

# CHAPTER 1

## INTRODUCTION

In the last two decades there have been an increasing interest in obtain sufficient conditions for the oscillation and nonoscillation of solutions for nonlinear second order differential equations with damping term. We refer to the recent papers [2 – 8 ] where further references can be found.

Because of the importance of the oscillation and nonoscillation of solutions of second order nonlinear differential equations with damping, such as in biological applications (description of fluctuations in population), various process in physics and engineering science.

Many researchers concerned with the oscillation of the general nonlinear second order differential equation

$$(r(t)x'(t))' + p(t)x'(t) + q(t)f(x(t)) = 0 \quad (*)$$

where  $r \in C^1([t_0, \infty); (0, \infty))$ ,  $p, q \in C([t_0, \infty); \mathbb{R})$ ,  $k_1 \in C^1(\mathbb{R}^2; \mathbb{R}^2)$ ,  $k_2 \in C(\mathbb{R}^2; \mathbb{R}^2)$ ,  $f \in C(\mathbb{R}; \mathbb{R})$  and  $xf(x) > 0$ ,  $x \neq 0$ . In the sequel, we shall use a class of functions

introduced by Philos [3]. Let  $D_0 = \{(t, s) : t > s \geq t_0\}$  and  $D = \{(t, s) : t \geq s \geq t_0\}$ . The function  $H \in C(D; \mathbb{R})$  is said to belongs to the class W if

- (i)  $H(t, t) = 0$  for  $t \geq t_0$  and  $H(t, s) > 0$  for  $(t, s) \in D_0$ ,
- (ii)  $H$  has a continuous nonpositive partial derivative on  $D_0$  with respect to the

second variable such that

$$\frac{\partial}{\partial s} H(t, s) = -h(t, s) \sqrt{H(t, s)} \text{ for } (t, s) \in D_0$$

where  $h$  is a nonnegative and continuous function on  $D$ .

With this technique many researcher got new criteria for Equation (\*).

**Theorem A:** (Rogovchenko [4]). Suppose that there exists  $f'(x)$  and  $f'(x) \geq K > 0$  for some  $K$ .

If there exists  $g \in C^1([t_0, \infty); (0, \infty))$  which

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \left[ H(t, s) \Psi(s) - \frac{a(s)r(s)}{4K} \left( h(t, s) + \frac{p(s)}{r(s)} \sqrt{H(t, s)} \right)^2 \right] ds = \infty,$$

where  $a(s) = \exp \left( -2 \int g(u) du \right)$

and  $\Psi(s) = K^{-1} a(s) \left( Kq(s) - p(s)g(s) - (r(s)g(s))' + r(s)g^2(s) \right)$ .

Then Equation (\*) is oscillatory.

**Theorem B :** (Kirane and Rogovchenko [2]). Assume that  $f$  is satisfied  $\frac{f(x)}{x} \geq K > 0$  for some  $K$  and  $x \neq 0$ . If there exists  $g \in C^1([t_0, \infty); (0, \infty))$  which

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \left[ H(t, s) \Psi(s) - \frac{a(s)r(s)}{4} Q^2(t, s) \right] ds = \infty,$$

where  $a(s) = \exp \left( -2 \int g(u) du \right)$ ,  $\Psi(s) = a(s) \left( Kq(s) - p(s)g(s) - (r(s)g(s))' + r(s)g^2(s) \right)$

and  $Q(t, s) = h(t, s) + \frac{p(s)}{r(s)} \sqrt{H(t, s)}$ . Then Equation (\*) is oscillatory.

In 2003 Rogovchenko [5] concerned with the nonlinear second order differential equation in the form

$$[r(t)k_1(x(t), x'(t))] + p(t)k_2(x(t), x'(t))x'(t) + q(t)f(x(t)) = 0, \quad (**)$$

which expanded from [2] with the two conditions

(i)  $k_1$  is satisfied  $k_1^2(u, v) \leq \alpha v k_1(u, v)$  for some constant  $\alpha$  and  $(u, v) \in \mathfrak{R}^2$ ,

(ii)  $k_2(u, v)$  have the same sign with  $v$  for  $(u, v) \in \mathfrak{R}^2$

This yields the following Theorem.

**Theorem C:** (Rogovchenko [5]). Suppose that  $f$  satisfied  $\frac{f(x)}{x} \geq K > 0$  for some  $K$

and  $x \neq 0$ . If there exists  $\rho \in C^1([t_0, \infty); (0, \infty))$  which

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \left[ KH(t, s) \rho(s) q(s) - \frac{\alpha r(s) \rho(s)}{4} Q^2(t, s) \right] ds = \infty,$$

where  $Q(t, s) = h(t, s) - \frac{\rho'(s)}{\rho(s)} \sqrt{H(t, s)}$ . Then Equation (\*\*) is oscillatory.

In 2004, Tiryaki and Zafer [8] considered Equation (\*\*) and replaced the condition (ii) with  $uvk_2(u, v) \geq \beta k_1^2(u, v)$  for some positive  $\beta$  and  $(u, v) \in \mathcal{R}^2$ .

**Theorem D:** (Tiryaki and Zafer [8]). Suppose that  $f$  satisfied  $\frac{f(x)}{x} \geq K > 0$  for some  $K$  and  $x \neq 0$ . If there exists  $\rho \in C^1([t_0, \infty); (0, \infty))$  which

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \left[ H(t, s) \rho(s) q(s) - \frac{\alpha r^2(s) \rho(s)}{4K[\alpha \beta \rho(s) + r(s)]} Q^2(t, s) \right] ds = \infty,$$

where  $Q(t, s) = h(t, s) - \frac{\rho'(s)}{\rho(s)} \sqrt{H(t, s)}$ . Then Equation (\*\*) is oscillatory.

In 2008, Rogovchenko and Tuncay [6] concerned with Equation (\*) and got two new criteria for oscillation.

**Theorem E:** (Rogovchenko and Tuncay [6]). Suppose that there exists  $f'(x)$  and  $f'(x) \geq \mu$  for some  $\mu > 0$ . If there exists  $g \in C^1([t_0, \infty); (0, \infty))$  which

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \left[ H(t, s) \psi(s) - \frac{\beta}{4\mu} v(s) r(s) h^2(t, s) \right] ds = \infty,$$

where  $v(t) = \exp \left( -2\mu \int \left( g(s) - \frac{p(s)}{2\mu r(s)} \right) ds \right)$ ,

and  $\psi(t) = v(t) \left( q(t) + \mu r(t) g^2(t) - p(t) g(t) - (r(t) g(t))' \right)$ . Then Equation (\*) is oscillatory.

**Theorem F:** Let  $f(x)$  satisfied  $\frac{f(x)}{x} \geq \mu$  for some  $\mu > 0$  and  $x \neq 0$ . If there exists

$g \in C^1([t_0, \infty); (0, \infty))$  and some  $\beta > 1$  such that

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \left[ H(t, s) \psi(s) - \frac{\beta}{4} v(s) r(s) h^2(t, s) \right] ds = \infty,$$

where  $v(t) = \exp \left( -2 \int \left( g(s) - \frac{p(s)}{r(s)} \right) ds \right)$ ,

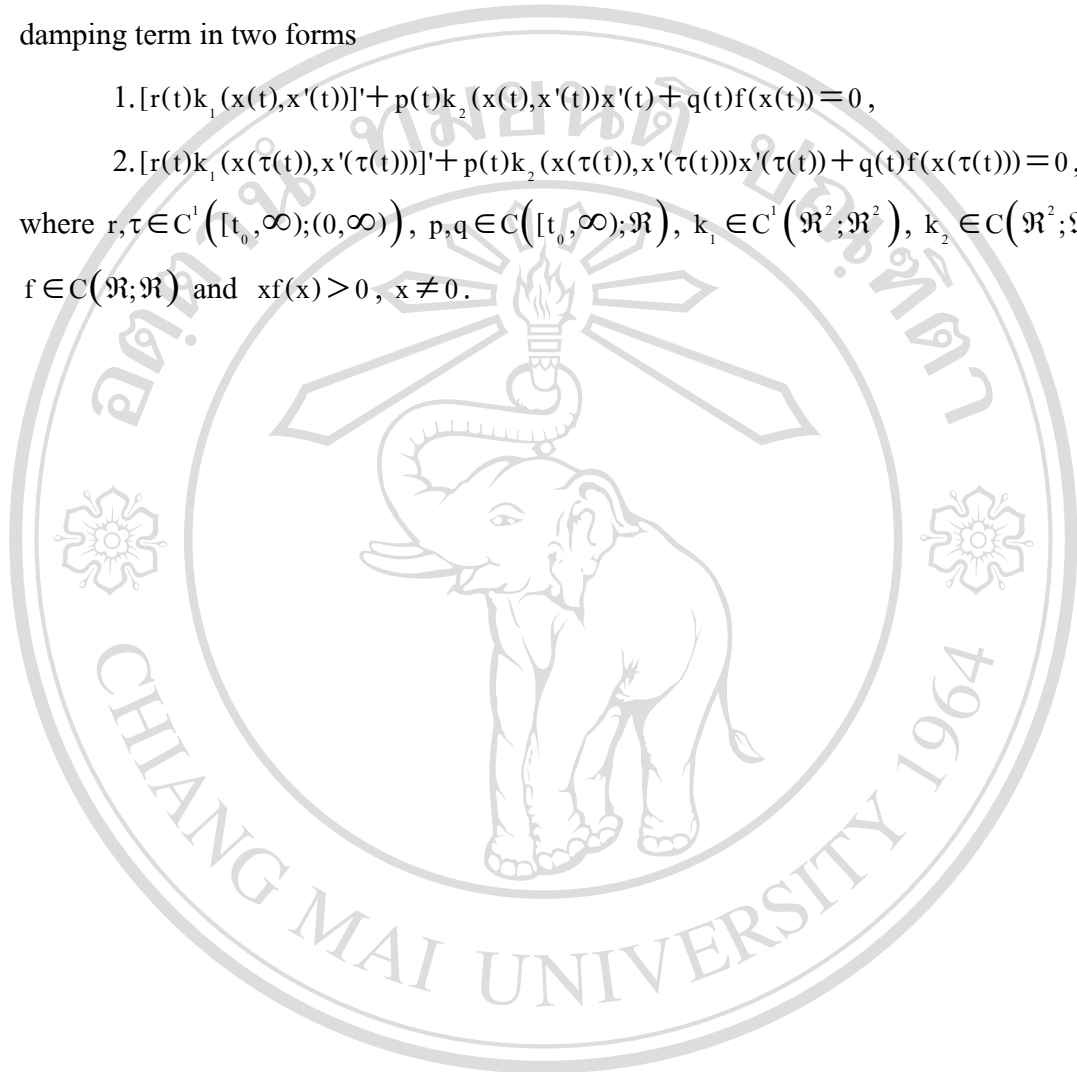
and  $\psi(t) = v(t)(\mu q(t) + r(t)g^2(t) - p(t)g(t) - (r(t)g(t))')$ . Then Equation (\*) is oscillatory.

In this Thesis, we concerned with nonlinear second order differential equation with damping term in two forms

$$1. [r(t)k_1(x(t), x'(t))] + p(t)k_2(x(t), x'(t))x'(t) + q(t)f(x(t)) = 0,$$

$$2. [r(t)k_1(x(\tau(t)), x'(\tau(t)))] + p(t)k_2(x(\tau(t)), x'(\tau(t)))x'(\tau(t)) + q(t)f(x(\tau(t))) = 0,$$

where  $r, \tau \in C^1([t_0, \infty); (0, \infty))$ ,  $p, q \in C([t_0, \infty); \mathbb{R})$ ,  $k_1 \in C^1(\mathbb{R}^2; \mathbb{R}^2)$ ,  $k_2 \in C(\mathbb{R}^2; \mathbb{R}^2)$ ,  $f \in C(\mathbb{R}; \mathbb{R})$  and  $xf(x) > 0$ ,  $x \neq 0$ .



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