapter 5, we give sufficient conditions for weak and strong convergence to a common fixed point for a finite family of nonxepansive mapping under some certain condition.