CHAPTER 1

INTRODUCTION

The researches on the time-delay systems have been the interesting issues because the existence of time-delay is an important source to make system unstable. Uncertainties shoud be considered for it can make a differrence to the dynamics of system. Stability analysis of systems with norm bounded and time-varying uncertainties can be found in literature, for examples, in [7], [12] and [13].

In many practical systems, models of system are described by neutral differential equations, in which the models depend on the delays of state and state derivatives.Heat exchanges and population ecology are examples of neutral systems. Therefore several researches have studied neutral systems and provided sufficient conditions to guarantee the asymptotic stability of neutral time delay systems, see [10-13] our references cited therein .

In 2006, X. Jiang and W-L. Han [7] studied the delay-dependent robust stability for uncertain linear systems with interval time-varying delay described by

$$\begin{aligned} \dot{x}(t) &= [A + \Delta A(t)]x(t) + [A_1 + \Delta A_1(t)]x(t - \tau(t)), \\ x(t) &= \phi(t), t \in [-\tau_M, 0], \end{aligned} \tag{1.1}$$

where $x(t) \in R^n$ is the state vector; $A, A_1 \in R^{n \times n}$ are constant matrices; $\Delta A(t)$ and $\Delta A_1(t)$ are unknown real matrices of appropriate dimensions and satisfy

$$[\Delta A(t) \quad \Delta A_1(t)] = DF(t)[E \quad E_1], \tag{1.2}$$

where D, E and E_1 are known real constant matrices of appropriate dimensions, and $F(t)$ is unknown matrix function with Lebesgue measurable elements satisfying

$$F^T(t)F(t) \leq I, \tag{1.3}$$

$\phi(t)$ is the initial condition of system (1.1). $\tau(t)$ is a continuous time-varying delay function satisfying

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M, \tag{1.4}$$

where τ_m and τ_M are two constant. The following result are obtained.

Corollary 1.1 [7] *For some given scalars τ_m and τ_M, system (1.1) is robustly stable for any $\tau(t)$ satifying (1.4), if there exist a scalar $\varepsilon > 0$, some matrices $P > 0$, $Q > 0, R > 0, S > 0$ and $N_i, M_i (i = 1, 2, 3)$ of appropriate dimensions such that*

$$\begin{pmatrix} \Theta & \Theta_1 & \Theta_2 \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{pmatrix} < 0, \tag{1.5}$$

where

$\Theta_1 = [D^T N_1 \ \ D^T N_2 \ \ D^T N_3 \ \ 0 \ \ 0]^T$,

$\Theta_2 = [\epsilon E \ \ \epsilon E_1 \ \ 0 \ \ 0 \ \ \delta \epsilon E_1]^T$.

In 2004, J. H. Park [10] studied the delay-dependent criterion for asymptotic stability of a class of neutral equations given by

$$\frac{d}{dt}[x(t) + px(t-\tau)] = -ax(t) + b\tanh x(t-\sigma), \qquad t \geq 0, \tag{1.6}$$

where a, b, τ and σ are positive real numbers, $\sigma \geq \tau$ and $|p| < 1$. With each solution $x(t)$ of equation (1.6), Assume the initial condition:

$$x(s) = \phi(s), \qquad s \in [-\sigma, 0], \qquad \text{where } \phi \in C_\sigma.$$

The following theorem is the main result in his study.

Theorem 1.1 [10] *For given $\sigma > 0$, every solution $x(t)$ of equation (1.6) satisfies $x(t) \to 0$ as $t \to \infty$, if the operator D is stable and there exist the positive scalars $\epsilon_1, \epsilon_2, \epsilon_3, \alpha$ and β such that the linear matrix inequality holds*

$$\Sigma(\alpha, \beta, \epsilon_1, \epsilon_2, \epsilon_3) = \begin{pmatrix} -2a + \alpha + \sigma\beta + \epsilon_1 b & \sqrt{b} & \sqrt{b} & b & -ap & -\sigma ab \\ * & -\epsilon_1 & 0 & 0 & 0 & 0 \\ * & * & -\epsilon_2 & 0 & 0 & 0 \\ * & * & * & -\epsilon_3 & 0 & 0 \\ * & * & * & * & \epsilon_2 bp^2 - \alpha & 0 \\ * & * & * & * & * & \epsilon_3 \end{pmatrix} < 0, \tag{1.7}$$

In 2006, Y. G. Sun and L. Wang [11] studied the asymptotic stability of a class of neutral differential equation given by

$$\frac{d}{dt}[x(t) + px(t-\tau)] = -ax(t) + b\tanh x(t-\sigma), \quad t \geq 0, \tag{1.8}$$

where a, τ and σ are positive constants, b and p are real numbers, $|p| < 1$. For each solution $x(t)$ of Equation (1.6), assume the following initial condition:

$$x = \phi(t), \qquad t \in [-\delta, 0], \qquad \text{where } \delta = \max\{\tau, \sigma\}, \phi \in C_\delta.$$

The following theorem is the main result in their studies.

Theorem 1.2 [11] *For given $a > 0, \tau > 0$ and $\sigma > 0$, equation (1.8) is asymptotically stable if the operator D is stable and there exist a constant α with $0 < |\alpha| < 1$, and positive constants β, γ, θ and η such that the following linear matrix inequality holds*:

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} & b & \alpha - a & b(\alpha - a) \\ * & -\beta - 2p\alpha & pb & -\alpha & -b\alpha \\ * & * & \theta\sigma^2 - \eta & b & b^2 \\ * & * & * & -\gamma & 0 \\ * & * & * & * & -\theta \end{pmatrix} < 0, \tag{1.9}$$

where

$\Omega_{11} = 2(\alpha - a) + \beta + \gamma\tau^2 + \eta, \qquad \Omega_{12} = p(\alpha - a) - \alpha.$

In 2008, B. Wang, X. Liu and S. Zhang [12] studied the stability analysis for uncertain neutral system with time-varying delay given by

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t-d) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t - \tau(t)), \\ x(t_0 + \theta) &= \varphi(\theta), \quad \theta \in [-h, 0], \end{aligned} \tag{1.10}$$

with

$\Delta A^T(t)\Delta A(t) \leq \rho_A I, \qquad \forall t \geq 0,$

$\Delta B^T(t)\Delta B(t) \leq \rho_B I, \qquad \forall t \geq 0,$

$\Delta A(t) = E_1 F_1(t) G_1, \qquad \forall t \geq 0,$

$\Delta B(t) = E_2 F_2(t) G_2, \qquad \forall t \geq 0,$

$F^T(t)_1 F_1(t) \leq I, \qquad \forall t \geq 0,$

$F^T(t)_2 F_2(t) \leq I, \qquad \forall t \geq 0,$

where $x(t) \in R^n$ is the state vector, $0 \leq d \leq h$ is a constant neutral delay, $0 \leq \tau(t) \leq h$ is a time-varying discrete delay, $\varphi(.) \in C_h$ is the initial condition, $C \in R^{n\times n}, B \in R^{n\times n}, A \in R^{n\times n}$ are constant matrices. E_1, E_2, G_1 and G_2 are known constant real matrices with appropriate dimensions. The following lemma is needed:

Lemma 1.1 [12] *Consider uncertain neutral system, for given positive scalar h, the operator $D(x_t)$ is stable if there exists a constant matrix $L > 0$ and positive scalars $\epsilon_1, \epsilon_2, \beta_1$ and β_2 such that*

$$\Omega = \begin{pmatrix} C^TLC - \beta_1 L & hC^TLB & hC^TLE_2 & 0 \\ * & \Lambda & 0 & h^2B^TLE_2 \\ * & * & -\epsilon_1 I & 0 \\ * & * & * & -\epsilon_2 I \end{pmatrix} < 0, \qquad (1.11)$$

$$\beta_1 + \beta_2 < 1, \qquad (1.12)$$

where

$\Lambda = h^2B^TLB - \beta_2 L + \epsilon_1 G_1^T G_1 + \epsilon_2 G_2^T G_2 + \rho_B h^2 L.$

The main result obtained in [12] is the following.

Theorem 1.3 [12] *Suppose that there exist positive scalars β_1 and β_2 satisfying (1.11) and (1.12). For given positive scalar h, the system (1.10) is stable if there exist positive scalars e_1, e_2 and constant matrixes $P_1 > 0, Q > 0$ and $N > 0$, satisfying the following LMI:*

$$\begin{pmatrix} \Pi & (A+B)^TP_1C & (A+B)^TP_1 & P_1E_1 & P_1E_2 \\ * & -N & 0 & C^TP_1E_1 & C^TP_1E_2 \\ * & * & -h^{-1}Q & P_1E_1 & P_1E_2 \\ * & * & * & -e_1 I & 0 \\ * & * & * & * & -e_2 I \end{pmatrix} < 0, \qquad (1.13)$$

where

$\Pi = P_1(A+B) + (A+B)^TP_1 + hB^TQB + N + e_1G_1^TG_1 + e_2G_2^TG_2 + \rho_B hQ.$

In 2008, K. W. Yu and C. H. Lien [13] studied the stability criteria for uncertain neutral systems with interval time-varying delays defined by the following state equations :

$$\dot{x}(t) = A_0 + A_1 x(t-h(t)) + A_2\dot{x}(t-\tau(t)) + \Delta f_0(t,x(t)) + \Delta f_1(t,x(t-h(t))) + \Delta f_2(t,\dot{x}(t-\tau(t))), \quad t \geq 0, \tag{1.14}$$

$$x(t) = \phi(t), \quad t \in [-H,0], \tag{1.15}$$

where $x \in R^n$, x_t is state at time t defined by $x_t(\theta) = x(t+\theta), \forall\theta \in [-H,0]$, $0 \leq h_m \leq h(t) \leq h_M, 0 \leq \tau(t) \leq \tau_M, \dot{h}(t) \leq h_D, \dot{\tau} \leq \tau_D < 1, h_M > 0, \tau_M > 0$, $H = \max\{h_M,\tau_M\}, A_0, A_1$ and $A_2 \in R^{n\times n}$ are constant matrices and the initial vector $\phi \in C_H$, $\Delta f_0(.), \Delta f_1(.)$ and $\Delta f_2(.)$ are some perturbations in system (1.14). There are two cases of perturbations.

(A_1) Nonlinear time-varying perturbation:

$$\|\Delta f_0(t,x(t))\| \leq \alpha_0\|x(t)\|, \quad \|\Delta f_1(t,x(t-h(t)))\| \leq \alpha_1\|x(t-h(t))\|, \quad \|\Delta f_2(t,\dot{x}(t-\tau(t)))\| \leq \alpha_2\|\dot{x}(t-\tau(t))\|, \tag{1.16}$$

where $\alpha_i, i \in \{0,1,2\}$ are given nonnegative constants.

(A_2) Parametric norm-bounded perturbations:

$$\Delta f_0(t,x(t)) = \Delta A_0(t)x(t), \quad \Delta f_1(t,x(t-h(t))) = \Delta A_1(t)x(t-h(t)),$$
$$\Delta f_2(t,\dot{x}(t-\tau(t))) = \Delta A_2(t)\dot{x}(t-\tau(t)),$$
$$[\Delta A_0(t) \quad \Delta A_1(t) \quad \Delta A_2(t)] = MF(t)[N_0 \quad N_1 \quad N_2], \tag{1.17}$$

where $M, N_i, i \in \{0,1,2\}$, are some given constant matrices, $F(t)$ is an unknown real time-varying functions with appropriate dimension and bounded as follows:

$$F^T(t)F(t) \leq I, \quad \forall t \geq 0.$$

The main result [13] is stated in the following.

Theorem 1.4 [13] *System (1.14) with* (A_1) *is asymptotically stable, if* $\|A_2\| + \alpha_2 < 1$ *and there exist some* $n \times n$ *matrices* $P_i > 0, i \in \{0,1,...,5\}$, *matrices* $Q_1, Q_2, Q_3 \in$

$R^{n\times n}$, *and some positive constants* $\epsilon_0, \epsilon_1, \epsilon_2$, *such that the following LMI conditions holds:*

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} & \Sigma_{17} & \Sigma_{18} \\ * & \Sigma_{22} & \Sigma_{23} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & \Sigma_{36} & \Sigma_{37} & \Sigma_{38} \\ * & * & * & \Sigma_{44} & \Sigma_{45} & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{55} & \Sigma_{56} & \Sigma_{57} & \Sigma_{58} \\ * & * & * & * & * & \Sigma_{66} & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{77} & 0 \\ * & * & * & * & * & * & * & \Sigma_{88} \end{pmatrix} < 0, \tag{1.18}$$

where

$\Sigma_{11} = \hat{\Sigma}_{11} + \epsilon_0\alpha_0^2 I$, $\hat{\Sigma}_{11} = P_0A_0 + A_0^TP_0 + Q_2^TA_0 + A_0^TQ_2 + P_1 + P_2 - P_3$, $\Sigma_{12} = P_3$,
$\Sigma_{13} = P_0A_1 + Q_2^TA_1 + A_0^TQ_3$, $\Sigma_{14} = P_0A_2 + Q_2^TA_2$, $\Sigma_{15} = A_0^TQ_1 - Q_2^T$,
$\Sigma_{16} = \Sigma_{17} = \Sigma_{18} = P_0 + Q_2^T$, $\Sigma_{22} = -P_1 - P_3 - P_4$, $\Sigma_{23} = P_4$,
$\Sigma_{33} = \hat{\Sigma}_{33} + \epsilon_1\alpha_1^2 I$, $\hat{\Sigma}_{33} = -(1-h_D)P_2 - P_4 + Q_3^TA_1 + A_1^TQ_3$, $\Sigma_{34} = Q_3^TA_2$
$\Sigma_{35} = A_2^TQ_1 - Q_3^T$, $\Sigma_{36} = \Sigma_{37} = \Sigma_{38} = Q_3^T$, $\Sigma_{44} = \hat{\Sigma}_{44} + \epsilon_2\alpha_2^2 I$,
$\hat{\Sigma}_{44} = -(1-\tau_D)P_5$, $\Sigma_{45} = A_2^TQ_1$, $\Sigma_{55} = P_5 + h_M^2P_3 + (h_M - h_m)^2P_4 - Q_1 - Q_1^T$,
$\Sigma_{56} = \Sigma_{57} = \Sigma_{58} = Q_1^T$, $\Sigma_{66} = -\epsilon_0 I$, $\Sigma_{77} = -\epsilon_1 I$, $\Sigma_{88} = -\epsilon_2 I$.

Theorem 1.5 [13] *System (1.14) with* (A_2) *is asymptotically stable, if there exist some* $n\times n$ *matrices* $P_i > 0, i = \{0,1,...,5\}$, *matrices* $Q_1, Q_2, Q_3 \in R^{n\times n}$, *and some positive constants* ϵ *and* η, *such that the following LMI conditions hold:*

$$\begin{pmatrix} -I + \eta N_2^TN_2 & A_2^T & 0 \\ A_2 & -I & M \\ 0 & M^T & -\eta I \end{pmatrix} < 0, \tag{1.19}$$

$$\begin{pmatrix} \hat{\Sigma}_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \hat{\Sigma}_{16} & \hat{\Sigma}_{17} \\ * & \Sigma_{22} & \Sigma_{23} & 0 & 0 & 0 & 0 \\ * & * & \hat{\Sigma}_{33} & \Sigma_{34} & \Sigma_{35} & \hat{\Sigma}_{36} & \hat{\Sigma}_{37} \\ * & * & * & \hat{\Sigma}_{44} & \Sigma_{45} & 0 & \hat{\Sigma}_{47} \\ * & * & * & * & \Sigma_{55} & \hat{\Sigma}_{56} & 0 \\ * & * & * & * & * & \hat{\Sigma}_{66} & 0 \\ * & * & * & * & * & * & \hat{\Sigma}_{77} \end{pmatrix} < 0, \qquad (1.20)$$

where $\hat{\Sigma}_{11}, \hat{\Sigma}_{33}, \hat{\Sigma}_{44}, \Sigma_{ij}, i, j = 1, ..., 5,$ *are defined in (1.18)*, $\hat{\Sigma}_{16} = P_0 M + Q_2^T M, \hat{\Sigma}_{17} = \epsilon N_0^T, \hat{\Sigma}_{36} = Q_3^T M, \hat{\Sigma}_{37} = \epsilon N_1^T, \hat{\Sigma}_{47} = \epsilon N_2^T, \hat{\Sigma}_{56} = Q_1^T M, \hat{\Sigma}_{66} = \hat{\Sigma}_{77} = -\epsilon I$

In summary, from [10-13] authors gave sufficient conditions for asymptotically stable of neutral system. In this thesis, we propose to study asymptotic stability of neutral system. In Chapter 3 we give sufficient conditions for delay-dependent criterion for asymptotic stability for uncertain neutral system. Numerical examples are illustrated to show the efficiency of our theoretical results. Conclusion is provided in Chapter 4.