CHAPTER 2

PRELIMINARIES

The aims of this chapter is to give some definitions, notations and dealing with some preliminaries and give some useful results that will be used throughout this thesis.

2.1 Elementary Properties

Let $\mathbb{Z}$ be the set of all integers.

Definition 2.1.1 Let Y be a nonempty subset of $\mathbb{Z}$. An element $d \in \mathbb{Z}^+$ is a *least common multiple* of Y (abbreviated by $lcm(Y)$) provided:

(1) $a|d$ for all $a \in Y$;

(2) $a|c$ for all $a \in Y \Rightarrow d|c$.

Definition 2.1.2 Let Y be a nonempty subset of $\mathbb{Z}$ and $Y \neq \{0\}$. An element $d \in \mathbb{Z}^+$ is a *greatest common divisor* of Y (abbreviated by $gcd(Y)$) provided:

(1) $d|a$ for all $a \in Y$;

(2) $c|a$ for all $a \in Y \Rightarrow c|d$.

Remark 2.1.3 Let A, B be nonempty subsets of $\mathbb{Z}$. If $A \subseteq B$ then $lcm(A)$ divides $lcm(B)$.

Proof. Assume that $A \subseteq B$. Let $lcm(A) = x$ and $lcm(B) = y$ for some $x, y \in \mathbb{Z}$. Let $a \in A \subseteq B$. Then $a \in B$, so $a|y$ for all $a \in A$ since $lcm(B) = y$. Thus y is a common multiple of A. Since $lcm(A) = x$, $x|y$. That is $lcm(A)$ divides $lcm(B)$. □

2.2 Semigroups of Transformations

Let X be a set, we denote the set of all total transformations of X by $T(X)$ and it is easy to see that this is a semigroup under composition of mappings: if $\alpha, \beta \in T(X)$ then $\alpha \circ \beta \in T(X)$ is defined by

$$x(\alpha \circ \beta) = (x\alpha)\beta, \quad x \in X.$$

Before we study the main result, let us fix some notations which will be useful when working with permutations.

Definition 2.2.1 Let X be a nonempty set. A *permutation* σ of X is a one-to-one function from X onto X.

Definition 2.2.2 The group of all permutations of X under composition of mappings is called the *symmetric* group on X and is denoted by S_X.

In this thesis, our study of permutation groups will focus on permutation groups on finite sets, i.e., X is a finite set. If $|X| = n$ say that $X = \{a_1, a_2, ..., a_n\}$, S_X is denoted by S_n. A permutation $\sigma \in S_n$ can be exhibited in the form

$$\begin{pmatrix} a_1 & a_2 & \dots & a_n \\ a_1\sigma & a_2\sigma & \dots & a_n\sigma \end{pmatrix},$$

consisting of two rows of elements in X; the top row has elements $a_1, a_2, ..., a_n$ usually (but not necessarily) in their natural order, and the bottom row has $a_i\sigma$ below a_i for each $i = 1, 2, ..., n$.