APPENDIX

(A.1.)

Since $V_{1,i}(x(t)) = x^T(t)P_ix(t) \geq \min_i\{\lambda_m(P_i)\} \parallel x(t) \parallel^2$.

we have $\alpha_1 = \min_i\{\lambda_m(P_i)\}$.

$V_{1,i}(x(t)) = x^T(t)P_ix(t) \leq \max_i\{\lambda_M(P_i)\} \parallel x(t) \parallel^2$,

$$
\begin{aligned}
V_{2,i}(x_t) &= \int_{t-h(t)}^{t} e^{2\beta(s-t)}x^T(s)Q_ix(s)ds \\
&\leq \max_i\{\lambda_M(Q_i)\} \parallel x_t \parallel^2 \int_{t-h(t)}^{t} e^{2\beta(s-t)}ds \\
&\leq h_M \max_i\{\lambda_M(Q_i)\} \parallel x_t \parallel^2 \qquad \because e^{2\beta(s-t)} \leq 1, \\
V_{3,i}(x_t) &= \int_{-h(t)}^{0}\int_{t+s}^{t} e^{2\beta(\xi-t)}x^T(\xi)R_ix(\xi)d\xi ds \\
&\leq \int_{-h(t)}^{0} [\max_i\{\lambda_M(R_i)\} \parallel x_t \parallel^2 \int_{t+s}^{t} e^{2\beta(\xi-t)}d\xi]ds \\
&\leq \int_{-h(t)}^{0} [\max_i\{\lambda_M(R_i)\} \parallel x_t \parallel^2 \int_{t+s}^{t} 1\ d\xi]ds \qquad \because e^{2\beta(s-t)} \leq 1 \\
&\leq \max_i\{\lambda_M(R_i)\} \parallel x_t \parallel^2 \int_{-h(t)}^{0} -s\ ds \\
&\leq \frac{h_M^2}{2} \max_i\{\lambda_M(R_i)\} \parallel x_t \parallel^2, \\
V_{4,i}(x_t) &= \int_{-h(t)}^{0}\int_{t+s}^{t} e^{2\beta(\xi-t)} \begin{bmatrix} x(\xi) \\ x(\xi-h(\xi)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(\xi) \\ x(\xi-h(\xi)) \end{bmatrix} d\xi ds \\
&\leq \int_{-h(t)}^{0} [2\max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix})\} \parallel x_t \parallel^2 \int_{t+s}^{t} e^{2\beta(\xi-t)}d\xi]ds \\
&\leq \int_{-h(t)}^{0} [2\max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix})\} \parallel x_t \parallel^2 \int_{t+s}^{t} 1\ d\xi]ds \qquad \because e^{2\beta(s-t)} \leq 1 \\
&\leq 2\max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix})\} \parallel x_t \parallel^2 \int_{-h(t)}^{0} -s\ ds \\
&\leq h_M^2 \max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix})\} \parallel x_t \parallel^2, \\
V_{5,i}(x_t) &= \int_{-h(t)}^{0}\int_{t+s}^{t} \dot{x}^T(\xi)T_i\dot{x}(\xi)d\xi ds \\
&= \int_{-h(t)}^{0}\int_{t+s}^{t} [A_ix(\xi) + B_ix(\xi-h(\xi)]^T T_i[A_ix(\xi) + B_ix(\xi-h(\xi)]d\xi ds
\end{aligned}
$$

$$
\begin{aligned}
&\leq \int_{-h(t)}^{0} 4\max_i\{\lambda_M(A_i^T T_i A_i), \lambda_M(A_i^T T_i B_i), \lambda_M(B_i^T T_i A_i), \lambda_M(B_i^T T_i B_i)\} \\
&\quad \times \| x_t \|^2 \int_{t+s}^{t} 1 d\xi ds \\
&\leq 4\max_i\{\lambda_M(A_i^T T_i A_i), \lambda_M(A_i^T T_i B_i), \lambda_M(B_i^T T_i A_i), \lambda_M(B_i^T T_i B_i)\} \\
&\quad \times \| x_t \|^2 \int_{-h(t)}^{0} -s \; ds \\
&\leq 2h_M \max_i\{\lambda_M(A_i^T T_i A_i), \lambda_M(A_i^T T_i B_i), \lambda_M(B_i^T T_i A_i), \lambda_M(B_i^T T_i B_i)\} \\
&\quad \times \| x_t \|^2 .
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
\alpha_2 &= \max_i\{\lambda_M(P_i)\} + h_M \max_i\{\lambda_M(Q_i)\} + \frac{h_M^2}{2}\max_i\{\lambda_M(R_i)\} \\
&\quad + h_M^2 \max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix})\} \\
&\quad + 2h_M^2 \max_i\{\lambda_M(A_i^T T_i A_i), \lambda_M(A_i^T T_i B_i), \lambda_M(B_i^T T_i A_i), \lambda_M(B_i^T T_i B_i)\}. \\
\alpha_3 &= \max_i\{\lambda_M(P_i)\} + h_M \max_i\{\lambda_M(Q_i)\} + \frac{h_M^2}{2}\max_i\{\lambda_M(R_i)\} \\
&\quad + h_M^2 \max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix})\}.
\end{aligned}
$$

(A.2.)

From $\dot{V}_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Omega_{1,i} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}$.

Find ξ_i

$$
\begin{aligned}
\dot{V}_i(x_t) &\leq \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Omega_{1,i} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} \\
&\leq \sum_{i=1}^{N} \lambda_i(t) \quad \max_i\{\lambda_M(\Omega_{1,i})\} \| \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} \|^2 \\
&\leq \sum_{i=1}^{N} \lambda_i(t) \quad 2\max_i\{\lambda_M(\Omega_{1,i})\} \| x_t \|^2 \\
&= \sum_{i=1}^{N} \lambda_i(t) \frac{2\max_i\{\lambda_M(\Omega_{1,i})\}}{\min_i\{\lambda_m(P_i)\}} \min_i\{\lambda_m(P_i)\} \| x_t \|^2 \\
&\leq \sum_{i=1}^{N} \lambda_i(t) \frac{2\max_i\{\lambda_M(\Omega_{1,i})\}}{\min_i\{\lambda_m(P_i)\}} V_{1,i}(x_t) \\
&\leq \sum_{i=1}^{N} \lambda_i(t) \frac{2\max_i\{\lambda_M(\Omega_{1,i})\}}{\min_i\{\lambda_m(P_i)\}} \; V_i(x_t)
\end{aligned}
$$

This implies that $V_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \parallel V_i(x_{t_0}) \parallel e^{\xi_i(t-t_0)}, t \geq t_0,$

where $\xi_i = \frac{2\max_i\{\lambda_M(\Omega_{1,i})\}}{\min_i\{\lambda_m(P_i)\}}$.

(A.3.)

$$\text{From } \dot{V}_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Omega_{2,i} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} - 2\beta V_{2,i}(x_t)$$
$$- (2\beta + \frac{1}{h_M})(V_{3,i}(x_t) + V_{4,i}(x_t)) - \frac{1}{2h_M} V_{5,i}(x_t)$$
$$- \int_{t-h(t)}^{t} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \dot{x}(s) \end{bmatrix}^T \Omega_{3,i} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \dot{x}(s) \end{bmatrix} ds.$$

Find ζ_i

Since $\Omega_{3,i} \geq 0$ we have

$$\dot{V}_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Omega_{2,i} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} - 2\beta(V_{2,i}(x_t) + V_{3,i}(x_t)$$
$$+ V_{4,i}(x_t)) - \frac{1}{2h_M} V_{5,i}(x_t)$$
$$\leq \sum_{i=1}^{N} \lambda_i(t) \quad - \min_i\{\lambda_m(-\Omega_{2,i})\} \parallel \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} \parallel^2 - 2\beta(V_{2,i}(x_t) + V_{3,i}(x_t)$$
$$+ V_{4,i}(x_t)) - \frac{1}{2h_M} V_{5,i}(x_t)$$
$$\leq \sum_{i=1}^{N} \lambda_i(t) \quad - \min_i\{\lambda_m(-\Omega_{2,i})\} \parallel x(t) \parallel^2 - 2\beta(V_{2,i}(x_t) + V_{3,i}(x_t) + V_{4,i}(x_t))$$
$$- \frac{1}{2h_M} V_{5,i}(x_t)$$
$$\leq \sum_{i=1}^{N} \lambda_i(t) \quad - \frac{\min_i\{\lambda_m(-\Omega_{2,i})\}}{\max_i\{\lambda_M(P_i)\}} x^T(t) P_i x(t) - 2\beta(V_{2,i}(x_t) + V_{3,i}(x_t) + V_{4,i}(x_t))$$
$$- \frac{1}{2h_M} V_{5,i}(x_t)$$
$$\leq \sum_{i=1}^{N} \lambda_i(t) \quad - \frac{\min_i\{\lambda_m(-\Omega_{2,i})\}}{\max_i\{\lambda_M(P_i)\}} V_1(x(t)) - 2\beta(V_{2,i}(x_t) + V_{3,i}(x_t) + V_{4,i}(x_t))$$
$$- \frac{1}{2h_M} V_{5,i}(x_t)$$

$$\leq \sum_{i=1}^{N} \lambda_i(t) \quad - \min\{\frac{\min_i\{\lambda_m(-\Omega_{2,i})\}}{\max_i\{\lambda_M(P_i)\}}, 2\beta, \frac{1}{2h_M}\}V_i(x_t)$$

This implies that $V_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \parallel V_i(x_{t_0}) \parallel e^{-\zeta_i(t-t_0)}, t \geq t_0,$

where $\zeta_i = \min\{\frac{\min_i\{\lambda_m(-\Omega_{2,i})\}}{\max_i\{\lambda_M(P_i)\}}, 2\beta, \frac{1}{2h_M}\}.$

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