CHAPTER 5

CONCLUSION

In this work, we study the approximation of the cone of copositive matrices in the cone K_n^1 and we extend the system of linear matrices inequalities LMI's in the cone K_n^2 for approximating optimal solution of the cone of copositive matrices. The main results are summarized as follows:

4.1 System of Linear Matrices Inequalities LMI's in the Cone K_n^1

Theorem 3.2.1. $M \in K_n^1$ if and only if there are n symmetric $n \times n$ matrices $M^{(i)} \in S_n$ for $i = 1, ..., n$ such that the system of LMI's is satisfied.

$$\begin{aligned} M - M^{(i)} &\in S_n^+ && , i = 1, ..., n, \\ M_{ii}^{(i)} &= 0 && , i = 1, ..., n, \\ M_{ii}^{(j)} + 2M_{ij}^{(i)} &= 0 && , i \neq j, \\ M_{jk}^{(i)} + M_{ik}^{(j)} + M_{ij}^{(k)} &\geq 0 && , i < j < k. \end{aligned} \tag{4.1}$$

We change the SDP approximations of the copositive cone to arrive at a series of LP approximations of copositive cone.

4.2 System of Linear Inequalities in the Cone C_n^1

Theorem 3.2.3. $M \in C_n^1$ if and only if there are n symmetric $n \times n$ matrices $M^{(i)} \in S_n$ for $i = 1, ..., n$ such that the following system of linear inequalities has a solution:

$$\begin{aligned} M - M^{(i)} &\in N_n && , i = 1, ..., n, \\ M_{ii}^{(i)} &= 0 && , i = 1, ..., n, \\ M_{ii}^{(j)} + 2M_{ij}^{(i)} &= 0 && , i \neq j, \\ M_{jk}^{(i)} + M_{ik}^{(j)} + M_{ij}^{(k)} &\geq 0 && , i < j < k. \end{aligned}$$

4.3 System of Linear Matrices Inequalities LMI's in the Cone K_n^2

Theorem 4.2.1. $M \in K_n^2$ if and only if there are n symmetric $n \times n$ matrices $M^{(ij)} \in S_n$ for $i = 1, ..., n$ and $j = 1, ..., n$ such that the following system of linear inequalities has a solution:

$$
\begin{array}{rcll}
M - M^{(ii)} & \in & S_n^+ & , i = 1, ..., n, \\
M_{ii}^{(ii)} & \geqslant & 0 & , i = 1, ..., n, \\
2M_{ii}^{(ij)} + 2M_{ij}^{(ii)} & \geqslant & 0 & i \neq j \\
M_{ii}^{(jj)} + M_{jj}^{(ii)} + 4M_{ij}^{(ij)} & \geqslant & 0 & i \neq j \\
2(M_{ii}^{(jk)} + 2M_{ij}^{(ik)} + 2M_{ik}^{(ij)} + M_{jk}^{(ii)}) & \geqslant & 0 & i \neq j, j \neq k, i \neq k \\
4(M_{ij}^{(kl)} + M_{ik}^{(jl)} + M_{il}^{(jk)} + M_{jk}^{(il)} + M_{jl}^{(ik)} + M_{kl}^{(ij)}) & \geqslant & 0 & i < j < k < l.
\end{array} \tag{4.2}
$$

We therefore change the SDP approximations of the copositive cone to arrive at a series of LP approximations of copositive cone.

4.4 System of Linear Inequalities in the Cone C_n^2

Theorem 4.2.2. $M \in C_n^2$ if and only if there are n symmetric $n \times n$ matrices $M^{(ij)} \in S_n$ for $i = 1, ..., n$ and $j = 1, ..., n$ such that the following system of linear inequalities has a solution:

$$
\begin{array}{rcll}
M - M^{(ii)} & \in & N_n^+ & , i = 1, ..., n, \\
M_{ii}^{(ii)} & \geqslant & 0 & , i = 1, ..., n, \\
2M_{ii}^{(ij)} + 2M_{ij}^{(ii)} & \geqslant & 0 & i \neq j \\
M_{ii}^{(jj)} + M_{jj}^{(ii)} + 4M_{ij}^{(ij)} & \geqslant & 0 & i \neq j \\
2(M_{ii}^{(jk)} + 2M_{ij}^{(ik)} + 2M_{ik}^{(ij)} + M_{jk}^{(ii)}) & \geqslant & 0 & i \neq j, j \neq k, i \neq k \\
4(M_{ij}^{(kl)} + M_{ik}^{(jl)} + M_{il}^{(jk)} + M_{jk}^{(il)} + M_{jl}^{(ik)} + M_{kl}^{(ij)}) & \geqslant & 0 & i < j < k < l.
\end{array}
$$

The higher degree approximation can be formulated in the very similar way. However, the size of the system of inequality grows extremely fast. Therefore, larger degree approximation may not be appropriated.