CHAPTER 1

INTRODUCTION

Let S be a semigroup and we call S a *regular semigroup* if for each $a \in S$, there exists $x \in S$ such that $axa = a$.

Let X be a nonempty partially ordered set and $O(X)$ denote the set of all order preserving total transformations of X. Then we see that $(O(X), \circ)$ is a semigroup under composition of mappings.

It is known [1, p.203, Exercise 6.1.7] that $O(X)$ is a regular semigroup if $(X, \leq)$ is a finite chain. In 2000, Y. Kemprasit and T. Changphas [3] showed that if X is a chain which is order isomorphic to a subset of $\mathbb{Z}$, the set of integers with their natural order, then $O(X)$ is regular. In 2004, S. Sornsanarm and R.P. Sullivan [4] studied $O(X)$ where X is a countable totally ordered set with the least element 0 and defined $T_r = \{\alpha \in O(X) : rank\ \alpha \leq r\}$ with $1 \leq r \leq \aleph_0$ and they showed that T_r is a regular semigroup.

Let $I_n = \{1, 2, ..., n\}$ and define $\leq$ on I_n by for all $a, b \in I_n$

$$a \leq b \text{ if and only if } a|b.$$

Then we see that $(I_n, \leq)$ is a partially ordered set which is not a chain and let $O(I_n)$ denote the set of all order preserving total transformations of I_n and we see that $(O(I_n), \circ)$ is a semigroup under composition of mappings. In, this thesis we are interested in the semigroup $O(I_n)$.

The purposes of this thesis are:

1. Study some properties of $(I_n, \leq)$.

2. To characterize regular elements on the semigroup $O(I_n)$.

3. To characterize maximal subgroups on the semigroup $O(I_n)$.

The thesis is divided into 4 chapters. Chapter 1 is an introduction to the research problem. In Chapter 2 we list some well-known results, definitions and notations those will be used in later chapter. Chapter 3 is the main results of this research. Precisely, in Section 3.1 we study some properties of $(I_n, \leq)$. In Section 3.2, we give a necessary and sufficient condition for regular element in

$O(I_n)$. In Section 3.3, we study a maximal subgroups of $O(I_n)$. The conclusion of this research is in Chapter 4.