CHAPTER 4

CONCLUSION

In this study, we found that:

4.1 Characterization

(1) Let $\alpha, \beta \in T(X,Y)$ such that $\alpha \neq \beta$. Then $\alpha \leq \beta$ if and only if the following statements hold:

(1) $X\alpha \subseteq Y\beta$;

(2) $\pi_\beta \subseteq \pi_\alpha$;

(3) if $x\beta \in X\alpha$, then $x\alpha = x\beta$.

(2) Let $\alpha, \beta \in T(X)$ such that $\alpha \neq \beta$. Then $\alpha \leq \beta$ if and only if the following statements hold:

(1) $X\alpha \subseteq X\beta$;

(2) $\pi_\beta \subseteq \pi_\alpha$;

(3) if $x\beta \in X\alpha$, then $x\alpha = x\beta$.

(3) Let $\alpha \in F, \beta \in T(X,Y)$ such that $\alpha \neq \beta$ and let $K(\alpha,\beta) = \{x \in X : x\alpha \neq x\beta\}$. If the following properties hold:

(1) $x\alpha \in Y\beta$ and $x\beta \notin X\alpha$ for all $x \in K(\alpha,\beta)$;

(2) $x\beta \neq y\beta$ for all $x, y \in K(\alpha,\beta)$ with $x \neq y$,

then $\alpha \leq \beta$.

4.2 Compatibility

(1) Let $|Y| > 1$ and $\gamma \in T(X,Y)$. Then γ is left compatible with $\leq$ on $T(X,Y)$ if and only if $Y = Y\gamma$.

(2) If $|Y| = 2$, then γ is right compatible with $\leq$ on $T(X,Y)$ for all $\gamma \in T(X,Y)$.

(3) Let $|Y| > 2$ and $\gamma \in T(X,Y)$. γ is right compatible with $\leq$ on $T(X,Y)$ if and only if $|Y\gamma| = 1$ or $\gamma|_Y$ is injective.

(4) If $|X| = 2$, then the following statements hold:
(1) γ is left compatible with $\leq$ on $T(X)$ if and only if γ is surjcetive;
(2) γ is right compatible with $\leq$ on $T(X)$ for all $\gamma \in T(X)$.

(5) Let $|X| \geq 3$ and $\gamma \in T(X)$. Then the following statements hold:
(1) γ is left compatible with $\leq$ on $T(X)$ if and only if γ is surjcetive;
(2) γ is right compatible with $\leq$ on $T(X)$ if and only if γ is injective or constant.

4.3 Maximal and Minimal Elements

(1) Let $\alpha \in T(X,Y)$. Then α is maximal if and only if $\alpha \notin F$ or α is surjective or α is injective.

(2) Let $\alpha \in T(X,Y)$. α is minimal if and only if $|X\alpha| = 1$.

(3) An element $\alpha \in T(X)$ is maximal with $\leq$ on $T(X)$ if and only if α is surjective or injective; α is minimal if and only if α is a constant map.

(4) If $|Y| \geq 2$, then $T(X,Y)$ has no maximum element.

(5) If $|Y| \geq 2$, then $T(X,Y)$ has no minimum element.

(6) Let $\alpha \in T(X,Y)$. Then there exists a maximal element $\beta \in T(X,Y)$ such that $\alpha \leq \beta$.

(7) Let $\alpha \in T(X,Y)$. Then there exists a minimal element $\beta \in T(X,Y)$ such that $\beta \leq \alpha$.

(8) Every element in $T(X,Y)$ must lie below some maximal and lie above some minimal elements.

4.4 Covering Elements

(1) Let $\alpha, \beta \in T(X, Y)$. Then β is an upper cover for α if and only if the following statements hold:

(i) $\alpha < \beta$;

(ii) $|Y\beta \setminus X\alpha| = 0$ or $|X\beta \setminus X\alpha| = 1$.

(2) Let $\alpha, \beta \in T(X, Y)$. Then α is a lower cover for β if and only if the following statements hold:

(i) $\alpha < \beta$;

(ii) $|Y\beta \setminus X\alpha| = 0$ or $|X\beta \setminus X\alpha| = 1$.

(3) Every nonmaximal element in $T(X, Y)$ has an upper cover.

(4) Every nonminimal element in $T(X, Y)$ has a lower cover.