# CHAPTER 4

# CONCLUSION

In this work, we study the exponential stability of zero solution for uncertain switched system with time-varying delay. We give sufficient conditions for switched system without uncertainties and switched system with uncertainties. The main results are summarized as follows:

For simplicity of later presentation, we use the following notations:

$\lambda^+ = \max_i\{\xi_i, \forall i \in S_u\}$, $\xi_i$ denotes the growth rates of the unstable modes.

$\lambda^- = \min_i\{\zeta_i, \forall i \in S_s\}$, $\zeta_i$ denotes the decay rates of the stable modes.

$T^+(t_0, t)$ denotes the total activation times of the unstable modes over $(t_0, t)$.

$T^-(t_0, t)$ denotes the total activation times of the stable modes over $(t_0, t)$.

$N(t)$ denotes the number of times the system is switched on $(t_0, t)$.

$l(t)$ denotes the number of times the unstable subsystems are activated on $(t_0, t)$.

$N(t) - l(t)$ denotes the number of times the stable subsystems are activated on $(t_0, t)$.

$$\psi = \frac{\max_i\{\lambda_M(P_i)\}}{\min_j\{\lambda_m(P_j)\}}.$$

$$\alpha_1 = \min_i\{\lambda_m(P_i)\},$$

$$\begin{aligned}\alpha_2 = {} & \max_i\{\lambda_M(P_i)\} + h_M \max_i\{\lambda_M(Q_i)\} + \frac{h_M^2}{2}\max_i\{\lambda_M(R_i)\} \\ & + h_M^2 \max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i}\end{bmatrix})\} \\ & + 2h_M^2 \max_i\{\lambda_M(A_i^T T_i A_i), \lambda_M(A_i^T T_i B_i), \lambda_M(B_i^T T_i A_i), \lambda_M(B_i^T T_i B_i)\},\end{aligned}$$

$$\begin{aligned}\alpha_3 = {} & \max_i\{\lambda_M(P_i)\} + h_M \max_i\{\lambda_M(Q_i)\} + \frac{h_M^2}{2}\max_i\{\lambda_M(R_i)\} \\ & + h_M^2 \max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i}\end{bmatrix})\}.\end{aligned}$$

$$\Omega_{1,i} = \begin{bmatrix} \Phi_{11,i} & \Phi_{12,i} \\ * & \Phi_{13,i}\end{bmatrix},$$

$$\Phi_{11,i} = A_i^T P_i + P_i A_i + Q_i + h_M R_i + h_M S_{11,i} + h_M A_i^T T_i A_i,$$

$$\Phi_{12,i} = B_i^T P_i + h_M S_{12,i} + h_M A_i^T T_i B_i,$$
$$\Phi_{13,i} = -(1-\mu)e^{-2\beta h_M} Q_i + h_M S_{22,i} + h_M B_i^T T_i B_i.$$

$$\Omega_{2,i} = \begin{bmatrix} \Phi_{21,i} & \Phi_{22,i} \\ * & \Phi_{23,i} \end{bmatrix},$$
$$\Phi_{21,i} = A_i^T P_i + P_i A_i + Q_i + h_M R_i + h_M S_{11,i} + h_M A_i^T T_i A_i + h_M X_{11,i} + Y_i + Y_i^T,$$
$$\Phi_{22,i} = B_i^T P_i + h_M S_{12,i} + h_M A_i^T T_i B_i + h_M X_{12,i} - Y_i + Z_i^T,$$
$$\Phi_{23,i} = -(1-\mu)e^{-2\beta h_M} Q_i + h_M S_{22,i} + h_M B_i^T T_i B_i + h_M X_{22,i} - Z_i - Z_i^T.$$

$$\Omega_{3,i} = \begin{bmatrix} X_{11,i} & X_{12,i} & Y_i \\ * & X_{22,i} & Z_i \\ * & * & \frac{T_i}{2} \end{bmatrix}.$$
$$\Xi_i = \begin{bmatrix} \Phi_{31,i} & \Phi_{32,i} \\ * & \Phi_{33,i} \end{bmatrix},$$
$$\Phi_{31,i} = A_i^T P_i + P_i A_i + Q_i + h_M R_i + h_M S_{11,i} + \varepsilon_{1i}^{-1} H_{1i}^T H_{1i} + \varepsilon_{1i} P_i E_{1i}^T E_{1i} P_i + \varepsilon_{2i} P_i E_{2i}^T E_{2i} P_i,$$
$$\Phi_{32,i} = B_i^T P_i + h_M S_{12,i},$$
$$\Phi_{33,i} = -(1-\mu)e^{-2\beta h_M} Q_i + h_M S_{22,i} + \varepsilon_{2i}^{-1} H_{2i}^T H_{2i}.$$

$$\Theta_i = \begin{bmatrix} \Phi_{41,i} & \Phi_{42,i} \\ * & \Upsilon_{43,i} \end{bmatrix},$$
$$\Phi_{41,i} = A_i^T P_i + P_i A_i + Q_i + h_M R_i + h_M S_{11,i} + \varepsilon_{3i}^{-1} \gamma_i I + \varepsilon_{3i} P_i P_i + \varepsilon_{4i}^{-1} H_{4i}^T H_{4i}$$
$$+ \varepsilon_{4i} P_i E_{4i}^T E_{4i} P_i + \varepsilon_{6i} P_i E_{5i}^T E_{5i} P_i,$$
$$\Phi_{42,i} = B_i^T P_i + h_M S_{12,i},$$
$$\Phi_{43,i} = -(1-\mu)e^{-2\beta h_M} Q_i + h_M S_{22,i} + \varepsilon_{3i}^{-1} \delta_i I + \varepsilon_{5i}^{-1} H_{5i}^T H_{5i}.$$



## 4.1 Exponential Stability of Linear Switched System with Time-Varying Delay

In this section, we deal with the problem for exponential stability of the zero solution of system (3.1) without the uncertainties and nonlinear perturbation ($\Delta A_i(t) = \Delta B_i(t) = 0,\ f_i(t, x(t), x(t-h(t))) = 0$).

**Theorem 3.1.1.** *The zero solution of system (3.1) with* $\Delta A_i(t) = \Delta B_i(t) = 0$ *and* $f_i(t, x(t), x(t-h(t))) = 0$ *is exponentially stable if there exist symmetric positive definite matrices* $P_i, Q_i, R_i, \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix}, T_i$ *and appropriate dimension matrices* $Y_i, Z_i$ *such that the following assumptions hold:*

*A*1. $(i)$ *For* $i \in S_u$,

$$\Omega_{1,i} > 0. \tag{3.9}$$

$(ii)$ *For* $i \in S_s$,

$$\Omega_{2,i} < 0 \ \ and \ \ \Omega_{3,i} \geq 0. \tag{3.10}$$

*A*2. *Assume that, for any* $t_0$ *the switching law guarantees that*

$$\inf_{t \geq t_0} \frac{T^-(t_0, t)}{T^+(t_0, t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*} \tag{3.11}$$

*where* $\lambda^* \in (0, \lambda^-)$. *Furthermore, there exists* $0 < \nu < \lambda^*$ *such that*

$(i)$ *If the subsystem* $i \in S_u$ *is activated in time intervals* $[t_{i_k-1}, t_{i_k}), k = 1, 2, ...,$ *then*

$$ln \ \psi - \nu(t_{i_k} - t_{i_k-1}) \leq 0, \ \ k = 1, 2, ... \tag{3.12}$$

$(ii)$ *If the subsystem* $j \in S_s$ *is activated in time intervals* $[t_{j_k-1}, t_{j_k}), k = 1, 2, ...,$ *then*

$$ln \ \psi + \zeta_j h_M - \nu(t_{j_k} - t_{j_k-1}) \leq 0, \ \ k = 1, 2, ... \tag{3.13}$$



## 4.2 Robust Exponential Stability of Linear Switched System with Time-Varying Delay

In this section, we deal with the problem for robust exponential stability of the zero solution of system (3.1) without nonlinear perturbation ($f_i(t, x(t), x(t-h(t))) = 0$).

**Theorem 3.2.1.** *The zero solution of system (3.1) with* $f_i(t, x(t), x(t-h(t))) = 0$ *is robust exponentially stable if there exist positive real numbers* $\varepsilon_{1i}, \varepsilon_{2i}$*, positive definite matrices* $P_i, Q_i, R_i$ *and* $\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix}$ *such that the following assumptions hold:*

*A*1. $(i)$ *For* $i \in S_u$,

$$\Xi_i > 0. \tag{3.27}$$

$(ii)$ *For* $i \in S_s$,

$$\Xi_i < 0. \tag{3.28}$$

*A*2. *Assume that, for any* $t_0$ *the switching law guarantees that*

$$\inf_{t \geq t_0} \frac{T^-(t_0, t)}{T^+(t_0, t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*} \tag{3.29}$$

*where* $\lambda^* \in (0, \lambda^-)$*. Furthermore, there exists* $0 < \nu < \lambda^*$ *such that*

$(i)$ *If the subsystem* $i \in S_u$ *is activated in time intervals* $[t_{i_k-1}, t_{i_k}), k = 1, 2, ...,$ *then*

$$ln\ \psi - \nu(t_{i_k} - t_{i_k-1}) \leq 0,\ k = 1, 2, ... \tag{3.30}$$

$(ii)$ *If the subsystem* $j \in S_s$ *is activated in time intervals* $[t_{j_k-1}, t_{j_k}), k = 1, 2, ...,$ *then*

$$ln\ \psi + \zeta_j h_M - \nu(t_{j_k} - t_{j_k-1}) \leq 0,\ k = 1, 2, ... \tag{3.31}$$



## 4.3 Robust Exponential Stability of Nonlinear Switched System with Time-Varying Delay

In this section, we deal with the problem for robust exponential stability of the zero solution of system (3.1).

**Theorem 3.3.1.** *The zero solution of system (3.1) is robust exponentially stable if there exist positive real numbers* $\varepsilon_{3i}, \varepsilon_{4i}, \varepsilon_{5i}$, *positive definite matrices* $P_i, Q_i, R_i$ *and* $\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix}$ *such that the following assumptions hold:*

*A*1. (*i*) *For* $i \in S_u$,

$$\Theta_i > 0. \tag{3.38}$$

(*ii*) *For* $i \in S_s$,

$$\Theta_i < 0. \tag{3.39}$$

*A*2. *Assume that, for any* $t_0$ *the switching law guarantees that*

$$\inf_{t \geq t_0} \frac{T^-(t_0,t)}{T^+(t_0,t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*} \tag{3.40}$$

*where* $\lambda^* \in (0, \lambda^-)$. *Furthermore, there exists* $0 < \nu < \lambda^*$ *such that*

(*i*) *If the subsystem* $i \in S_u$ *is activated in time intervals* $[t_{i_k-1}, t_{i_k}), k = 1, 2, ...$, *then*

$$ln\ \psi - \nu(t_{i_k} - t_{i_k-1}) \leq 0,\ \ k = 1, 2, ... \tag{3.41}$$

(*ii*) *If the subsystem* $j \in S_s$ *is activated in time intervals* $[t_{j_k-1}, t_{j_k}), k = 1, 2, ...$, *then*

$$ln\ \psi + \zeta_j h_M - \nu(t_{j_k} - t_{j_k-1}) \leq 0,\ \ k = 1, 2, ... \tag{3.42}$$