Chapter 7
Conclusion

7.1 A Non-linear Heat Equation

We modified the new operator to solve non-linear heat equation of the form

$$\frac{\partial}{\partial t}u(x,t) - c^2(-\otimes)^k u(x,t) = f(x,t,u(x,t))$$

where $\otimes^k$ is the operator iterated k-times, defined by

$$\otimes^k = \left[\left(\sum_{i=1}^{p}\frac{\partial^2}{\partial x_i^2}\right)^3 - \left(\sum_{j=p+1}^{p+q}\frac{\partial^2}{\partial x_j^2}\right)^3\right]^k$$

where $p+q=n$ is the dimension of the Euclidean space. On suitable conditions for f, p, q, k and the spectrum, we obtain the unique solution $u(x,t)$ of such equation. Moreover, if we put $p=0, k=1$, we obtain the solution of non-linear triharmonic heat equation.

7.2 The Generalized $\otimes^k$ operator related to Triharmonic Wave Equation

We modified the new operator and use the new method to find the solution of the generalized wave equation of the form

$$\frac{\partial^2}{\partial t^2}u(x,t) + c^2(\otimes)^k u(x,t) = 0$$

with the initial conditions

$$u(x,0) = f(x), \quad \frac{\partial}{\partial t}u(x,0) = g(x).$$

where $\otimes^k$ is the operator iterated k-times, defined by

$$\otimes^k = \left[\left(\sum_{i=1}^{p}\frac{\partial^2}{\partial x_i^2}\right)^3 - \left(\sum_{j=p+1}^{p+q}\frac{\partial^2}{\partial x_j^2}\right)^3\right]^k$$

By the new method we obtain asymptotic solution $u(x,t) = O(\epsilon^{-n/3k})$. In particular, if we put $k=1$ and $p=0$, the $u(x,t)$ reduces to the solution of the wave equation

$$\frac{\partial^2}{\partial t^2}u(x,t) - c^2(\triangle)^3 u(x,t) = 0.$$

which is related to the triharmonic wave equation.
We can applied the new method to solve the equation in partial differential equation.

7.3 On The Green Function of the operator $(\otimes + m^6)^k$ Related to Diamond Operator

We define and study the Green function of the $(\otimes + m)^k$ operator and we obtain the solution which is related to the elementary solution of the Klein-Gordon operator, the Helmholtz operator and the Diamond operator of the form $(\diamondsuit + m^4)^k$. We can applied the Green function of the $(\otimes + m)^k$ operator to find the solution of the equation $(\otimes + m)^k K(x) = f(x)$. By the convolution method we obtain the solution in the form $K(x) = G(x) * f(x)$ where $G(x)$ is the Green function of the $(\otimes + m)^k$ operator.

7.4 On The $\boxtimes^k$ Operator and Nonlinear $\boxtimes^k$ Operator Related to Wave Equation

We defined the $\boxtimes^k$ operator iterated k times where defined by

$$\boxtimes^k = \left(\left(\sum_{i=1}^{p} \frac{\partial^2}{\partial x_i^2} \right)^6 - \left(\sum_{j=p+1}^{p+q} \frac{\partial^2}{\partial x_j^2} \right)^6 \right)^k .$$

Firstly, we studied Green function of the $\boxtimes^k$ operator and applied the Green function to find the solution of the equation $\boxtimes^k u(x) = f(x)$ where $u(x)$ is an unknown function and $f(x)$ s the generalized function. By the convolution method we obtain the solution in the classical form $u(x) = G(x) * f(x)$ where $G(x)$ is the Green function of the $\boxtimes^k$ operator. Moreover, on the suitable condition of p and q we found that $u(x)$ is related to the solution of the Laplace equation and wave equation. Finally, we study the solution of the nonlinear equation $\boxtimes^k u(x) = f(x, \Box^{k-1} L^k \circledast^k u(x)$. We can find the solution of the nonlinear equation by the convolution method. It is found that the existence of the solution $u(x)$ of such an equation depends on the condition of f and $\Box^{k-1} L^k \circledast^k u(x)$. Moreover a solution $u(x)$ related the inhomogeneous equation depends on the condition of p, q and k.