

## CHAPTER 4 CONCLUSION

The main aim of this study was to find the oscillation criteria of second order nonlinear differential equations with damping in two forms

$$(I) [r(t)k_1(x(t), x'(t))] + p(t)k_2(x(t), x'(t))x'(t) + q(t)f(x(t)) = 0,$$

$$(II) [r(t)k_1(x(\tau(t)), x'(\tau(t)))] + p(t)k_2(x(\tau(t)), x'(\tau(t)))x'(\tau(t)) + q(t)f(x(\tau(t))) = 0.$$

We started with the class of functions introduced by Philos [3], some criteria theorems for second order nonlinear differential equations from many researchers [2-8].

Learned how to use some techniques to reach our goal. At last, we had 4 theorems for the first equation

**Theorem 1:** Let  $(C_1)$  and  $(C_2)$  hold. Assume that  $f'(x)$  exists and

$$f'(x) \geq \mu$$

for some  $\mu > 0$  and for all  $x \neq 0$ . Suppose that there exists a function  $g \in C^1([t_0, \infty); \mathbb{R})$

such that for some  $\beta \geq 1$  and  $H \in W$

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \left( H(t, s)\psi(s) - \frac{\beta}{4\mu\alpha_1} v(s)r(s)(h(t, s))^2 \right) ds = \infty$$

where

$$v(t) = \exp\left[-2\mu\alpha_1 \int_{t_0}^t \left(g(s) - \frac{\alpha_2 p(s)}{2\mu\alpha_1 r(s)}\right) ds\right]$$

and

$$\psi(t) = v(t)[q(t) + \mu\alpha_1 r(t)g^2(t) - \alpha_2 p(t)g(t) - (r(t)g(t))']$$

Then Equation (I) is oscillatory.

**Theorem 2:** Let  $(C_1)$ ,  $(C_2)$  and (3) hold. Suppose that

$$0 < \inf_{s \geq t_0} \left[ \liminf_{t \rightarrow \infty} \frac{H(t,s)}{H(t,t_0)} \right] \leq \infty$$

Assume that there exists function  $g \in C^1([t_0, \infty); \mathfrak{R})$  and  $\phi \in C([t_0, \infty); \mathfrak{R})$  such that for all  $t \geq T \geq t_0$  and for some  $\beta \geq 1$

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t,t_0)} \int_T^t \left[ H(t,s)\psi(s) - \frac{\beta}{4\mu\alpha_1} v(s)r(s)(h(t,s))^2 \right] ds \geq \phi(T)$$

where  $v(t), \psi(t)$  are as in Theorem 1, and suppose further that

$$\limsup_{t \rightarrow \infty} \int_{t_0}^t \frac{(\phi_+(s))^2}{v(s)r(s)} ds = \infty$$

where  $\phi_+(T) = \max\{\phi(t), 0\}$ . Then Equation (I) is oscillatory.

**Theorem 3:** Let  $(C_1) - (C_2)$ , (3) and (11) hold. Suppose that there exist functions  $g \in C^1([t, \infty), \mathfrak{R})$  and  $\phi \in C([t_0, \infty), \mathfrak{R})$  such that for all  $t \geq T \geq t_0$  and for some  $\beta > 1$ ,

$$\liminf_{t \rightarrow \infty} \frac{1}{H(t,T)} \int_T^t \left[ H(t,s)\psi(s) - \frac{\beta}{4\mu\alpha_1} v(s)r(s)(h(t,s))^2 \right] ds \geq \phi(t)$$

where  $v(t), \psi(t)$  and  $\phi(t)$  are the same as in Theorem 2. Assume also that (13) holds.

Then Equation (I) is oscillatory.

**Theorem 4:** Let  $(C_1), (C_2)$  hold. Suppose that  $f(x)$  satisfies

$$\frac{f(x)}{x} \geq \mu$$

for some  $\mu > 0$  and for all  $x \neq 0$ . Assume that there exists a function  $g \in C^1([t, \infty), \mathfrak{R})$  such that for some  $\beta \geq 1$  and  $H \in W$ ,

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t,t_0)} \int_{t_0}^t \left[ H(t,s)\psi(s) - \frac{\beta}{4\alpha_1} v(s)r(s)(h(t,s))^2 \right] ds = \infty$$

where

$$v(t) = \exp \left[ -2\alpha_1 \int_{t_0}^t \left( g(s) - \frac{\alpha_2 p(s)}{2\alpha_1 r(s)} \right) ds \right]$$

and

$$\psi(t) = v(t) \left[ \mu q(t) + \alpha_1 r(t) (g(t))^2 - \alpha_2 p(t) q(t) - (r(t) g(t))' \right]$$

Then Equation (I) is oscillatory.

And 2 theorems for the second one.

**Theorem 5:** Assume that  $f'(x)$  exists and

$$f'(x) \geq \mu$$

For some  $\mu > 0$  and for all  $x \neq 0$ . Suppose that there exists a function  $g \in C^1([t_0, \infty), \mathbb{R})$

such that for some  $\beta \geq 1$  and  $H \in W$ ,

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \left( H(t, s) \psi(s) - \frac{\beta}{4\mu\alpha_1} \frac{v(s)r(s)}{\tau'(s)} (h(t, s))^2 \right) ds = \infty$$

where

$$v(t) = \exp \left[ -2\mu\alpha_1 \int_{t_0}^t \left( \tau'(s)g(s) - \frac{\alpha_2 p(s)}{2\mu\alpha_1 r(s)} \right) ds \right]$$

and

$$\psi(t) = v(t) \left[ q(t) + \mu\alpha_1 r(t) \tau'(t) g^2(t) - \alpha_2 p(t) g(t) - (r(t) g(t))' \right]$$

Then equation (II) is oscillatory.

**Theorem 6:** Let  $(C_1) - (C_3)$  hold. Suppose that  $f(x)$  satisfied

$$\frac{f(x)}{x} \geq \mu$$

for some  $\mu > 0$  and for all  $x \neq 0$ . Suppose that there exists a function  $g \in C^1([t, \infty), \mathbb{R})$

such that for some  $\beta \geq 1$  and  $H \in W$ ,

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \left[ H(t, s) \psi(s) - \frac{\beta}{4\alpha_1} v(s)r(s)\tau'(t)(h(t, s))^2 \right] ds = \infty$$

where

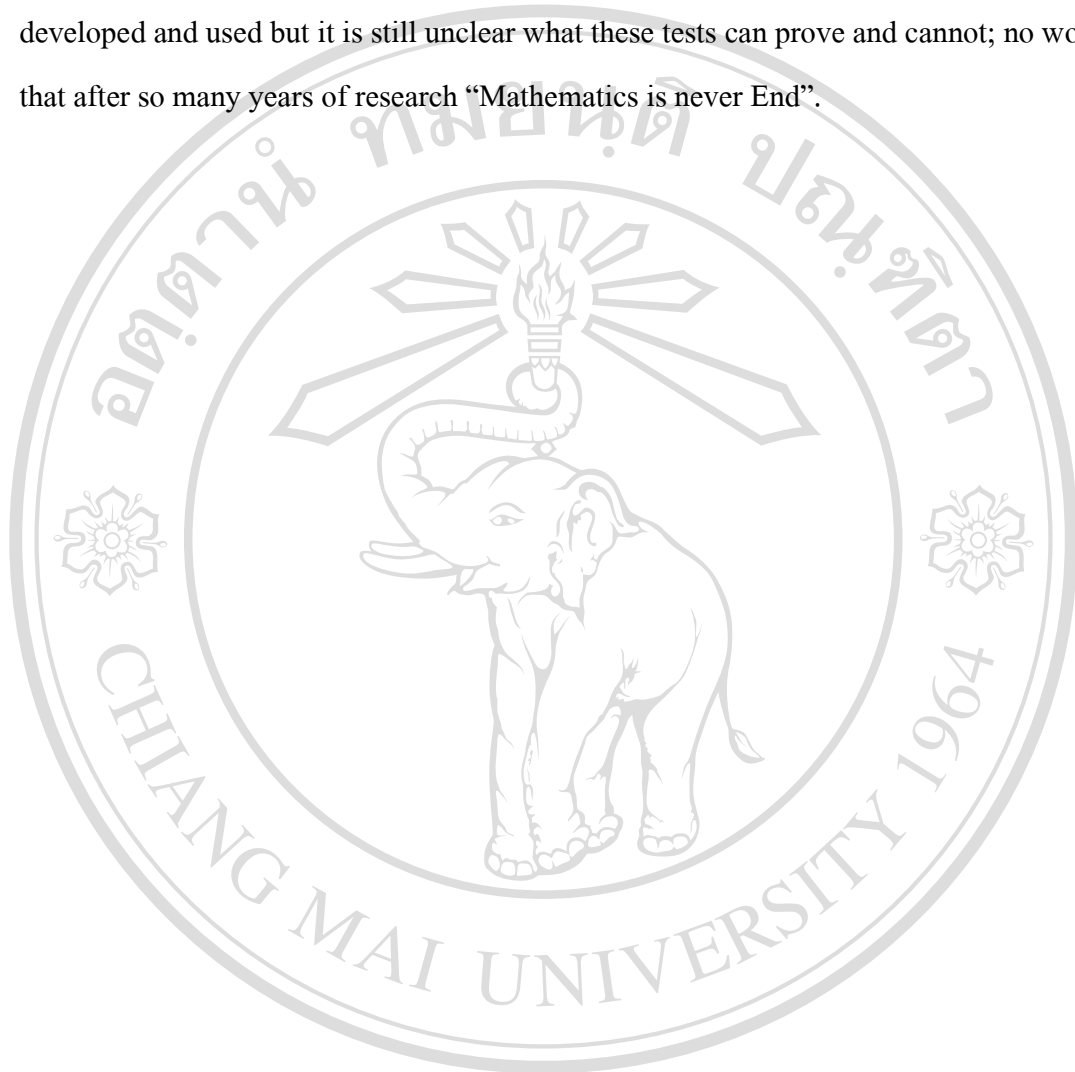
$$v(t) = \exp \left( -2\alpha_1 \int_{t_0}^t \left( \tau'(s)g(s) - \frac{\alpha_2 p(s)}{2\alpha_1 r(s)} \right) ds \right)$$

and

$$\psi(t) = v(t) \left[ \mu q(t) + \alpha_1 \tau'(t) r(t) (g(t))^2 - \alpha_2 p(t) q(t) - (r(t) g(t))' \right]$$

Then Equation (II) is oscillatory.

It is fairly easy to present and justify those theorems but the further model of equations is still questionable. Many proving methods and procedures have been developed and used but it is still unclear what these tests can prove and cannot; no wonder that after so many years of research “Mathematics is never End”.



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