APPENDIX

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Harmonic Analysis of Dynamic Hysteresis Response of $BaTiO_3$ Bulk Ceramics

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In this work, the dynamic ferroelectric hysteresis properties in response to dynamic electric field of $BaTiO_3$ bulk ceramics was investigated using the harmonic analysis approach. Fourier transformation was used to analyze the periodic polarization signal on frequency domain via each discrete harmonic. From the results, the hysteresis area is found to depend only on the first harmonic of the real part. On the other hand, the remnant polarization depends on all odd harmonics of the real part. Further, the coercive field can be found from the phase-lag between the inverse Fourier signals re-calculated from the first harmonic of the real part and that of the imaginary part. The hysteresis properties from the harmonic analysis match well with those of the original measurement. This suggests that the harmonic analysis is one of the powerful techniques which can be used to predict hysteresis behavior.

Keywords Q1

1. Introduction

The ferroelectric hysteresis properties (i.e. hysteresis area A, remnant polarization P_r and coercive field E_c) in response to the electric field parameters (i.e. filed amplitude E_0 and field frequency f) has recently gained an intense interest in view of both technological and fundamental importance [1–10]. This is since if the understanding of how the hysteresis properties relating to the electric field perturbation is fully obtained, one may use this knowledge in designing state of the art ferroelectric applications with high efficiency. Nevertheless, most previous investigations, either from experimental [1–6] or theoretical viewpoints [7–10], focused mainly on the use of simple empirical power law scaling (non-linear fit) to relate the hysteresis properties to the external field and relevant perturbation. In general, a large number of input-output data is required to perform the non-linear fit. For a limited number of input-output data, one generally encounters non-convergence problems. To avoid, one has to extract each exponent separately while assuming that the other are constants and repeat the procedure backward. However, this method may not be useful if the exponents are not truly constants. For instance, α may be a function of E_0, and β may

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be a function of f. In this case, the simple power law scaling is no longer simple and alternative applicable techniques are required for substitution.

In this study, we purpose an alternative approach that can be used to model the dynamic hysteresis behavior. Since the hysteresis is periodic signal in time, Fourier transformation can be used to analyze the hysteresis in frequency domain. Nevertheless, from literatures, very less details have ascribed its full use [11, 12]. In this work, an extensive harmonic investigation on hysteresis data in modeling hysteresis properties is proposed using $BaTiO_3$ bulk ceramics as an application.

2. Methodology

The $BaTiO_3$ bulk ceramics were prepared by a conventional mixed-oxide method having diameter = 8 mm, thickness = 1 mm and T_C = 124.5°C. Its dynamic hysteresis was measured at room temperature (25°C) by Sawyer-Tower circuit with electric field signal (sine wave) generated by a function generator (HP 3310A). The hysteresis loop was recorded after reaching steady state. Details of the experimental setup were explained elsewhere [1]. Then, with the measured hysteresis data, Fourier transformation, i.e. $F(k) = \sum_{n=0}^{N-1} f(n)\exp(-i2\pi nk/N)$ where N is number of data points in 1 field period, was applied on the data.

However, due to electronic noises, the time interval between hysteresis data point in each loop tend to vary which limits the use of Fourier analysis. Therefore, we needed to perform the average on the polarization data having the same electric field from many loops. This is done by dividing the hysteresis signal into 3 intervals i.e. the interval that electric field increases from zero to maximum, decreases from maximum to minimum, and increases from minimum to zero. After that, the electric and polarization data were averaged and smoothed using the Loess smoothing method. The Cubic spine interpolation was then used to generate the discrete electric field data in a sine waveform and its corresponding polarization signal with equal time-interval (consisting of 1024 data points per loop). Then, the fast Fourier transformation was used to transform the periodic polarization signal from time domain to frequency domain. After the transformation, the relationship between hysteresis properties and electric field parameters to each harmonic of Fourier transformation was investigated.

3. The Analysis, Results and Discussions

In this work, the Fourier spectrums of the hysteresis signal were investigated. It should be noted that all even harmonics of the transformed polarization signal should be zero since the hysteresis loop is half-wave symmetry. However, as the hysteresis loop measured from the experiment is not a perfect symmetry loop (due to electric noise), the even harmonics exist but they are comparatively small and can ignore (see Fig. 1, bottom left). Additionally, it is found that the polarization signal re-calculated from the inverse Fourier transformation of the real part is out of phase with the electric field, but from the imaginary part is in-phase with the electric field (not shown). As the hysteresis loop is originated from the phase mismatch (lag) between the polarization and electric field signals, the hysteresis area can then be extracted from inverse Fourier transformation of odd harmonics of the real part of the transformed polarization. Further, it is also found that the hysteresis area obtained from the inverse Fourier transformation using only the first harmonic of real part are almost equal to the measured hysteresis area. On the other hand, the hysteresis area from the

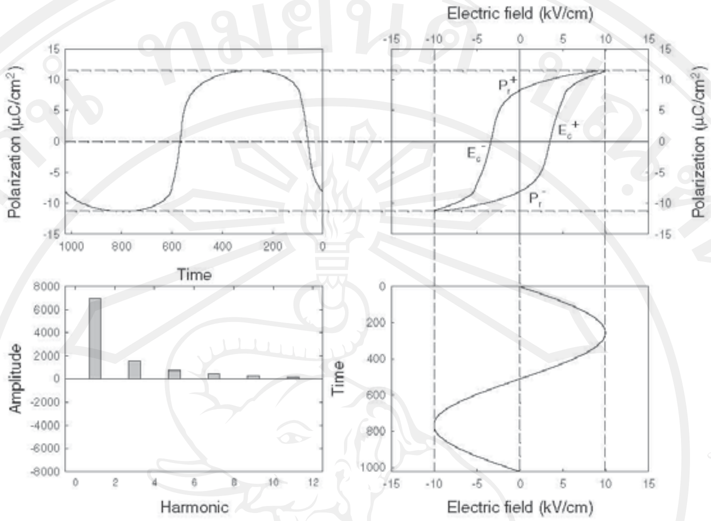

Figure 1. The hysteresis loop (top right), measured at $E_0 = 10$ kV/cm and $f = 10$ Hz, with its corresponding electric field signal (bottom right), polarization signal (top left), and spectrums of Fourier transformation (bottom left).

inverse Fourier transformation using all higher harmonics of real part are cancel or very small in comparison to that of the measured hysteresis area .

In addition, the inverse Fourier transformation using only the first harmonic of the real part shows ellipse-like hysteresis loops with an area $A = \pi E_0 P_1$ (where P_1 is polarization obtained from the inverse Fourier transformation of the first harmonic of real part). Since

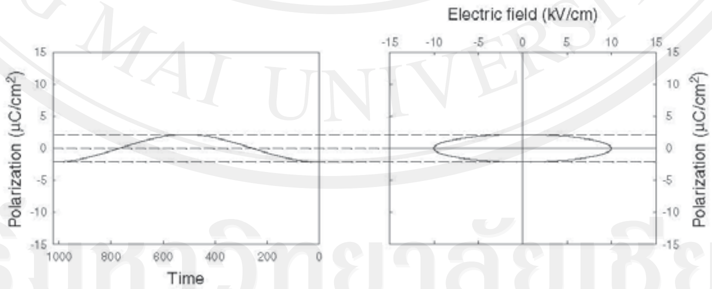

Figure 2. The hysteresis loop re-calculated from the inverse Fourier transform using only the first harmonic of the real part in the bottom left of Fig. 1 (right) and its polarization signal (left).

P_1 can be written in term of the Fourier coefficient of the first harmonic a_1 i.e. $P_1 = 2a_1/N$, the hysteresis area can be rewritten as

$$A = -\frac{2\pi}{N} E_0 a_1. \tag{1}$$

To verify the validity of Eq. (1), numerical comparison was evaluated. In Fig. 1, the measured hysteresis loop has $A = 133.5459$ mCV/cm^3 but from Eq. (1) $A = 133.6468$ mCV/cm^3, which agree very well. As a result, this emphasizes that the hysteresis area depends on the field amplitude and the first harmonic of the real part of the transformed polarization signal in following the relationship stated by Eq. (1).

On considering the remnant polarization P_r, as mentioned above, the polarization re-calculated from the inverse Fourier transformation of the real part is out of phase with the electric field so the electric field and polarization do not vanish at the same time. On other hand, the polarization from inverse Fourier transformation of imaginary part is in-phase with electric field, so electric field and polarization vanish together. From this harmonic analysis, it is found that that the remnant polarization P_r can be calculated from the sum of their associated amplitude of odd harmonics of the real part, e.g. see Fig. 3.

Additionally, P_n can be written in terms of Fourier coefficients of the odd harmonics of the real part a_n, so $P_n = 2a_n/N$ where n are odd numbers. Consequently, the relationship between positive/negative remnant polarizations and the harmonic of Fourier transformation have the form

$$P_r^{\pm} = \mp \frac{2}{N} \sum_{n=1}^{N/2} a_n. \tag{2}$$

To verify this, the hysteresis loop in Fig. 1 has the average remnant polarization $(P_r^+ - P_r^-)/2$ of 8.1887 μC/cm^2 while the average remnant polarization calculated from Eq. (2) is 8.1887 μC/cm^2, which are exactly the same (up to 4 digits). Therefore, it can be concluded that the remnant polarization depends on all odd harmonics of real part of the transformed polarization signal via Eq. (2).

On the coercive field E_c investigation, since it describes the magnitude of the electric field needed to cancel the polarization magnitude, E_c can be found from the phase-lag ϕ

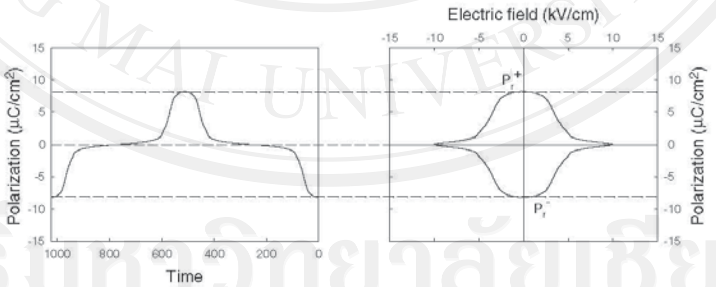

Figure 3. The hysteresis loop re-calculated from the inverse Fourier transformation using all harmonics of the real part in the bottom left of Fig. 1 (right) and its polarization signal (left).

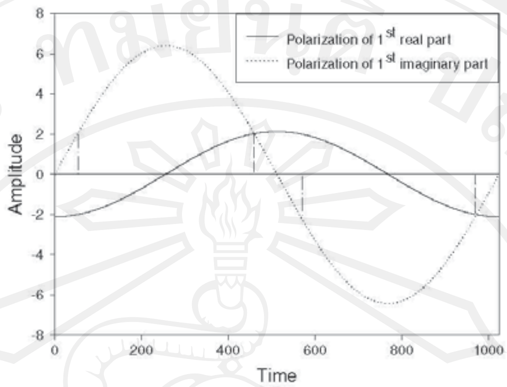

Figure 4. The polarization signals re-calculated from the inverse Fourier transformation of the first harmonic of the real part (solid) and of the imaginary part (dot), obtained from hysteresis loop measured at $E_0 = 10$ kV/cm and $f = 10$ Hz. The dash lines show the phase-angle at where both signals are equal.

between electric field signal and polarization signal. The relationship between this phase-lag and the harmonic of Fourier transformation was then investigated. The results show that the phase-lag between electric field signal and polarization signal can be found from π minus with the phase-angle that the polarization re-calculated from the inverse Fourier transformation of the first harmonic of real part equals to that of the imaginary part in magnitude, i.e. $\varphi_1 = \tan^{-1}(-a_1/b_1) = -\tan^{-1}(a_1/b_1)$ where a_1 and b_1 are Fourier coefficients of the first harmonic of the real part and of the imaginary part respectively, e.g. see Fig. 4. As $\phi = \pi - \varphi_1$ and $\sin\varphi_1 = \sin(\pi - \varphi_1)$ for $\varphi_1 \leq \pi$, then $\sin\phi = \sin\varphi_1$. Therefore, the positive/negative coercive fields can be found from

$$E_c^{\pm} = \pm E_0 \sin\phi = \mp E_0 \sin(\tan^{-1}(a_1/b_1)). \quad (3)$$

Note that the hysteresis loop in Fig. 1 has the average coercive field $(E_c^+ - E_c^-)/2$ of 3.4594 kV/cm while average coercive field calculated from Eq. (3) is 3.4367 kV/cm. This therefore confirms that the coercive field depends on the electric field amplitude and the first harmonic of real part and of imaginary part in following Eq. (3).

4. Conclusion

This study has performed the harmonic analysis on the dynamic hysteresis behavior of the $BaTiO_3$ bulk ceramics. It is found that important hysteresis properties can be modeled by considering parameters obtained from the Fourier transformation. As a result, this study provides an alternative approach which can be used to predict and model the complex hysteresis behavior, which can be enhanced to gain further understanding of the ferroelectric materials.

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The Fourier Analysis of Ferromagnetic Hysteresis Properties in Two Dimensional Ising Model

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Abstract

This work performed Monte Carlo simulation of ferromagnetic Ising spins in 2 dimensions to investigate the hysteresis properties, of both symmetric and asymmetric types, under the effects of external perturbations using the Fourier analysis. The hysteresis loops were analyzed using the discrete Fourier transformation. From the results, the hysteresis areas of both symmetric and asymmetric hysteresis depend on the amplitude of magnetic field and the Fourier coefficient of the first harmonic of real part. Nevertheless, the remnant magnetization of symmetric hysteresis depends only on the Fourier coefficients of odd harmonics of real part, while that of asymmetric hysteresis depends on both the Fourier coefficients of the zero and odd harmonics of real part. Additionally, the coercive field of symmetric hysteresis depends on the amplitude of magnetic field and the Fourier coefficients of the first harmonics of both real and imaginary parts. This suggests that the hysteresis behavior can be classified and modeled in frequency domain using the Fourier analysis.

1. Introduction

Recently, ferromagnetic materials bring to many useful applications e.g. transformer and memory. The modeling of hysteresis behavior by relate the hysteresis properties (i.e. hysteresis area A, remnant magnetization m_r and coercive field h_c) under the effects of external perturbations (i.e. temperature T, amplitude h_0 and frequency f of the magnetic field) has an intense investigated. This topic has been studied in view of theoretical [1-3] and experimental [4-6], focusing mostly on the simple power law scaling to relate hysteresis properties and external perturbations. Nevertheless, some materials may not response well to the field in some conditions which can severely affect to the scaling. As can be seen, the simple power law scaling is no longer simple, and the hysteresis behavior modeling requires alternative suitable techniques. Therefore, this work proposes an alternative technique that can be used to model the Ising hysteresis behavior of both symmetric and asymmetric types in steady state using the discrete Fourier transformation [7-11]. Ising model was verified by both theoretical [12, 13] and experimental [14, 15] studies is magnetic systems assumes spins have only two states (spin up and spin down). The spins Hamiltonian has a form

$$H = -\frac{1}{2}\sum_{\langle i,j\rangle} J_{ij} s_i s_j - h(t)\sum_i s_i , \tag{1}$$

where $-\frac{1}{2}\sum_{\langle i,j\rangle} J_{ij} s_i s_j$ is the interaction energy between spin s_i and its nearest neighbor spins s_j, and $-h(t)\sum_i s_i$ is the interaction energy between magnetic field and spin s_i. Magnetic field is sine wave i.e. $h(t) = h_0\sin(2\pi ft)$, where h_0 and f are amplitude and frequency of magnetic field respectively. In general, J was set in unit of energy so J/k_B is unit of temperature. Monte Carlo simulation on Ising spins used to generate hysteresis under the effects of external perturbations. Then, the discrete Fourier transformation was used to transform the periodic magnetization signal from time domain to frequency domain. The discrete Fourier transformation has a form

$$F(k)=\sum_{n=0}^{N-1} f(n)\exp(-i2\pi nk/N). \qquad (2)$$

Since, $\exp(-i\theta) = \cos(\theta) - i\sin(\theta)$, Eq. (3) can be rewritten as

$$F(k)=\sum_{n=0}^{N-1} f(n)\cos(2\pi nk/N)-i\sum_{n=0}^{N-1} f(n)\sin(2\pi nk/N). \qquad (3)$$

In Eq.(3), cosine term is Fourier coefficient of the k^{th} harmonic of real part (a_k) and sine term is Fourier coefficient of the k^{th} harmonic of imaginary part (b_k) i.e. $F(k) = a_k + ib_k$, where N is number of data points per period. After the transformation, the relationship between hysteresis properties and external perturbations and the Fourier coefficients was investigated.

2. Methodology

The hysteresis under the effects of external perturbations was generated by Monte Carlo simulation using the initial random spins $s_i(0)$ describes on the values 1 for spin up and -1 for spin down. Then, a spin was taken in random and the probability of spin flipping was calculated that has a form

$$p_i(t)=\exp\left(\frac{-\Delta E_i(t)}{T}\right). \qquad (4)$$

In Eq. (4), $\Delta E_i(t)$ is the energy difference of spin flipping i.e. $\Delta E_i(t) = 2s_i(t)[h(t)+\sum s_j(t)]$. After that, a random number [0,1) was generated and compared with the probability of spin flipping. If a random number equal or less then the probability of spin flipping, spin $s_i(t)$ can be flip. Conversely, if a random number more than the probability of spin flipping, spin $s_i(t)$ cannot be flip. The time that uses to consider all spins in system is *mcs* (Monte Carlo steps per site) was set in unit of time so mcs^{-1} is unit of frequency of magnetic field. Details on the simulation can be found elsewhere [1-3]. In this study, there are 1024 points per period in hysteresis loop, so magnetization was calculated every $\Delta t = 1/(1024f)$ mcs that has a form

$$m(t)=\frac{1}{N}\sum_{i=0}^{N-1} s_i(t), \qquad (5)$$

where N is the number of spins in system. The hysteresis generation is going on until 1,000 hysteresis loops to sure that it steady in time. Then, 10,000 hysteresis loops were recorded and averaged. After that, the discrete Fourier transformation was used to transform the magnetization from time domain to frequency domain using Eq. (3). In the transformation, the time intervals between data points need to be equal and the numbers of data points per period should be enough for specifying signal. The discrete Fourier transformation has then long times to calculate. To maximize the calculation efficiency, the fast Fourier transformation (the number of data points per period should be power of two) was used to analyze the hysteresis behavior. After the transformation, the relationship between hysteresis properties and external perturbations and the Fourier coefficients was investigated.

3. Analysis, Results and Discussions

In the study of Ising ferromagnetic hysteresis behavior of both symmetric and asymmetric types in 2 dimensions (e.g. see Fig. 1), the Fourier coefficients $\left(\sqrt{a_k^2 + b_k^2}\right)$ of both hysteresis types are somewhat different (e.g. see Fig. 2). For symmetric hysteresis, the Fourier coefficients present only odd harmonics since symmetric hysteresis is half wave symmetry i.e. $f(t) = -f(t + P/2) = -f(t + P/2)$, where P is signal period. For asymmetric hysteresis, the Fourier coefficients present all harmonics, but that of the zero harmonic is extremely larger than the other harmonics. Since the zero harmonic implies that some spins are inactive to temperature and magnetic field, and its Fourier coefficient is extremely larger than the other harmonics, so it is shown that both temperature and magnetic field affect slightly on spins flipping.

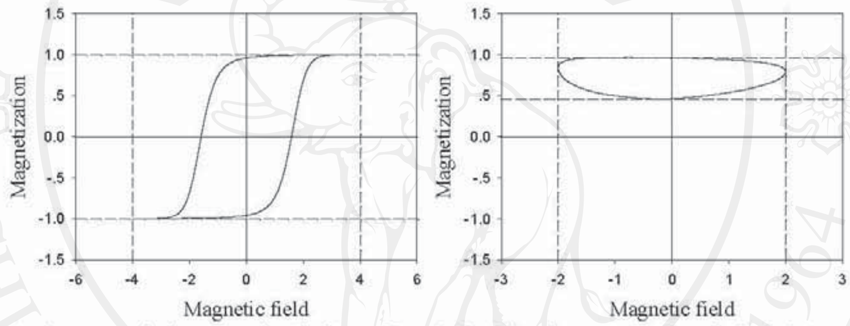

Figure 1. Symmetric hysteresis (left) at T = 2.00 J/k_B, h_0 = 4.00 J and f =0.01 mcs^{-1}, and asymmetric hysteresis (right) at T = 1.00 J/k_B, h_0 = 2.00 J and f = 0.10 mcs^{-1}.

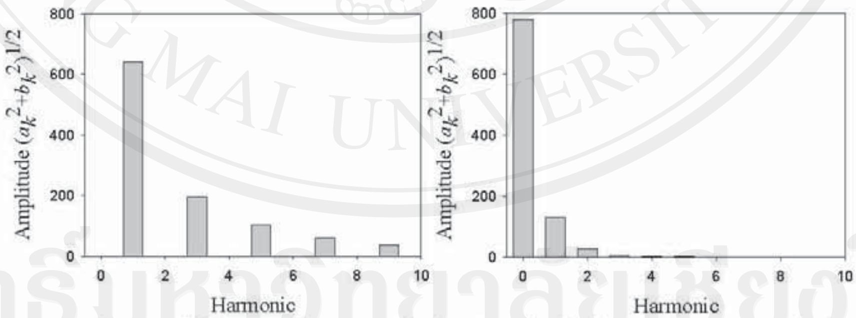

Figure 2. Fourier coefficients of symmetric hysteresis (left), and asymmetric hysteresis (right) in Fig. 1.

On considering of the hysteresis area (A), the magnetization re-calculated from inverse Fourier transformation of the imaginary part (sine term in Eq. (3)) is in-phase with magnetic field, so hysteresis area is equal to zero. Then, the hysteresis area re-calculated from inverse Fourier transformation of the real part (cosine term in Eq. (3)) was investigated in details. It is found that the hysteresis area of both hysteresis types re-calculated from inverse Fourier transformation of the first harmonic of real part is almost equal to the real hysteresis area. On the other hand, the hysteresis areas from inverse Fourier transformation of other harmonics of real part nearly cancel out and are very small in comparison to the real hysteresis area. Further, the hysteresis re-calculated from inverse Fourier transformation of the first harmonic of real part is ellipse-like (e.g. see Fig. 3). The hysteresis area is then equal to $\pi h_0 m_1^{\mathrm{Re}}$ (where m_1^{Re} is the magnetization amplitude re-calculated from inverse Fourier transformation of the first harmonic of real part). Additionally, m_1^{Re} can be written as $m_1^{\mathrm{Re}} = -2a_1/N$. The hysteresis area of both symmetric and asymmetric hysteresis can be rewritten as

$$A = -\frac{2\pi}{N} h_0 a_1. \tag{6}$$

Thus, the hysteresis area depends on the amplitude of magnetic field and the Fourier coefficient of the first harmonic of real part.

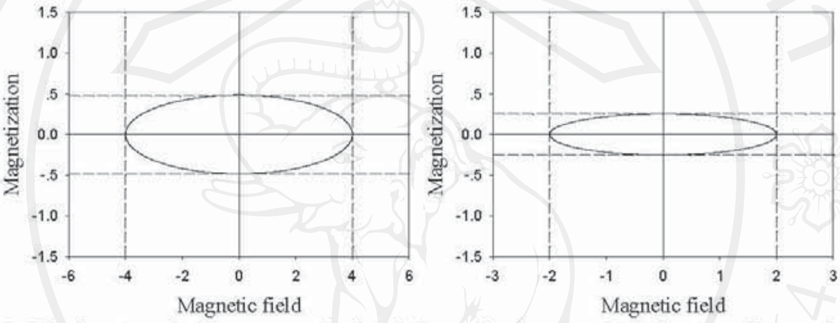

Figure 3. The hysteresis loops re-calculated from the inverse Fourier transform of the first harmonic of real part of symmetric hysteresis (left), and asymmetric hysteresis (right) in Fig. 1.

On determining of the remnant magnetization (m_r), the magnetization re-calculated from inverse Fourier transformation of the real part (cosine term in Eq. (3)) is out of phase with magnetic field, so magnetization and magnetic field magnitude does not vanish at the same time. Since the magnetization re-calculated from inverse Fourier transformation of the imaginary part (sine term in Eq. (3)) is in-phase with magnetic field, magnetization and magnetic field magnitude vanishes together. The remnant magnetization re-calculated from the inverse Fourier transformation of the real part was then investigated in details. From the results, the remnant magnetization of symmetric hysteresis can be calculated from the sum of the magnetization amplitude re-calculated from inverse Fourier transformation of odd harmonics of real part (m_k^{Re}), as shown in Fig. 4(left). Additionally, m_k^{Re} can be written as $m_k^{\mathrm{Re}} = -2a_k/N$, where k are odd integers. Therefore, the remnant magnetization of symmetric hysteresis can be written as

$$m_r^{\pm} = \mp \frac{2}{N} \sum_{k=1}^{N/2} a_k . \tag{7}$$

Thus, the remnant magnetization of symmetric hysteresis depends only on the Fourier coefficients of odd harmonics of real part. On the other hand, the remnant magnetization of asymmetric hysteresis can be calculated partly same as symmetric hysteresis. Specifically, it requires to shift the magnetization (re-calculated from inverse Fourier transformation of the zero harmonic of real part (m_0^{Re}) as shown in Fig. 4(right)) with the amount $m_0^{\mathrm{Re}} = a_0/N$, Therefore, the remnant magnetization of asymmetric hysteresis can be written as

$$m_r^{\updownarrow} = \frac{2}{N}\left(\frac{a_0}{2} \mp \sum_{k=1}^{N/2} a_k\right), \tag{8}$$

where k are odd integers. Thus, the remnant magnetization of asymmetric hysteresis depends on both the Fourier coefficients of the zero and odd harmonics of real part.

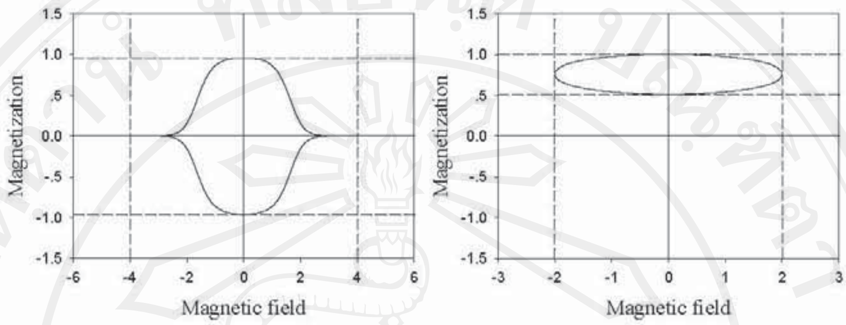

Figure 4. The hysteresis loops re-calculated from the inverse Fourier transformation of odd harmonics of real part in of symmetric hysteresis (left), and asymmetric hysteresis (right) in Fig. 1.

On the investigation of the coercive field (h_c), it can be calculated from the magnetic field at the phase-lag (ϕ) between magnetic field and magnetization, so h_c can be written as

$$h_c^{\pm} = \pm h_0 \sin(\phi). \quad (9)$$

Further, this phase-lag (ϕ) is equal to the phase-angle that the magnetizations of all harmonics (magnetization re-calculated from inverse Fourier transformation of the real and imaginary part) are cancelled. Since the Fourier coefficients of symmetric hysteresis present only odd harmonics, the magnetizations re-calculated from inverse Fourier transformation of each harmonic of the real and imaginary part are cancelled at two phase-angles. They can be found from where the magnitude of the magnetization re-calculated from inverse Fourier transformation of the first harmonic of the real part (m_1^{Re}) is equal to that of the imaginary part (m_1^{Im}) but with opposite signs (e.g. see Figs. 5) i.e. $-m_1^{\mathrm{Re}}\cos(\phi) = m_1^{\mathrm{Im}}\sin(\phi)$. Since, m_1^{Re} and m_1^{Im} can be written as of $m_1^{\mathrm{Re}} = -2a_1/N$ and $m_1^{\mathrm{Im}} = 2b_1/N$ respectively, so $a_1\cos(\phi) = b_1\sin(\phi)$, i.e. $\phi = \tan^{-1}(a_1/b_1)$. Therefore, the coercive field can be rewritten as

$$h_c^{\pm} = \pm h_0 \sin(\tan^{-1}(a_1 / b_1)). \quad (10)$$

Thus, the coercive field of symmetric hysteresis depends on the amplitude of magnetic field and the Fourier coefficients of the first harmonics of both real and imaginary parts.

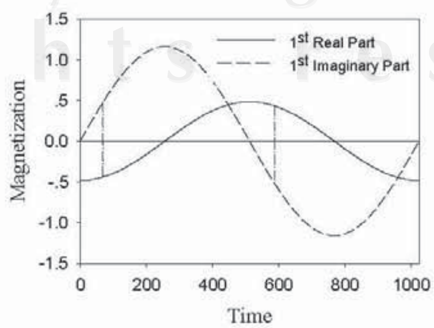

Figure 5. The magnetization signals re-calculated from the inverse Fourier transformation of the first harmonic of real (solid) and imaginary part (dash) in Fig. 1. The dash-dot line shows the phase-angle at both signals are equal in magnitude but opposite signs.

4. Conclusion

This work performed the Monte Carlo simulation extraction of Ising ferromagnetic hysteresis in 2 dimensions to investigate the hysteresis properties of both symmetric and asymmetric types under the effects of external perturbations, using the discrete Fourier transformation. It is found that the hysteresis properties can be modeled by considering harmonics of the Fourier transformation. This study is therefore proposing a fundamental knowledge in the modeling of ferromagnetic materials.

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Characteristic of Ising Mean-Field Hysteresis in 2 Dimensions: The Fourier Investigation

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This work performed the mean-field extraction of Ising ferromagnetic hysteresis in 2 dimensions to investigate the hysteresis properties, of both symmetric and asymmetric types, under the effects of external perturbations. The hysteresis loops were analyzed using the Fourier transformation. The results show that both temperature and magnetic field have considerable effects on hysteresis properties. The Fourier-harmonic characteristics of both hysteresis types are somewhat different. Specifically, the hysteresis areas of both symmetric and asymmetric hysteresis depend on the amplitude of magnetic field and the amplitude of the first real-part-harmonic. Nevertheless, the remnant magnetization of symmetric hysteresis depends only on the amplitude of odd real-part-harmonics, while that of the asymmetric hysteresis depends on both the amplitude of the zero and odd real-part-harmonics. Additionally, the coercivity of the symmetric hysteresis depends on the amplitude of magnetic field and the amplitude of the first harmonics of both real and imaginary parts. From these evidences, the hysteresis properties of the considered systems can be classified and modeled in frequency domain using the Fourier analysis approach.

Keywords: hysteresis, Ising, mean-field analysis, Fourier transformation

1. INTRODUCTION

Ferromagnetic material generally changes the direction and magnitude of magnetization with external perturbations i.e. temperature and magnetic field. Therefore, it leads to many useful applications. This is since the design of ferromagnetic applications requires specific understanding of hysteresis behavior. For examples, memory application should have remnant magnetization large enough for the reader-head to sense and has coercivity large enough for the recorded data to become stable against small noises. On the other hand, transformer application should have small hysteresis area to minimize energy dissipation. As can be seen, the study of hysteresis properties under the affects of external perturbations is of an intense interest. However, previous studies on this topic used only the simple power law scaling to relate hysteresis properties to external perturbations [2,3,6]. Nevertheless, the relationships in some materials are sometime too complicate for the simple power law scaling. Therefore, this work offers an alternative technique to consider ferromagnetic hysteresis behavior of both symmetric and asymmetric types, using the Fourier transformation [1,4] and 2 dimensions Ising model via mean-field picture as a steady case. In the Ising model, spins can be have only two states (spin up and spin down), where the Hamiltonian has a form

$$H = -\frac{1}{2}\sum_{i \neq j} J_{ij} s_i s_j - h(t)\sum_i s_i \cdot \qquad (1)$$

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In Eq. (1), $-\frac{1}{2}\sum_{i \neq j} J_{ij} s_i s_j$ is the interaction between spin s_i and its nearest neighbor spins s_j and $-h(t)s_i$ is the interaction between magnetic field and spin s_i. In this study, $h(t)$ is sine wave i.e. $h(t) = h_0\sin(2\pi ft)$ where h_0 and f are amplitude and frequency of magnetic field respectively. In general, $J = 1$ was set so J has become the unit of energy and J/k_B is unit of temperature. The mean-field analysis assumes magnetization can be calculated from the averaged field of all spins, which has a form [5]

$$\tau \frac{dm(t)}{dt} = -m(t) + \tanh \frac{1}{k_B T}\left(h(t) + \sum_i J_i m(t)\right) \cdot \qquad (2)$$

In Eq. (2), $\tau = 1$ was set so τ is unit of time. Temperature (T) was set in multiplex of Curie temperature (T_c) so $k_B T_c$ has then become the unit of magnetic field amplitude (h_0) and T_c is unit of temperature (T). After that, this equation was solved using the fourth order Range-Kutta. Then, the Fourier transformation was used to transform the periodic magnetization signal from time domain to frequency domain. The Fourier transformation has a form

$$F(k) = \sum_{n=0}^{N-1} f(n)\exp(-i2\pi nk/N) \cdot \qquad (3)$$

Since, $\exp(-i\theta) = \cos(\theta) - i\sin(\theta)$, Eq. (3) can be rewritten as

$$F(k) = \sum_{n=0}^{N-1} f(n)\cos(2\pi nk/N)$$

$$-i\sum_{n=0}^{N-1} f(n)\sin(2\pi nk/N). \qquad (4)$$

In Eq.(4), cosine term is amplitude of the k^{th} real-part-harmonic (a_k) and sine term is amplitude of the k^{th} imaginary part-harmonic (b_k) i.e. $F(k) = a_k + ib_k$ where, N is number of data points per period. After the transformation, the relationship between hysteresis properties and external perturbation parameters and the amplitude of each harmonic of Fourier transformation was investigated.

2. EXPERIMENT

In the analysis of ferromagnetic Ising mean-field hysteresis behavior in 2 dimensions of both symmetric and asymmetric types, the hysteresis under the affects of external perturbations was generated by solving the mean-field equation, Eq. (2). It was solved, via the fourth order Range-Kutta, using the initial magnetization $m(0) = 1$. The hysteresis generation is going on until the hysteresis loop is steady in time which can be observed from the dynamic order parameter

$$Q(n) = \frac{1}{P}\int_0^P m(t)dt, \qquad (5)$$

where P is the field period and n is the loop index. In this study, hysteresis is assumed steady when $\Delta Q \le 10^{-5}$. Then, 1000 hysteresis loops were recorded and averaged. After that, the Fourier transformation was used to transform the periodic magnetization signal from time domain to frequency domain using Eq. (3). In the transformation, the time intervals between data points are required to be equal, and the numbers of data points per period should be enough for specifying signal, unless the Fourier transformation may be no longer useful. To maximize the calculation efficiency, the fast Fourier transformation (the number of data points per period should be power of two) was considered to analyze the hysteresis behavior. After the transformation, the relationship between hysteresis properties and external perturbation parameters and the amplitude of each harmonic of Fourier transformation was investigated.

3. RESULTS AND DISCUSSIONS

In the analysis of ferromagnetic Ising mean-field hysteresis behavior in 2 dimensions, the spectrum of Fourier transformation of the both symmetric and asymmetric hysteresis was investigated. From the transformation, the Fourier-harmonic characteristics ($\sqrt{a_k^2 + b_k^2}$) of both hysteresis types are partly different as shown in Fig. 1, 2. The Fourier-harmonics of symmetric hysteresis presents only odd harmonics of the transformed magnetization (see Fig. 1). This is expected as symmetric hysteresis is half-wave symmetry. On the other hand, the Fourier-harmonics of asymmetric hysteresis presents all harmonics of the transformed magnetization, but the zero harmonic has amplitude extremely larger than other harmonics (see Fig. 2). In general, the zero harmonic presents that some spins are inactive to temperature and magnetic field. In addition, its amplitude is extremely larger than other harmonics. Therefore, it can be said that both temperature and magnetic field less affect on spins flip.

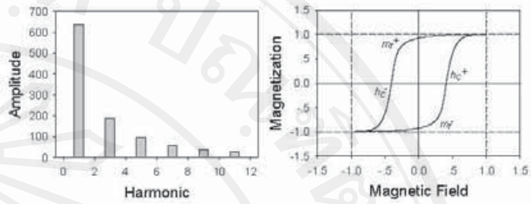

FIGURE 1. The symmetric hysteresis loop (right) at $T = 0.625\ T_c$, $h_0 = 1.000\ k_BT_c$ and $f = 0.010\ \tau^{-1}$, and spectrums of Fourier transformation (left).

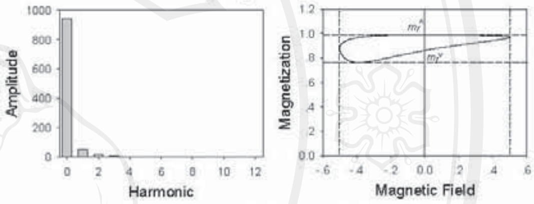

FIGURE 2. The asymmetric hysteresis loop (right) at $T = 0.375\ T_c$, $h_0 = 0.500\ k_BT_c$ and $f = 0.100\ \tau^{-1}$, and spectrums of Fourier transformation (left).

Additionally, it is found that the magnetization re-calculated from inverse Fourier transformation of the real part is out of phase with magnetic field, but from the imaginary part is in phase with magnetic field (see fig. 3). Consequently, it is useful to find the relationship between hysteresis properties and the amplitude of each harmonic of Fourier transformation.

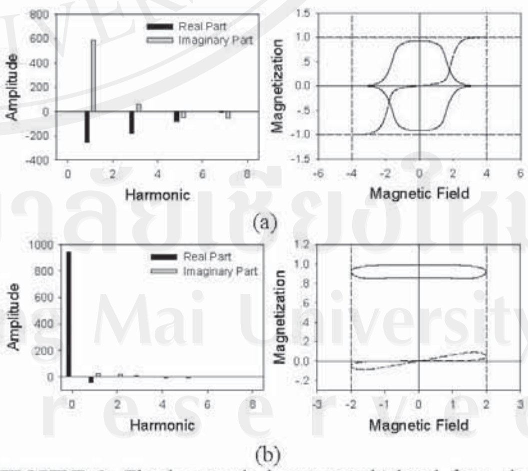

FIGURE 3. The hysteresis loops re-calculated from the inverse Fourier transform using the real (solid) and imaginary (dash) part harmonics in (a) Fig. 1 and (b) Fig. 2.

From the study of symmetric and asymmetric hysteresis, the hysteresis area (A) refers to the external energy require to cycle the magnetic dipole moments in ferromagnetic materials. As mentioned above, it is safe to imply that the hysteresis area can be extracted from inverse Fourier transformation of odd real-part-harmonics. In addition, the hysteresis area of both hysteresis types re-calculated from inverse Fourier transformation of the first real-part-harmonic is almost equal to the real hysteresis area. On the other hands, the hysteresis area from inverse Fourier transformation of all other real-part-harmonics are cancel and very small in comparison to the real hysteresis area.

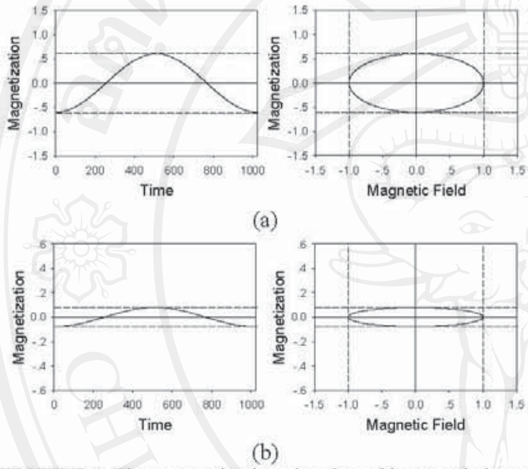

FIGURE 4. The magnetization signals and hysteresis loops re-calculated from the inverse Fourier transform using only the first real-part-harmonic in (a) Fig. 1 and (b) Fig. 2.

Further, it is found from Fig. 4 that the hysteresis re-calculated from inverse Fourier transformation of the first real-part-harmonic is ellipse-like. Since the area of typical ellipse equal to πab where a is length of major axis and b is length of minor axis, the hysteresis area is then compound with quantity $\pi h_0 m_1^{real\ part}$ (where h_0 is the magnetic field amplitude and $m_1^{real\ part}$ is the magnetization amplitude re-calculated from inverse Fourier transformation of the first real-part-harmonic), and it was found that they are equal (with less than percent different). Additionally, $m_1^{real\ part}$ can be written in the term of the first real-part-harmonic (a_1) i.e. $m_1^{real\ part} = 2a_1/N$ where N is number of data points per period, hysteresis area of symmetric and asymmetric hysteresis can be then rewritten as

$$A = -\frac{2\pi}{N} h_0 a_1. \qquad (6)$$

Therefore, it can imply that the hysteresis area depends on the magnetic field amplitude and the amplitude of the first real-part-harmonic.

On considering of the remnant magnetization (m_r) of the symmetric and asymmetric hysteresis loops, the magnitude of the magnetization remains at vanish magnetic field was investigated. As mentioned above, the magnetization re-calculated from inverse Fourier transformation of the real part (cosine term in Eq. (4)) is out of phase with magnetic field, so magnetic field and magnetization magnitude do not vanish at the same time. On the other hand, the magnetization re-calculated from inverse Fourier transformation of the imaginary part (sine term in Eq. (4)) is in-phase with magnetic field, so magnetic field and magnetization magnitude vanishes together. Then the remnant magnetization re-calculated from the inverse Fourier transformation of the real-part-harmonics was investigated in details. From the results, the remnant magnetization of symmetric hysteresis can be calculated from the sum of the magnetization amplitude re-calculated from inverse Fourier transformation of odd real-part-harmonics ($m_k^{real\ part}$), as shown in Fig. 5(a), so it can be written as

$$m_r^{\pm} = \pm\sum_{k=1}^{N/2} m_k^{real\ part}. \qquad (7)$$

Since, $m_k^{real\ part}$ can be written in terms of the amplitude of odd real-part-harmonics (a_k) i.e. $m_k^{real\ part} = -2a_k/N$ where N is number of data points per period and n are odd integers. Therefore, the remnant magnetization of symmetric hysteresis can be rewritten as

$$m_r^{\pm} = \mp\frac{2}{N}\sum_{k=1}^{N/2} a_k, \qquad (8)$$

Thus, this implies that the remnant magnetization of symmetric hysteresis depends on the amplitude of odd real-part-harmonics. On the other hand, the remnant magnetization of asymmetric hysteresis can be calculated partly the same way as symmetric hysteresis. Specifically, it requires to shift the magnetization (re-calculated from inverse Fourier transformation of the zero real-part-harmonic as shown in Fig. 5(b)) with the amount $m_0^{real\ part} = a_0/N$, which can be rewritten as

$$m_r^{\updownarrow} = \frac{2}{N}\left[\frac{a_0}{2} \mp \sum_{k=1}^{N/2} a_k\right], \qquad (9)$$

where k are odd integers. Thus, the remnant magnetization of asymmetric hysteresis also depends on the amplitude of the zero and odd real-part-harmonics.

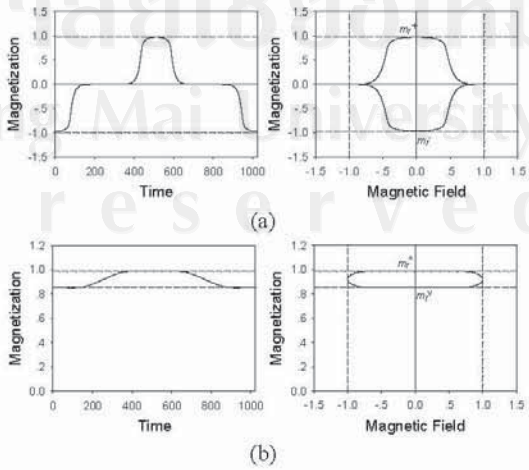

FIGURE 5. The magnetization signals and hysteresis loops re-calculated from the inverse Fourier transform using the real-part-harmonics in (a) Fig. 1 and (b) Fig. 2.

On the investigation of the coectivity (h_c) of symmetric hysteresis, the magnitude of the magnetic field that cancels the remnant magnetization in ferromagnetic material was considered. In general, the coercivity can be found from the magnetic field magnitude at the phase-lag (ϕ) between magnetic field and magnetization (not shown), so it can be written as

$$h_c^{\pm} = \pm h_0 \sin(\phi) . \quad (10)$$

Further, this phase-lag is also equal to the phase-angle (ϕ) that the combinations of magnetizations of all harmonics (magnetization re-calculated from inverse Fourier transformation of the real and imaginary part) are cancel. Since, Fourier-harmonics of symmetric hysteresis presents only odd harmonics, so the magnetizations re-calculated from inverse Fourier transformation of each harmonic of real and imaginary part are cancel at two co-phase-angles. One of these two phase-angles can be found from where the magnitude of the magnetization re-calculated from inverse Fourier transformation of the first harmonic of real part equal to that of the imaginary part but with opposite signs (see Figs. 6) i.e.

$$-m_1^{real\,part}\cos(\phi) = m_1^{imaginary part}\sin(\phi), \quad (11)$$

where $m_1^{real\ part}$ and $m_1^{imaginary\ part}$ are amplitude of magnetizations re-calculated from inverse Fourier transformation of the first harmonic of the real and imaginary part respectively. Since, $m_1^{real\ part}$ and $m_1^{imaginary\ part}$ can be written in the term of the first harmonic of real a_1 and imaginary b_1 part respectively i.e. $m_1^{real\ part} = -2a_1/N$ and $m_1^{imaginary\ part} = 2b_1/N$, where N is number of data points per period, Eq. (10) can be rewritten as $a_1\cos\phi = b_1\sin\phi$, i.e. $\phi = tan^{-1}(a_1/b_1)$. Therefore, the coercivity can be rewritten as

$$h_c^{\pm} = \pm h_0 \sin(\tan^{-1}(a_1 / b_1)) . \quad (12)$$

Thus, the coercivity depends on the electric field amplitude and the amplitude of the first harmonic of real part and imaginary part.

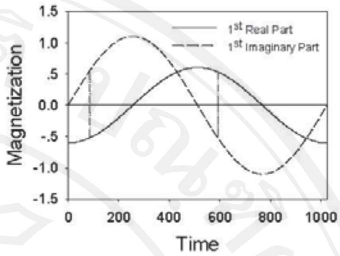

FIGURE 6. The magnetization signals re-calculated from the inverse Fourier transformation of the first real-part-harmonics (solid) and imaginary-part-harmonics (dash) of Fig. 1. The dash-dot line shows the phase-angle at both signals are equal in magnitude but opposite signs.

4. CONCLUSION

This work performed the mean-field extraction of Ising ferromagnetic hysteresis in 2 dimensions to investigate the hysteresis properties of both symmetric and asymmetric types under the effects of external perturbations, using the Fourier transformation. It is found that the hysteresis properties can be modeled by considering harmonics of the Fourier transformation. This study is therefore proposing a fundamental knowledge in the modeling of ferromagnetic materials.

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Conference Presentations and Publications Related to Thesis

International Publications:

1. **K. Kanchiang**, R. Yimnirun, N. Wongdamnern, A. Ngamjarurojana, S. Ananta, and Y. Laosiritaworn, "Harmonic Analysis of Dynamic Hysteresis Response of $BaTiO_3$ Bulk Ceramics" *Ferroelectrics*, (2009).
2. **K. Kanchiang**, R. Yimnirun, S. Ananta, and Y. Laosiritaworn, "The Fourier Analysis of Ferroelectric Hysteresis Properties in Two Dimensional Ising Model" *Ferroelectrics*, (2010).

National Conferences:

1. **K. Kanchiang**, R. Yimnirun, S. Ananta, and Y. Laosiritaworn, "Characteristic of Ising Mean-Field Hysteresis in 2 Dimensions: The Fourier Investigation" Siam Physics Congress, Kanchanaburi, Thailand (2010).

International Conferences:

1. **K. Kanchiang**, R. Yimnirun, N. Wongdamnern, A. Ngamjarurojana, S. Ananta, and Y. Laosiritaworn, "Harmonic Analysis of Dynamic Hysteresis Response of $BaTiO_3$ Bulk Ceramics" The 12th International Meeting on Ferroelectricity and 18th IEEE International Symposium on Applications of Ferroelectrics, Xi-an, China (2009).
2. **K. Kanchiang**, R. Yimnirun, S. Ananta, and Y. Laosiritaworn, "The Fourier Analysis of Ferroelectric Hysteresis Properties in Two Dimensional Ising Model" The 10th Russia/CIS/Baltic/Japan Symposium on Ferroelectricity, Tokyo Institute of Technology, Yokohama, Japan (2010).