CHAPTER 1

INTRODUCTION

There has been worldwide interest in the study of switched systems in recent years. These systems have been widely applied to the walking robots, biological cell growth and division, air and ground transportation systems, machine industrial systems and so on.

Switching system is a class of hybrid systems consisting of discrete or continuous subsystems and a switching rule indicating the active subsystem at each instant of time. For switched systems, one of the most important and challenging problems is to find the switching laws, i.e., what switching laws can guarantee the switched systems stable. Therefore several researchers have studied switched system and provided sufficient conditions to guarantee the stability of switched systems with delay.

In 2004, M.Wu, Y.He, J.H. She and G.P. Liu [17] have studied the delay-dependent robust stability for time-varying delay systems described by

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]x(t - d(t)), \ t > 0, \\ x(t) = \phi(t), \ t \in [-\tau, 0], \end{cases} \tag{1.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector. The time delay $d(t)$ is a time-varying continuous function that satisfies

$$0 \leq d(t) \leq \tau, \quad \dot{d}(t) \leq \mu < 1,$$

where τ and μ are constants and the initial condition, $\phi(t)$ is a continuous vector valued initial function of $t \in [-\tau, 0]$. $A, B \in \mathbb{R}^{n \times n}$ are given constant matrices. The uncertainties are assumed to be of the form

$$[\Delta A(t) \ \Delta B(t)] = DF(t)[E_a \ E_b],$$

where D, E_a and E_b are constant matrices with appropriate dimensions and $F(t)$ is an unknown, real matrices satisfying $F^T(t)F(t) \leq I$.

The following theorems are the main results in their studies.

Theorem 1.1 [17] *Given scalars $\tau > 0$ and $\mu < 1$, the system (1.1) with $\Delta A(t) = \Delta B(t) = 0$ is asymptotically stable if there exist symmetric positive definite matrices P, Q and Z, a symmetric semipositive definite matrix $X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \geq 0$, and any appropriately dimensioned matrices Y and T such that the following LMIs are true*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \tau A^T Z \\ * & \Phi_{22} & \tau B^T Z \\ * & * & -\tau Z \end{bmatrix} < 0 \quad and \quad \Psi = \begin{bmatrix} X_{11} & X_{12} & Y \\ * & X_{22} & T \\ * & * & Z \end{bmatrix} \geq 0, \tag{1.2}$$

where

$\Phi_{11} = PA + A^T P + Y + Y^T + Q + \tau X_{11}$,

$\Phi_{12} = PB - Y + T^T + \tau X_{12}$,

$\Phi_{22} = -T - T^T - (1-\mu)Q + \tau X_{22}$,

and $*$ denotes the symmetric terms in a symmetric matrix.

Theorem 1.2 [17] *Given scalars $\tau > 0$ and $\mu < 1$, the system (1.1) is robust asymptotically stable if there exist symmetric positive definite matrices P, Q and Z, a symmetric semipositive definite matrix $X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \geq 0$, and any appropriately dimensioned matrices Y and T such that the following LMIs are true*

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \tau A^T Z & PD \\ * & \Omega_{22} & \tau B^T Z & 0 \\ * & * & -\tau Z & \tau Z D \\ * & * & * & -I \end{bmatrix} < 0 \quad and \quad \Psi = \begin{bmatrix} X_{11} & X_{12} & Y \\ * & X_{22} & T \\ * & * & Z \end{bmatrix} \geq 0, \tag{1.3}$$

where

$\Omega_{11} = PA + A^T P + Y + Y^T + Q + \tau X_{11} + E_a^T E_a$,

$\Omega_{12} = PB - Y + T^T + \tau X_{12} + E_a^T E_b$,

$\Omega_{22} = -T - T^T - (1-\mu)Q + \tau X_{22} + E_b^T E_b$.

Y. Zhang, X. Liu and X. Shen [20] have given the following definition about the switched systems.

Definition 1.1 [20] *Consider the differential system:*

$$\dot{x}(t) = [A_i + \Delta A_i(t)]x(t),\ \ t > 0, \tag{1.4}$$

where $A_i,\ i \in I_k = \{1, 2, ..., k\}$ *are constant matrices,* $\Delta A_i(t)$ *are uncertain matrices which are of the form*

$$\Delta A_i(t) = D_i F_i(t) E_i, \tag{1.5}$$

where D_i, E_i *are unknown constant matrices of appropriate dimension and* $F_i(t)$ *are unknown matrices satisfying*

$$F_i^T(t) F_i(t) \leq I.$$

The system (1.4) is called the uncertain linear switched system with k subsystems.
$I(t) : \mathbb{R}^+ \rightarrow I_k = \{1, 2, ..., k\}, t \in [t_N, t_{N+1}), N = 0, 1, 2, ...$
$I(t)$ is the switching signal determines which subsystem is activated at certain time interval.
Assumption of the switched systems:
(1) The system will switch forever (will not stop switching).
(2) For any consecutive interval $[t_N, t_{N+1}), [t_{N+1}, t_{N+2})$*, the active subsystems are different.*
(3) A solution of the switched system is a continuous function.

Definition 1.2 [20] *The system (1.4) is said to be robustly stable if the trivial solution of system (1.4) is asymptotically stable for all uncertainties satisfying (1.5).*

Definition 1.3 [20] $T_0 = \inf\{t_i - t_{i-1}\}$ *is called the dwell time of switched system.*

In 2005, H. Huang, Y. Hu and H.X. Li [8] have studied the robust stability of switched Hopfield neural networks with time-varying delay under uncertainty given by

$$\dot{u}(t) = \sum_{i=1}^{N} \xi_i(t)[-(C_i + \Delta C_i(t))u(t) + (B_i + \Delta B_i(t))g(u(t - \tau(t)))] \tag{1.6}$$

Define the indicator function $\xi(t) = [\xi_1(t), \xi_2(t), ..., \xi_N(t)]^T$, where

$$\xi_i(t) = \begin{cases} 1, & \text{when the switched system is described by the } i\ th \text{ mode,} \\ 0, & \text{otherwise,} \end{cases}$$

with $i = 1, 2, ..., N$. The following assumptions are further made:

$(H1)$ There exists a positive diagonal matrix $K = diag(k_1, k_2, ..., k_n) > 0$, such that the activations g_j satisfy

$$| g_j(x) | \leq k_j | x |, \ \forall x \in \mathbb{R}, \ j = 1, 2, ..., n.$$

$(H2)$ The time-varying delay $\tau(t)$ is bounded on $\mathbb{R}$ (i.e., $0 \leq \tau(t) \leq \tau$) and is a differentiable function with $\dot{\tau}(t) \leq \zeta < 1$, where τ and ζ are positive constants.

$(H3)$ The parametric uncertainties $\Delta C_i(t), \Delta B_i(t)$ are time variant and unknown, but norm bounded. The uncertainties are of the following form:

$$[\Delta C_i(t) \ \Delta B_i(t)] = DF(t)[E_i^C \ E_i^B],$$

in which D, E_i^C, E_i^B are known real constant matrices with appropriate dimensions. The uncertain matrix $F(t)$ satisfies

$$F^T(t)F(t) \leq I, \ \forall t \in \mathbb{R}.$$

They gave the following condition for robust stability of system (1.6).

Theorem 1.3 [8] *Assume that the activation function g, time-varying delay $\tau(t)$ and parametric uncertainties $\Delta C_i(t), \Delta B_i(t)$ satisfy $(H1), (H2)$ and $(H3)$, respectively. Then the switched Hopfield neural network (1.6) is globally exponentially stable if there exist a matrix $P > 0$, a diagonal $Q > 0$ and two positive scalars ρ, σ, such that the following LMIs hold for $i = 1, 2, ..., N$:*

$$\Sigma_i = \begin{bmatrix} \Theta_i & PB_i & 0 & \sqrt{\rho + \sigma} PD \\ * & -(1-\zeta)Q & \sqrt{\sigma^{-1}}(E_i^B)^T & 0 \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \qquad (1.7)$$

in which $\Theta_i = -PC_i - C_iP + \rho^{-1}(E_i^C)^T E_i^C + KQK$.

In 2008, M.S. Alwan and X. Liu [1] have studied the linear switched systems with time delay described by

$$\dot{x}(t) = A_i x(t) + B_i x(t-\tau),\ t \in [t_{k-1}, t_k), \tag{1.8}$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, τ is constant time-delay. A_i, B_i are $n \times n$ constant matrices. Let $S_u = \{1, 2, ..., r\}$ and $S_s = \{r+1, r+2, ..., N\}$ be the set of indices of the unstable and stable modes, respectively. $i \in S = S_u \cup S_s$.

The following theorem is the main result in his study.

Theorem 1.4 [1] *The trivial solution of system (1.8) is globally exponentially stable if the following assumptions hold:*

A1 (i) For $i \in S_u$,

$$Re[\lambda(A_i)] > 0 \text{ and } Re[\lambda(A_i + B_i)] > 0.$$

(ii) For $i \in S_s$, A_i is Hurwitz (i.e., $Re[\lambda(A_i)] < 0$) and

$$-\left(\frac{\lambda_m(Q_i) - \beta_i^*}{\lambda_M}\right) + \frac{\beta_i^*}{\lambda_m} < 0.$$

where $\beta_i^* = \| P_i B_i \|, \lambda_M = max\{\lambda_M(P_i), \forall i \in S\}, \lambda_m = min\{\lambda_m(P_i), \forall i \in S\}$,

$$A_i^T P_i + P_i A_i = -Q.$$

A2 Let $\lambda^+ = max\{\xi_i, \forall i \in S_u\}$ and $\lambda^- = min\{\zeta_i, \forall i \in S_s\}$ with ξ_i and ζ_i being respectively the growth rates of the unstable modes and the decay rates of the stable modes, and $T^+(t_0, t)$, $T^-(t_0, t)$ denote the total activation times of the unstable and stable modes over (t_0, t), respectively. Assume that, for any t_0, the switching law guarantees that

$$\inf_{t \geq t_0} \frac{T^-(t_0, t)}{T^+(t_0, t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*},$$

where $\lambda^* \in (0, \lambda^-)$, furthermore, there exists $0 < \nu < \lambda^*$ such that

(i) for $i \in S_u$

$$ln\mu - \nu(t_k - t_{k-1}) \leq 0,\ k = 1, 2, ..., r.$$

(ii) for $i \in S_s$

$$ln\mu + \zeta_i \tau - \nu(t_k - t_{k-1}) \leq 0,\ k = r+1, r+2, ..., N.$$

Moreover, M.S. Alwan and X. Liu [1] have studied the weakly nonlinear switched systems with time delay described by

$$\dot{x}(t) = A_i x(t) + g_i(t, x, x_t), \tag{1.9}$$

where $i \in S = S_s \cup S_u$ and $g_i(t, 0, 0) \equiv 0$ for all t and $i \in S$.

The sufficient conditions to guarantee exponential stability of the trivial solution are stated in the following theorem.

Theorem 1.5 [1] *The trivial solution of system (1.9) is globally exponentially stable if the following assumptions hold:*

A1 (i) For $i \in S_u$,

$$Re[\lambda(A_i)] > 0.$$

(ii) For $i \in S_s$, A_i *is Hurwitz.*

A2 For each $i \in S$, *there exist positive constants* a_i *and* b_i *such that*

$$2x^T(t)P_i g_i(t, x, x_t) \leq a_i \parallel x(t) \parallel^2 + b_i \parallel x_t \parallel_\tau^2,$$

where $\parallel x_t \parallel_\tau = \sup_{t-\tau \leq \theta \leq t} \parallel x(\theta) \parallel$.

A3 For $i \in S_s$,

$$-\left(\frac{\lambda_m(Q_i) - a_i}{\lambda_M}\right) + \frac{b_i}{\lambda_m} < 0.$$

A4 Assumption A2 of theorem 1.4 holds.

In 2008, J. Liu, X. Liu and W.C. Xie [15], have studied the delay-dependent robust control for uncertain switched systems with time-delay defined by the following state equations:

$$\begin{cases} \dot{x}(t) = [A_i + \Delta A_i(t)]x(t) + [\tilde{A}_i + \Delta\tilde{A}_i(t)]x(t-h) \\ \qquad\qquad + [B_i + \Delta B_i(t)]u(t), \ t > 0, \\ x(t) = \phi(t), \ t \in [-h, 0], \end{cases} \tag{1.10}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input and h is the constant time-delay. $A_i, \tilde{A}_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are constant real matrices. $I : \mathbb{R}^+ \to I_k \triangleq \{1, 2, ..., k\}$ is the switching signal, which is assumed to be piecewise constant on $\mathbb{R}^+$ and has only finite discontinuous instants in any bounded interval. $\Delta A_i, \Delta\tilde{A}_i$ and ΔB_i are time-dependent uncertainties of the form

$$\Delta A_i = D_{1,i}\Xi_{1,i}(t)E_{1,i},\ \Delta\tilde{A}_i = D_{2,i}\Xi_{2,i}(t)E_{2,i},\ \Delta B_i = D_{3,i}\Xi_{3,i}(t)E_{3,i},$$

where $D_{j,i}$ and $E_{j,i}, (i \in I_k, j = 1,2,3)$ are known constant real matrices of appropriate dimensions. $\Xi_{j,i}(t), (i \in I_k, j = 1,2,3)$ are unknown matrix functions of time t satisfying $\| \Xi_{j,i}(t) \| \leq 1$.

The main result obtained in [15] is the following.

Theorem 1.6 [15] *Given $h > 0$ and $\gamma > 1$,*

(i) if there exist some positive numbers $\varepsilon_{j,i}(j = 1,2,...,8, i \in I_k)$, positive definite matrices P_i and $T_i(i \in I_k)$, non-singular matrices G_i such that the following matrix inequality is satisfied:

$$\Omega_i = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ * & \Sigma_{22} & \Sigma_{23} \\ * & * & \Sigma_{33} \end{bmatrix} < 0. \tag{1.11}$$

(ii) if

$$T_0 \geq \frac{1}{\rho} ln \frac{\lambda_1}{\alpha\lambda_2}, \tag{1.12}$$

where $0 < \alpha < 1$ is some real number,

$$\rho = \tfrac{1}{2}\min\{\frac{\lambda}{\lambda_1}, \frac{1}{h\gamma}, \frac{\lambda_2}{\lambda_3}\},$$

and T_0 is the dwell time, then for any given switching law subject to (1.12), the uncertain switched system (1.10) is robustly stable,

where

$$\Sigma_{11} = P_i(A_i + G_i) + (A_i + G_i)^T P_i + \varepsilon_{1,i}^{-1} P_i D_{1,i} D_{1,i}^T P_i + \varepsilon_{1,i} E_{1,i}^T E_{1,i} + \varepsilon_{3,i}^{-1} P_i D_{2,i} D_{2,i}^T P_i + \varepsilon_{5,i} E_{1,i}^T E_{1,i} + \gamma h G_i^T P_i G_i + T_i,$$

$$\Sigma_{22} = -h^{-1}(\gamma - 1)P_i + \varepsilon_{5,i}^{-1} P_i D_{1,i} D_{1,i}^T P_i + \varepsilon_{7,i}^{-1} P_i D_{2,i} D_{2,i}^T P_i,$$

$$\Sigma_{33} = -T_i + \varepsilon_{3,i} E_{2,i}^T E_{2,i} + \varepsilon_{7,i} E_{2,i}^T E_{2,i},$$

$$\Sigma_{12} = (A_i + G_i)^T P_i,$$

$$\Sigma_{13} = P_i(\tilde{A}_i - G_i),$$

$$\Sigma_{23} = P_i(\tilde{A}_i - G_i),$$

$$\lambda_1 = \max_i\{\lambda_M(P_i)\},$$

$$\lambda_2 = \min_i\{\lambda_m(P_i)\},$$

$\lambda_3 = \min_i\{\lambda_M((G_i^T G_i)^{-1}T_i)\}$,
$\lambda = \max_i\{\lambda_m(-\Omega_i)\}$.

In 2009, L.V. Hien, Q.P. Ha and V.N. Phat [6] have studied the stability of uncertain switched linear systems with time delay described by

$$\begin{cases} \dot{x}(t) = [A_\sigma + \Delta A_\sigma(t)]x(t) + [D_\sigma + \Delta D_\sigma(t)]x(t-h(t)),\ t \geq 0, \\ x(t) = \phi(t),\ t \in [-h_M, 0], \end{cases} \tag{1.13}$$

where $x(t) \in \mathbb{R}^n$ is the system state; $\sigma(\cdot) : \mathbb{R}^n \to \mathcal{I} := \{1, 2, ..., N\}$ is the switching function, which is piece-wise constant function depending on the state at each time and will be designed. $A_\sigma, D_\sigma \in \{[A_i, D_i], i = 1, 2, ..., N\}, A_i, D_i$ are given matrices and $\phi(t) \in C([-h, 0], \mathbb{R}^n)$ is the initial function with $\| \phi \| = \sup_{s\in[-h,0]} \| \phi(s) \|$. The uncertainties satisfy the following conditions:

$$\Delta A_i(t) = E_{0i}F_{0i}(t)H_{0i},\ \Delta D_i(t) = E_{1i}F_{1i}(t)H_{1i},$$

where $E_{ki}, H_{ki},\ k = 0, 1,\ i = 1, 2, ..., N$ are given constant matrices with appropriate dimensions. $F_{ki}(t)$ are unknown, real matrices satisfying

$$F_{ki}^T(t)F_{ki}(t) \leq I,\ k = 0, 1,\ i = 1, 2, ..., N,\ \forall t \geq 0.$$

The time-varying delay function $h(t)$ is assumed to satisfy the following condition:

$$0 \leq h(t) \leq h,\ \dot{h}(t) \leq \mu < 1,\ t \geq 0,$$

where h and μ are given constants.

The following definitions are presented to formulate the problem.

Definition 1.4 [6] *Given $\beta > 0$. The system (1.13) is $\beta-$exponentially stable if there exists a switching function $\sigma(\cdot)$ and positive number γ such that any solution $x(t, \phi)$ of the system satisfies*

$$\| x(t, \phi) \| \leq \gamma e^{-\beta t} \| \phi \|,\ \forall t \in \mathbb{R}^+,$$

for all the uncertainties.

Definition 1.5 [6] *The system of matrices* $\{L_i\}, i = 1, 2, ..., N$, *is said to be strictly complete if for every* $x \in \mathbb{R}^n \setminus \{0\}$ *there is* $i \in \{1, 2, ..., N\}$ *such that* $x^T L_i x < 0$.

Remark 1.1 [6] *A sufficient condition for the strictly completeness of the system* $\{L_i\}$ *is that there exist* $\xi_i \geq 0, i = 1, 2, ..., N$ *such that* $\sum_{i=1}^{N} \xi_i > 0$ *and*

$$\sum_{i=1}^{N} \xi_i L_i < 0.$$

For given numbers β, h and μ, symmetric positive definite matrix P we set

$\tau = (1 - \mu)^{-1}, \eta = \tau e^{2\beta h} + 2\beta,$

$S_i = E_{0i}E_{0i}^T + e^{2\beta h}E_{1i}E_{1i}^T, Q = \sum_{i=1}^{N} D_i^T P D_i, R = \sum_{i=1}^{N} H_{1i}^T H_{1i},$

$L_i(P) + A_i^T P + PA_i + H_{0i}^T H_{0i} + PS_iP + Q_\tau R + \eta P,$

$\alpha_1 = \lambda_{min}(P), \alpha_2 = \lambda_{max}(P) + h[\sum_{i=1}^{N} \lambda_{max}(D_i^T P D_i) + \tau \sum_{i=1}^{N} \lambda_{max}(H_{1i}^T H_{1i})].$

The main result [6] is stated in the following.

Theorem 1.7 [6] *The system (1.13) is* $\beta-$*exponentially stable if there exists a symmetric positive definite matrix* P *such that the system of matrices* $\{L_i(P)\}, i = 1, 2, ..., N$ *is strictly complete.*

Moreover, the solution $x(t, \phi)$ *of the system satisfies*

$$\| x(t, \phi) \| \leq \gamma e^{-\beta t} \| \phi \|, \ \forall t \in \mathbb{R}^+.$$

The switching rule can be chosen as

$$\sigma(x(t)) = arg\ min\{x^T(t)L_i(P)x(t)\}, \ t \geq 0.$$

In 2009, V.N. Phat, T. Botmart and P. Niamsup [16] have studied the stability of uncertain switched linear systems with time delay given by

$$\begin{cases} \dot{x}(t) = A_\alpha x(t) + D_\alpha x(t - h(t)) + f_\alpha(x(t), x(t - h(t))), \ t \geq 0, \\ x(t) = \phi(t), \ t \in [-h, 0], \end{cases} \tag{1.14}$$

where $x(t) \in \mathbb{R}^n$ is the state. $A_\alpha, D_\alpha \in \mathbb{R}^{n \times n}$ are given constant matrices, $f_\alpha(x, y) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is the given nonlinear perturbation; $\phi(t) \in C([-h, 0], \mathbb{R}^n)$ is the initial function with the norm $\| \phi \| = \sup_{s\in[-h,0]} \| \phi(s) \|$. $\alpha(x) : \mathbb{R}^n \to \Omega := \{1, 2, ..., N\}$ is the switching rule. The delay function satisfying:

$(D.1)$ $0 \le h(t) \le h, \dot{h}(t) \le \delta < 1, \ \forall t \ge 0$, or

$(D.2)$ $0 \le h(t) \le h, \ \forall t \ge 0$.

The nonlinear perturbation $f_i(\cdot), \ i = 1, 2, ..., N$, satisfies the following condition

$$\exists a_i > 0, b_i > 0, :\| f_i(x, y) \| \le a_i \| x \| + b_i \| y \|, \ \forall x, y. \tag{1.15}$$

For given positive numbers h, a, b, β and δ, we set $\mu = (1 - \delta)^{-1}$,

$$L_i(P) = (A_i + D_i)^T P + P(A_i + D_i) + 2\beta P + 2he^{2\beta h} PD_i(A_iA_i^T + \mu D_iD_i^T)D_i^T P$$
$$+ 2he^{2\beta h}(a^2 + \mu b^2)PD_iD_i^T P + (a^2 + b^2e^{2\beta h}\mu)P^2 + 2(h + 1)I,$$

$$S_i^P = \{x \in \mathbb{R}^n : x^T L_i(P)x < 0\}, \ \bar{S}_1^P = S_1^P, \bar{S}_i^P = S_i^P \setminus \bigcup_{j=1}^{i-1} \bar{S}_j^P, i = 2, ..., N,$$

$$M = \sqrt{\frac{\lambda_{max}(P)+h+2h^2}{\lambda_{min}(P)}}, a = max\{a_1, a_2, ..., a_N\}, b = max\{b_1, b_2, ..., b_N\}.$$

The following theorem is the main result in their studies.

Theorem 1.8 [16] *Assume the conditions $(D.1)$ and (1.15). Switched nonlinear system (1.14) is exponentially stable if there exist a positive number β and a symmetric positive definite matrix P such that one of the following conditions holds:*

(i) The system of matrices $\{L_i(P)\}, \ i = 1, 2, ..., N$, is strictly complete.

(ii) There exists $\tau_i \ge 0, \ i = 1, 2, ..., N$, with $\sum_{i=1}^{N} \tau_i > 0$ such that

$$\sum_{i=1}^{N} \tau_i L_i(P) < 0.$$

The switching rule is chosen in case (i) as $\alpha(x(t)) = i$ whenever $x(t) \in \bar{S}_i$ and in case (ii) as

$$\alpha(x(t)) = arg \ min\{x^T(t)L_i(P)x(t)\}.$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\| x(t, \phi) \| \le Me^{-\beta t} \| \phi \|, \ \forall t > 0.$$

In summary, to study the stability of switched systems with time delay from [1], authors used there idea consider switching between stable and unstable modes, switching laws depend on dwell-time technique while [15] studied the stability of time-varying delay switched systems which switching especially stable modes, switching laws depend on dwell-time technique. In [6] the switching laws depend on the solution of the system. From [9],[10],[13],[17] and [19], authors used an interesting techniques: construct the Lyapunov function and add some free matrix variables. Motivated by these result [1],[6] and [15] so in this thesis, we propose to study the exponentially stable of uncertain switched systems described by

$$\begin{cases} \dot{x}(t) = [A_\sigma + \Delta A_\sigma(t)]x(t) + [B_\sigma + \Delta B_\sigma(t)]x(t - h(t)) \\ \qquad + f_\sigma(t, x(t), x(t - h(t))),\ t > 0, \\ x(t) = \phi(t),\ t \in [-h_M, 0], \end{cases} \tag{1.16}$$

where $x(t) \in \mathbb{R}^n$ is the state vector. $\sigma(\cdot) : \mathbb{R}^n \to S = \{1, 2, ..., N\}$ is the switching function. Let $i \in S = S_u \cup S_s$ such that $S_u = \{1, 2, ..., r\}$ and $S_s = \{r + 1, r + 2, ..., N\}$ be the set of the unstable and stable modes, respectively. N denotes the number of subsystems. $A_i, B_i \in \mathbb{R}^{n\times n}$ are given constant matrices. $\Delta A_i(t), \Delta B_i(t)$ are uncertain matrices satisfy the following conditions:

$$\Delta A_i(t) = E_{1i}F_{1i}(t)H_{1i},\ \Delta B_i(t) = E_{2i}F_{2i}(t)H_{2i}, \tag{1.17}$$

where $E_{ji}, H_{ji},\ j = 1, 2,\ i = 1, 2, ..., N$ are given constant matrices with appropriate dimensions. $F_{ji}(t)$ are unknown, real matrices satisfying:

$$F_{ji}^T(t)F_{ji}(t) \leq I,\ j = 1, 2,\ i = 1, 2, ..., N,\ \forall t \geq 0, \tag{1.18}$$

where I is the identity matrix of appropriate dimension.
The nonlinear perturbation $f_i(t, x(t), x(t - h(t))),\ i = 1, 2, ..., N$ satisfies the following condition:

$$\| f_i(t, x(t), x(t - h(t))) \| \leq \gamma_i \| x(t) \| + \delta_i \| x(t - h(t)) \| . \tag{1.19}$$

The time-varying delay function $h(t)$ is assumed to satisfy one of the following conditions:

(i) when $\Delta A_i(t) = 0$ and $\Delta B_i(t) = 0$ and $f_i(t, x(t), x(t - h(t))) = 0$

$$0 \leq h_m \leq h(t) \leq h_M,\ \dot{h}(t) \leq \mu,\ t \geq 0, \tag{1.20}$$

(ii) when $\Delta A_i(t) \neq 0$ or $\Delta B_i(t) \neq 0$ or $f_i(t, x(t), x(t-h(t))) \neq 0$

$$0 \leq h_m \leq h(t) \leq h_M, \ \dot{h}(t) \leq \mu < 1, \ t \geq 0, \tag{1.21}$$

where h_m, h_M and μ are given constants.

In Chapter 3, we give sufficient conditions for exponentially stable of zero solution of uncertain switched system with time-varying delay. Numerical examples are illustrated to show the efficiency of our theoretical results. Conclusion is provided in Chapter 4.