CHAPTER 4
RESULTS AND DISCUSSION

4.1 Results

The mode shapes and the natural frequencies of some lamination schemes of each study (Table 4.1) are evaluated to achieve the research objective. The results can be divided into three parts

(1) The mode shapes and the natural frequencies of straight nodal lines of symmetrically laminated composite rectangular plates with various boundary conditions.

(2) The mode shapes and the natural frequencies of curved nodal lines of symmetrically laminated composite rectangular plates with various boundary conditions.

(3) The higher mode shapes and the higher natural frequencies of straight nodal lines of symmetrically laminated composite rectangular plates with CCCC boundary conditions.

Before considering the above results (Topic 4.1.2-4.1.4), which are applied by the extended Kantorovich method, it is necessary to consider the accuracy of the extended Kantorovich method. To establish the accuracy of the extended Kantorovich method, the results of the validation of the frequency parameters of isotropic, orthotropic and laminated composite rectangular plates are presented in Topic 4.1.1.

4.1.1 Results of the comparison of the extended Kantorovich method with the known solutions

The frequency parameters of isotropic, orthotropic, and laminated composite rectangular plates with various boundary conditions are evaluated by the extended Kantorovich method, and the results obtained are compared with the corresponding results by Sakata *et al* (1996), Rajalingham *et at* (1997) and Reddy (2004), as illustrated in Tables 4.2-4.5. As expected for the special case, the bending stiffness of the composite plate are considered as $D_{11} = D_{22} = (D_{12} + D_{66}) = D$ for isotropic rectangular plates, and $D_{11} = D_{22}$, $(D_{12} + D_{66}) = 0.5D_{11}$ for orthotropic rectangular plates. For [0/90/90/0] laminated composite rectangular plates, the mechanical properties are $G_{12} = 0.5E_2$ and $\nu_{12} = 0.25$. By Table 4.3, the lateral displacement is divided into 4 categories as SS, AS, SA and AA, the first and second character presents property of the mode about the *x* and *y* coordinate direction respectively. The character "S" presents the lateral displacement symmetry with the coordinate direction and "A" presents the lateral displacement antisymmetry with the coordinate direction. The results of frequency parameters demonstrated that

(1) The maximum difference percentage of the frequency parameters of isotropic rectangular plates with various boundary conditions (Table 4.2) obtained by the extended Kantorovich method and Sakata *et al* (1996) is 0.230%.

(2) The maximum difference percentage of the first hundred frequency parameters of isotropic square plates with CCCC boundary conditions (Table 4.3) obtained by the extended Kantorovich method and Rajalingham *et al* (1997) is 0.002%.

(3) The maximum difference percentage of the frequency parameters of orthotropic rectangular plates with various boundary conditions (Table 4.4) obtained by the extended Kantorovich method and Sakata *et al* (1996) is 0.042%.

(4) The maximum difference percentage of [0/90/90/0] laminated composite rectangular plates with SSSS boundary conditions (Table 4.5) obtained by the extended Kantorovich method and Reddy (2004) is zero.

4.1.2 Results of the mode shapes and the natural frequencies of straight nodal lines

The first nine mode shapes and the first nine natural frequencies of [0/90/90/0] laminated composite square plates with various boundary conditions obtained by the extended Kantorovich method and the finite element method are illustrated in Table 4.6 and Figure 4.1, respectively. The graph of Figure 4.1 is lines on 2 axes, the left axe is the natural frequencies and the right axe is the difference percentage of the extended Kantorovich method from the finite element method. The results of mode shapes and the natural frequencies demonstrate that

(1) The first nine mode shapes of [0/90/90/0] laminated composite square plates with various boundary conditions (Table 4.6) obtained by the extended Kantorovich method are similar to the finite element method. The nodal lines, the contour represent the zero lateral displacement, are parallel to the *x* and *y* coordinate direction.

(2) The difference percentage of the first nine natural frequencies of [0/90/90/0] laminated composite square plates with various boundary conditions (Figure 4.1) obtained by the extended Kantorovich method and the finite element method tend to have direct proportional with natural frequency-order. The minimum difference percentage occurs at the (1, 1) mode shape is approximately 0.128%. The maximum difference percentage occurs at the (3, 3) mode shape is approximately 6.244%.

Additional the first nine mode shapes and the first nine natural frequencies of [0/0/0/0] and [90/90/90/90] laminated composite rectangular plates with various boundary conditions are illustrated in Appendix B.

4.1.3 Results of the mode shapes and the natural frequencies of curved nodal lines

The first five mode shapes and the first five natural frequencies of [45/-45/-45/45] laminated composite square plates with various boundary conditions obtained by the combination of the extended Kantorovich method and the Rayleigh-Ritz method and the finite element method are illustrated in Table 4.7 and Figure 4.2, respectively. The results of mode shapes and the natural frequencies demonstrate that

(1) The first five mode shapes of [45/-45/-45/45] laminated composite square plates with various boundary conditions (Table 4.7) obtained by the combination of the extended Kantorovich method and the Rayleigh-Ritz method are similar to the finite element method. The nodal lines are parallel and perpendicular to the fiber orientation.

(2) The difference percentage of the first five natural frequencies of [45/-45/-45/45] laminated composite square plates with various boundary conditions (Figure 4.2) obtained by the combination of the extended Kantorovich method and the Rayleigh-Ritz method and the finite

element method tend to have direct proportional with natural frequency-order. The minimum difference percentage occurs at the first mode shape is approximately 1.431%. The maximum difference percentage occurs at the fifth mode shape is approximately 11.727%.

Additional the first five mode shapes and the first five natural frequencies of [45/45/45/45] laminated composite square plates with various boundary conditions are illustrated in Appendix C.

4.1.4 Results of the higher mode shapes and the higher natural frequencies of straight nodal lines

The first hundred mode shapes and the first hundred natural frequencies of [0/90/90/0] laminated composite square plates with CCCC boundary conditions by the extended Kantorovich method and the finite element method are illustrated in Table 4.8 and Figure 4.3, respectively. The results of the mode shapes and the natural frequencies demonstrate that

(1) The first hundred mode shapes of [0/90/90/0] laminated composite square plates with CCCC boundary conditions (Table 4.8) obtained by the extended Kantorovich method are similar to the finite element method. The nodal lines are parallel to the x and y coordinate direction.

(2) The difference percentage of the first hundred natural frequencies of [0/90/90/0] laminated composite square plates with CCCC boundary conditions (Figure 4.3) obtained by the extended Kantorovich method and the finite element method tend to have direct proportional with natural frequency-order. The minimum difference percentage occurs at the (1, 2) mode shape is approximately 0.488%. The maximum difference percentage occurs at the (10, 1) mode shape is approximately 13.002%.

Additional the first hundred mode shapes and the first hundred natural frequencies of [0/90/90/0] laminated composite rectangular plates with CCCC boundary conditions are illustrated in Appendix D.

Table 4.1 Schematic of numerical results of each study.

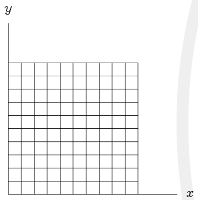
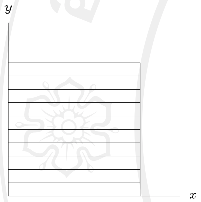
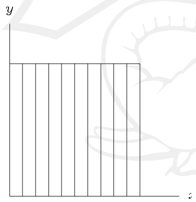
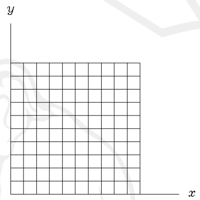
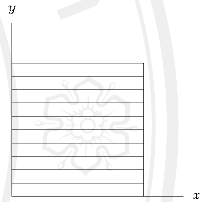
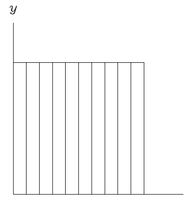
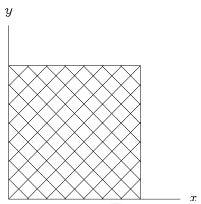
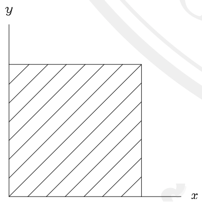
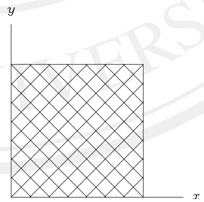
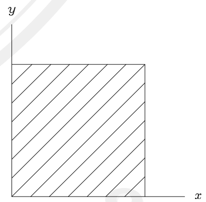
Study			Numerical results	
No.	Detail		Chapter 4	Appendix
1	Lamination schemes	 Cross-ply  Unidirectional 0°  Unidirectional 90°	 [0/90/90/0]	 [0/0/0/0]  [90/90/90/90]
	Boundary conditions	15	CCCC, CCCS, CCSS, CFCC, CFCF, CFCS, CFSC, CFSF, CFSS, CSCS, CSSS, FSCS, FSFS, SSFS, SSSS	
	Number of frequency		9	
2	Lamination schemes	 Angle-ply 45°  Unidirectional 45°	 [45/-45/-45/45]	 [45/45/45/45]
	Boundary conditions	15	CCCC, CCCS, CCSS, CFCC, CFCF, CFCS, CFSC, CFSF, CFSS, CSCS, CSSS, FSCS, FSFS, SSFS, SSSS	
	Number of frequency		5	

Table 4.1 Schematic of numerical results of each study (Continued).

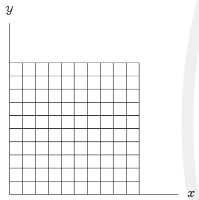
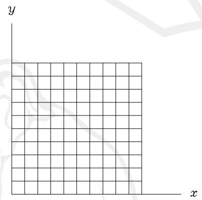
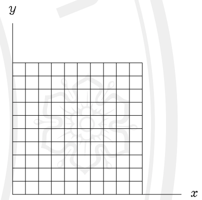
Study			Numerical results	
No.	Detail		Chapter 4	Appendix
3	Lamination schemes	 Cross-ply	 [0/90/90/0]	
	Boundary conditions	CCCC	CCCC	
	Number of frequency	100	100	

Table 4.2 The first nine frequency parameters $\omega ab\sqrt{m/D}$ of isotropic rectangular plates.

(1) Boundary conditions CCCC

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	98.324	98.324	35.999	35.999	24.581	24.581
(1, 2)	255.937	255.939	73.405	73.405	31.833	31.833
(1, 3)	492.994	492.996	131.902	131.902	44.779	44.779
(2, 1)	127.333	127.333	73.405	73.405	63.985	63.985
(2, 2)	284.324	284.325	108.236	108.236	71.081	71.081
(2, 3)	521.416	521.414	165.023	165.023	83.281	83.281
(3, 1)	179.115	179.115	131.902	131.902	123.250	123.249
(3, 2)	333.125	333.125	165.023	165.023	130.353	130.353
(3, 3)	569.511	569.510	220.059	220.059	142.377	142.377

(2) Boundary conditions CCCS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	73.405	73.405	31.833	31.833	24.144	24.144
(1, 2)	210.526	210.526	63.340	63.340	30.253	30.253
(1, 3)	427.358	427.357	116.367	116.366	41.756	41.756
(2, 1)	108.236	108.236	71.081	71.081	63.742	63.742
(2, 2)	242.668	242.667	100.803	100.803	70.145	70.143
(2, 3)	458.533	458.531	151.906	151.906	81.484	81.296
(3, 1)	165.023	165.023	130.353	130.353	123.080	123.081
(3, 2)	296.368	296.366	159.486	159.487	129.704	129.693
(3, 3)	510.650	510.647	209.335	209.336	140.996	140.937

(3) Boundary conditions CCSS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	71.081	71.081	27.059	27.059	17.770	17.770
(1, 2)	209.374	209.373	60.667	60.667	25.201	25.201
(1, 3)	426.598	426.596	114.632	114.633	37.977	37.977
(2, 1)	100.803	100.803	60.667	60.667	52.343	52.343
(2, 2)	238.347	238.347	92.844	92.844	59.600	59.587
(2, 3)	455.604	455.578	145.938	145.937	71.944	71.886
(3, 1)	151.906	151.906	114.632	114.633	106.649	106.649
(3, 2)	287.772	287.542	145.938	145.937	113.901	113.894
(3, 3)	504.478	504.312	198.151	198.116	126.119	126.078

Table 4.2 The first nine frequency parameters $\omega ab\sqrt{m/D}$ of isotropic rectangular plates (Continued).

(4) Boundary conditions CSCS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	54.743	54.743	28.951	28.951	23.816	23.816
(1, 2)	170.347	170.346	54.743	54.743	28.951	28.951
(1, 3)	366.818	366.817	102.216	102.216	39.089	39.089
(2, 1)	94.585	94.585	69.327	69.327	63.535	63.535
(2, 2)	206.698	206.697	94.585	94.585	69.327	69.327
(2, 3)	401.081	401.079	140.205	140.205	79.525	79.525
(3, 1)	154.777	154.776	129.096	129.096	122.929	122.930
(3, 2)	265.196	265.196	154.777	154.776	129.096	129.096
(3, 3)	457.440	457.439	199.811	199.811	139.623	139.622

(5) Boundary conditions CSSS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	51.674	51.674	23.646	23.646	17.332	17.332
(1, 2)	168.959	168.959	51.674	51.674	23.646	23.646
(1, 3)	365.951	365.950	100.270	100.270	35.051	35.051
(2, 1)	86.134	86.134	58.646	58.646	52.098	52.098
(2, 2)	201.726	201.725	86.134	86.134	58.646	58.646
(2, 3)	397.770	397.768	133.791	133.791	69.913	69.913
(3, 1)	140.845	140.846	113.227	113.228	106.479	106.479
(3, 2)	255.470	255.469	140.845	140.846	113.227	113.228
(3, 3)	450.483	450.482	188.114	188.113	124.633	124.633

(6) Boundary conditions SSSS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	49.348	49.348	19.739	19.739	12.337	12.337
(1, 2)	167.784	167.783	49.348	49.348	19.739	19.739
(1, 3)	365.177	365.175	98.696	98.696	32.076	32.076
(2, 1)	78.957	78.957	49.348	49.348	41.946	41.946
(2, 2)	197.393	197.392	78.957	78.957	49.348	49.348
(2, 3)	394.786	394.784	128.305	128.305	61.685	61.685
(3, 1)	128.305	128.305	98.696	98.696	91.294	91.294
(3, 2)	246.741	246.740	128.305	128.305	98.696	98.696
(3, 3)	444.134	444.132	177.653	177.653	111.033	111.033

Table 4.3 The first hundred frequency parameters $\omega ab\sqrt{m/D}$ of isotropic square plates with CCCC boundary conditions.

i	SS		SA		AS		AA	
	EKM	Rajalingham (1997)	EKM	Rajalingham (1997)	EKM	Rajalingham (1997)	EKM	Rajalingham (1997)
1	35.999	35.998	73.405	73.405	73.405	73.405	108.236	108.235
2	131.902	131.902	165.023	165.023	165.023	165.023	242.670	242.667
3	131.902	131.902	210.526	210.526	210.526	210.526	242.670	242.667
4	220.059	220.058	296.366	296.366	296.366	296.366	371.376	371.375
5	309.036	309.037	340.590	340.590	340.590	340.590	458.533	458.531
6	309.036	309.037	427.359	427.356	427.359	427.356	458.533	458.531
7	393.357	393.355	467.291	467.290	467.291	467.290	583.745	583.748
8	393.357	393.355	510.645	510.647	510.645	510.647	583.745	583.748
9	562.177	562.178	596.363	596.366	596.363	596.366	754.034	754.035
10	565.452	565.452	677.744	677.745	677.744	677.745	754.034	754.035
11	565.452	565.452	720.483	720.486	720.483	720.486	792.461	792.462
12	648.021	648.020	723.306	723.308	723.306	723.308	877.329	877.329
13	648.021	648.020	805.348	805.350	805.348	805.350	877.329	877.329
14	813.746	813.747	927.708	927.706	927.708	927.706	1083.300	1083.301
15	813.746	813.747	931.500	931.503	931.500	931.503	1083.300	1083.301
16	900.913	900.917	969.996	969.994	969.996	969.994	1128.750	1128.751
17	900.913	900.917	1054.160	1054.163	1054.160	1054.163	1128.750	1128.751
18	982.562	982.561	1098.270	1098.275	1098.270	1098.275	1250.910	1250.915
19	982.562	982.561	1179.610	1179.610	1179.610	1179.610	1250.910	1250.915
20	1062.230	1062.226	1217.160	1217.163	1217.160	1217.163	1371.470	1371.470
21	1146.340	1146.348	1259.110	1259.105	1259.110	1259.105	1455.010	1455.006
22	1146.340	1146.348	1342.710	1342.710	1342.710	1342.710	1455.010	1455.006
23	1392.390	1392.385	1546.110	1546.110	1546.110	1546.110	1740.980	1740.982
24	1392.390	1392.385	1587.780	1587.780	1587.780	1587.780	1740.980	1740.982
25	1720.190	1720.193	1914.540	1914.550	1914.540	1914.550	2108.400	2108.395

Table 4.4 The first nine frequency parameters $\omega ab\sqrt{m/D_{22}}$ of orthotropic rectangular plates.

(1) Boundary conditions CCCC

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	95.391	95.391	33.917	33.917	23.848	23.848
(1, 2)	251.966	251.965	69.687	69.687	29.626	29.625
(1, 3)	488.703	488.701	127.613	127.613	41.396	41.396
(2, 1)	118.502	118.502	69.687	69.687	62.991	62.991
(2, 2)	269.987	269.987	98.440	98.440	67.497	67.497
(2, 3)	505.072	505.073	151.291	151.290	76.419	76.419
(3, 1)	165.584	165.583	127.613	127.613	122.175	122.175
(3, 2)	305.675	305.677	151.291	151.290	126.269	126.268
(3, 3)	535.625	535.624	197.291	197.290	133.906	133.906

(2) Boundary conditions CCCS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	69.687	69.687	29.626	29.625	23.447	23.447
(1, 2)	205.994	205.994	59.270	59.270	28.057	28.057
(1, 3)	422.631	422.632	111.711	111.711	38.276	38.276
(2, 1)	98.440	98.440	67.497	67.497	62.794	62.794
(2, 2)	226.799	226.800	90.838	90.838	66.672	66.672
(2, 3)	440.758	440.757	137.574	137.574	74.533	74.532
(3, 1)	151.291	151.290	126.269	126.268	122.045	122.045
(3, 2)	267.125	267.127	145.991	145.990	125.677	125.730
(3, 3)	474.395	474.396	186.338	186.337	132.680	132.654

(3) Boundary conditions CCSS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	67.497	67.497	24.610	24.610	16.874	16.874
(1, 2)	205.044	205.045	56.700	56.700	22.710	22.710
(1, 3)	422.039	422.040	110.188	110.189	34.394	34.394
(2, 1)	90.838	90.838	56.700	56.700	51.261	51.261
(2, 2)	222.923	222.923	82.584	82.584	55.731	55.731
(2, 3)	438.331	438.325	131.767	131.766	64.637	64.637
(3, 1)	137.574	137.574	110.188	110.189	105.510	105.510
(3, 2)	258.547	258.547	131.767	131.766	109.581	109.581
(3, 3)	468.869	468.802	174.753	174.753	117.216	117.201

Table 4.4 The first nine frequency parameters $\omega ab\sqrt{m/D_{22}}$ of orthotropic rectangular plates (Continued).

(4) Boundary conditions CSCS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	50.349	50.349	26.809	26.809	23.173	23.172
(1, 2)	165.148	165.147	50.349	50.349	26.809	26.809
(1, 3)	361.599	361.598	97.171	97.171	35.547	35.547
(2, 1)	84.652	84.652	65.989	65.989	62.635	62.635
(2, 2)	189.301	189.300	84.652	84.652	65.989	65.989
(2, 3)	381.751	381.750	125.380	125.380	72.902	72.901
(3, 1)	141.681	141.682	125.236	125.263	121.934	121.933
(3, 2)	234.630	234.629	141.681	141.682	125.263	125.263
(3, 3)	418.843	418.842	176.791	176.791	131.549	131.548

(5) Boundary conditions CSSS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	47.325	47.325	21.163	21.163	16.497	16.497
(1, 2)	163.988	163.988	47.325	47.325	21.163	21.163
(1, 3)	360.921	360.919	95.437	95.437	31.345	31.345
(2, 1)	75.783	75.783	54.927	54.927	51.073	51.073
(2, 2)	184.718	184.717	75.783	75.783	54.927	54.927
(2, 3)	378.989	378.987	119.030	119.030	62.774	62.774
(3, 1)	127.025	127.026	109.055	109.055	105.383	105.384
(3, 2)	224.931	224.930	127.025	127.026	109.055	109.055
(3, 3)	412.583	412.581	164.600	164.600	115.967	115.966

(6) Boundary conditions SSSS

(i, j)	b = 0.5a		b = a		b = 2a	
	EKM	Sakata (1996)	EKM	Sakata (1996)	EKM	Sakata (1996)
(1, 1)	45.228	45.228	17.095	17.095	11.307	11.307
(1, 2)	163.073	163.073	45.228	45.228	17.095	17.095
(1, 3)	360.344	360.342	94.150	94.150	28.455	28.455
(2, 1)	68.379	68.379	45.228	45.228	40.768	40.768
(2, 2)	180.914	180.913	68.379	68.379	45.228	45.228
(2, 3)	376.602	376.600	113.822	113.822	54.114	54.114
(3, 1)	113.822	113.822	94.150	94.150	90.086	90.085
(3, 2)	216.458	216.457	113.822	113.822	94.150	94.150
(3, 3)	407.056	407.054	153.852	153.852	101.763	101.763

Table 4.5 The first frequency parameters $(\omega ab/\pi^2)\sqrt{m/D_{22}}$ of [0/90/90/0] laminated composite rectangular plates with SSSS boundary conditions.

a/b	$E_1 = 10E_2$		$E_1 = 20E_2$		$E_1 = 40E_2$	
	EKM	Reddy (2004)	EKM	Reddy (2004)	EKM	Reddy (2004)
0.5	8.515	8.515	9.355	9.355	9.917	9.917
1.0	2.519	2.519	2.638	2.638	2.721	2.721
1.5	1.531	1.531	1.536	1.536	1.539	1.539
2.0	1.246	1.246	1.229	1.229	1.216	1.216
2.5	1.138	1.138	1.119	1.119	1.105	1.105
3.0	1.087	1.087	1.071	1.071	1.059	1.059

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates.

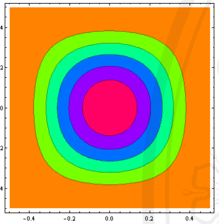
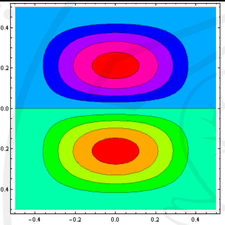
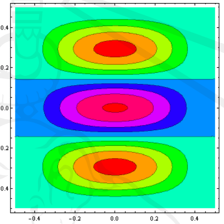
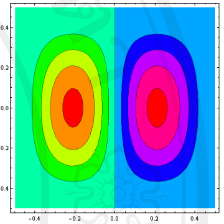
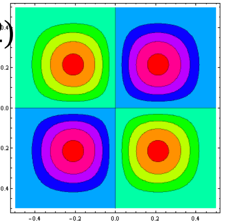
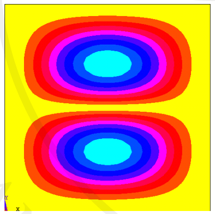
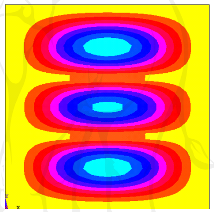
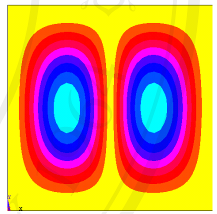
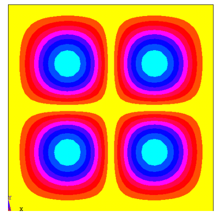
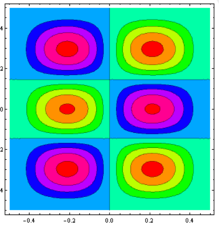
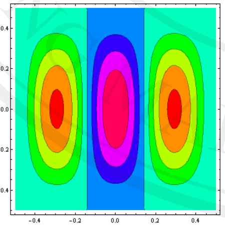
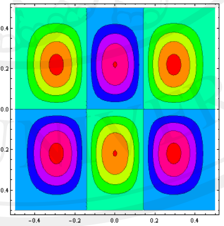
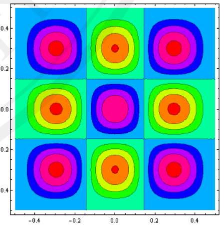
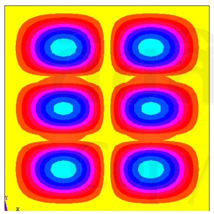
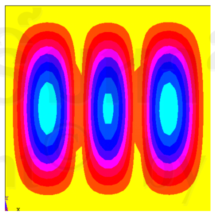
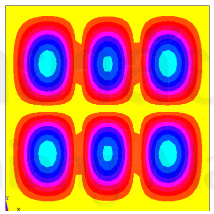
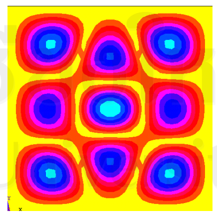
BCs	Method	Plate mode				
CCCC	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

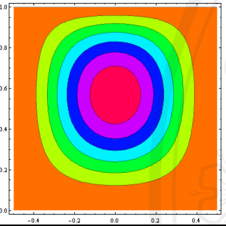
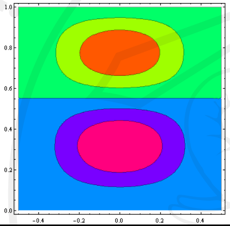
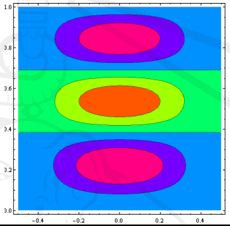
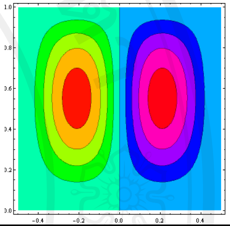
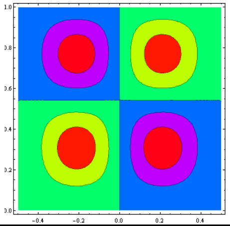
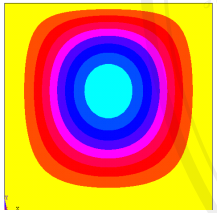
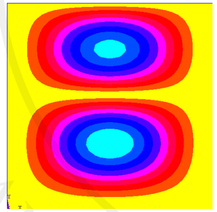
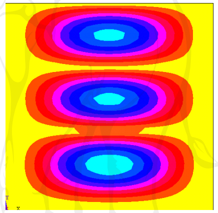
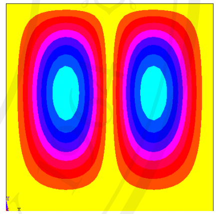
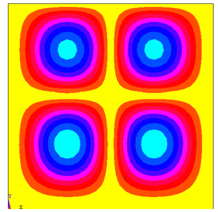
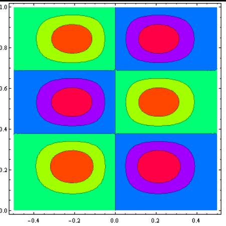
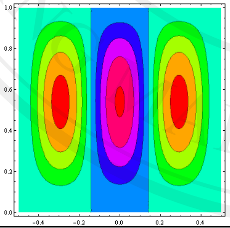
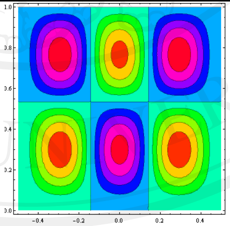
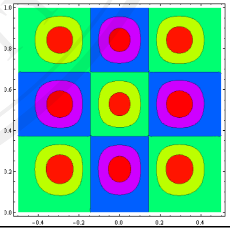
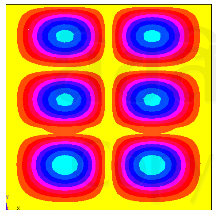
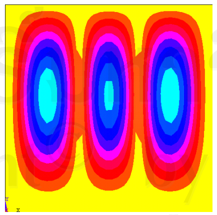
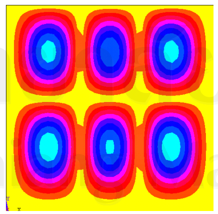
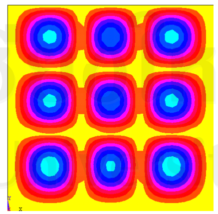
BCs	Method	Plate mode				
CCCS	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

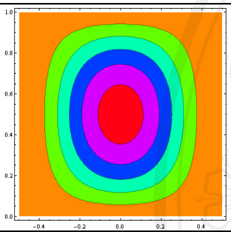
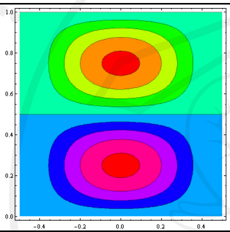
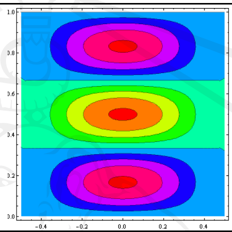
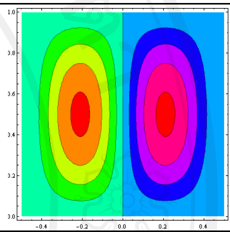
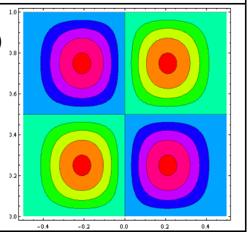
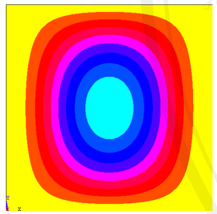
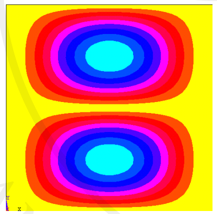
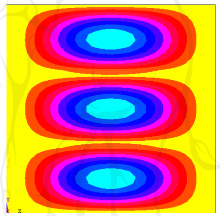
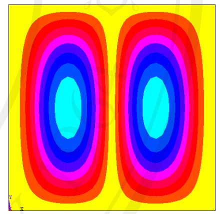
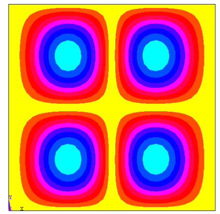
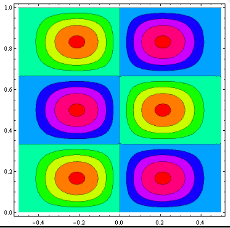
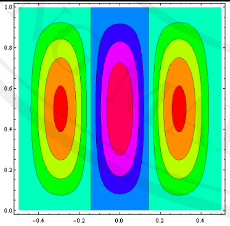
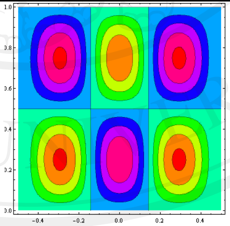
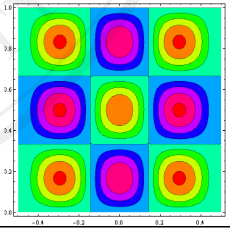
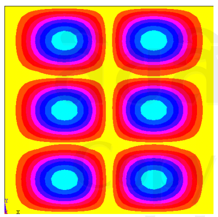
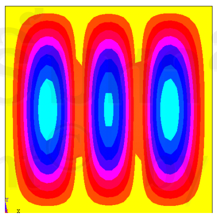
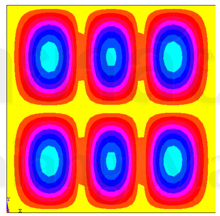
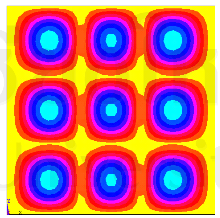
BCs	Method	Plate mode				
CSCS	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

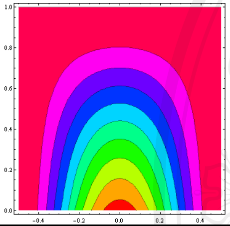
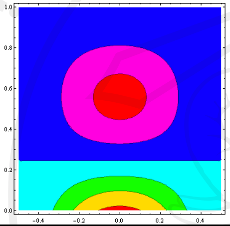
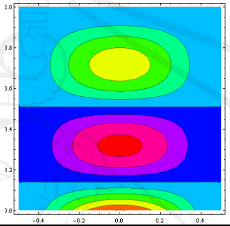
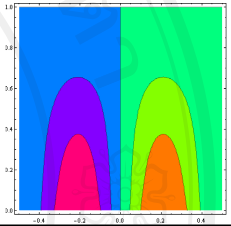
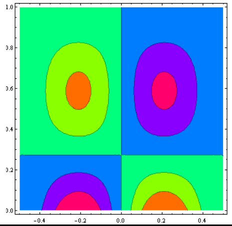
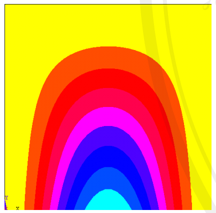
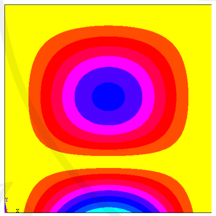
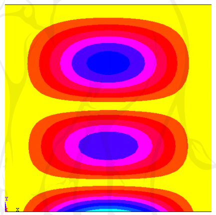
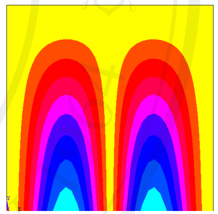
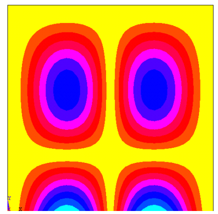
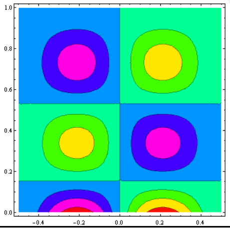
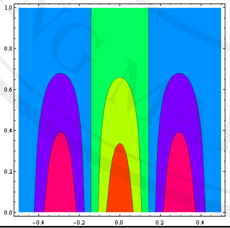
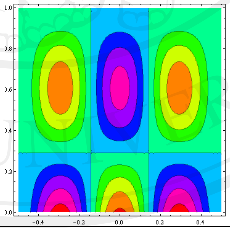
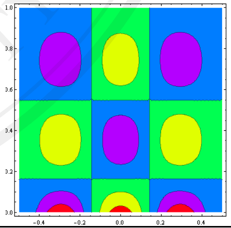
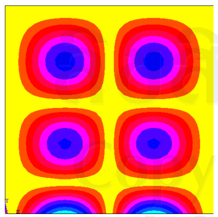
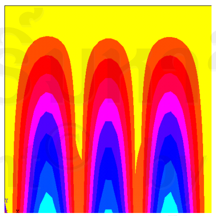
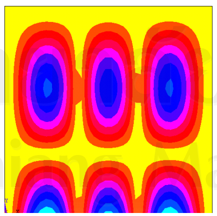
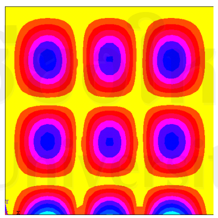
BCs	Method	Plate mode				
CFCC	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

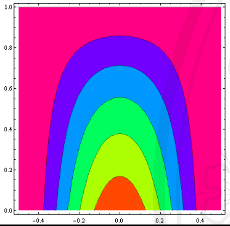
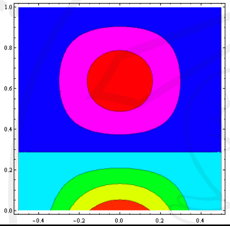
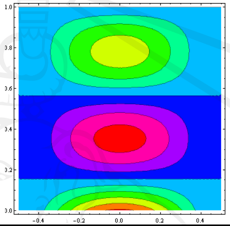
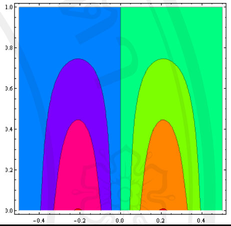
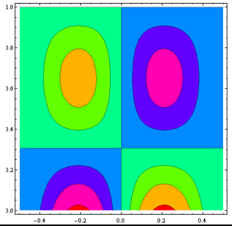
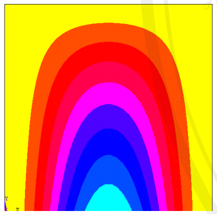
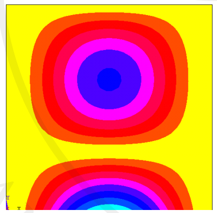
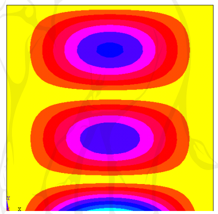
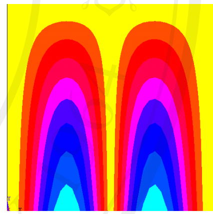
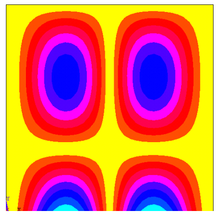
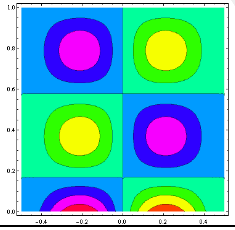
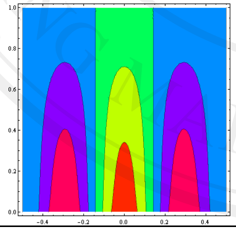
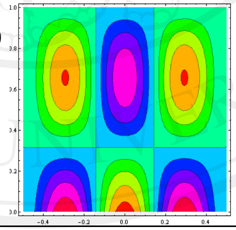
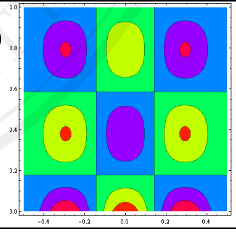
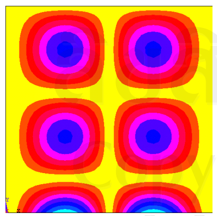
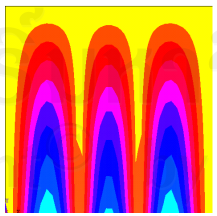
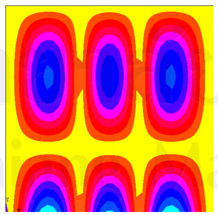
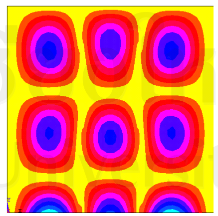
BCs	Method	Plate mode				
CFCS	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

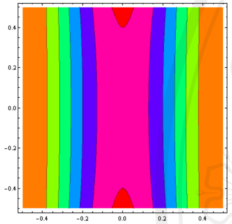
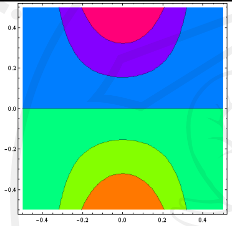
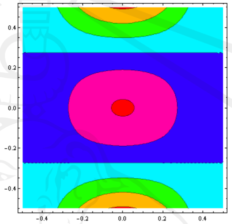
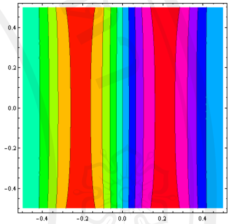
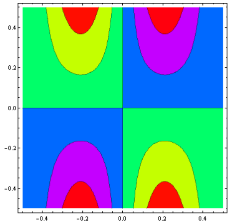
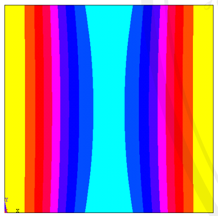
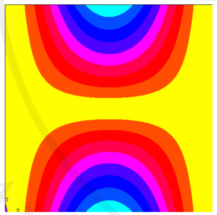
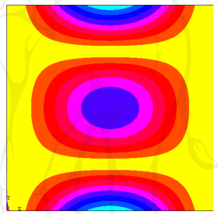
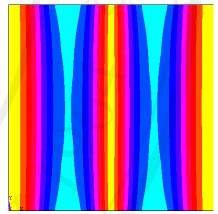
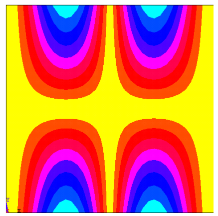
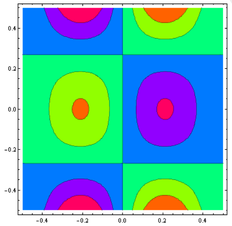
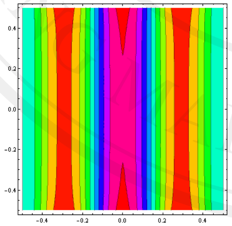
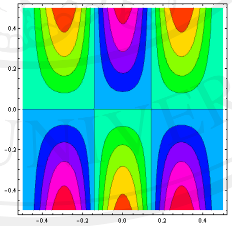
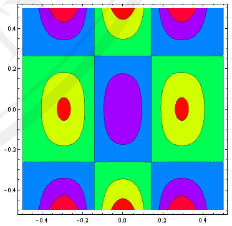
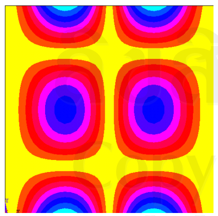
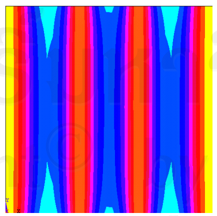
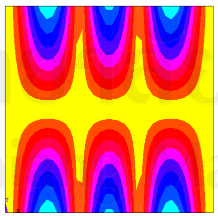
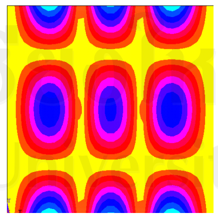
BCs	Method	Plate mode				
CFCF	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

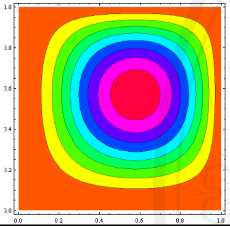
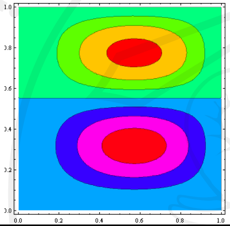
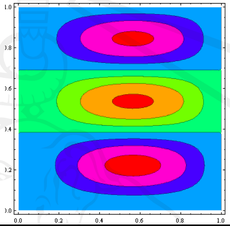
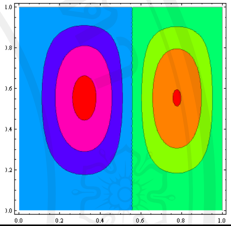
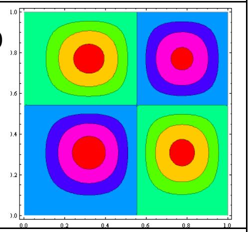
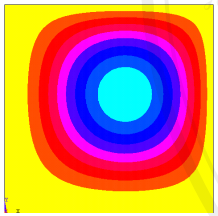
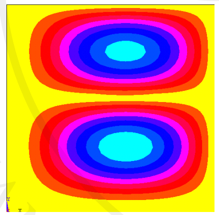
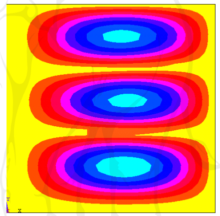
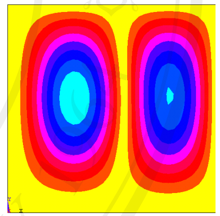
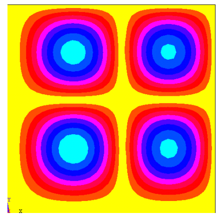
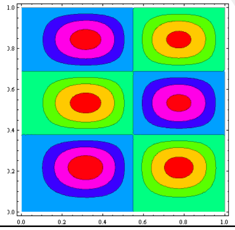
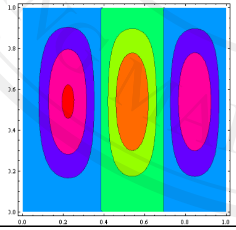
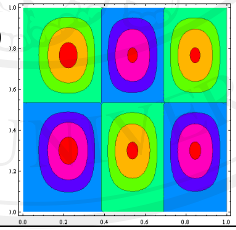
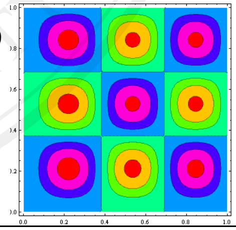
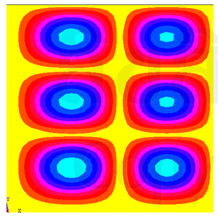
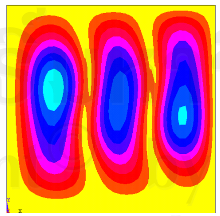
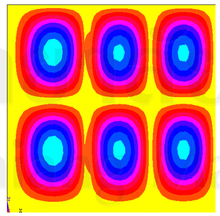
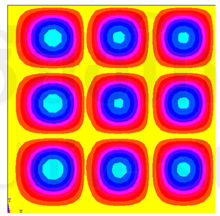
BCs	Method	Plate mode				
CCSS	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

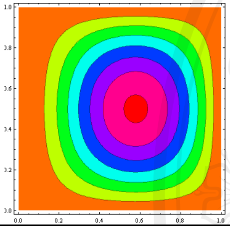
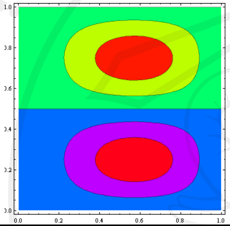
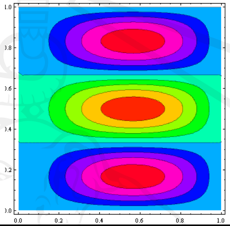
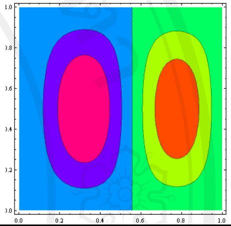
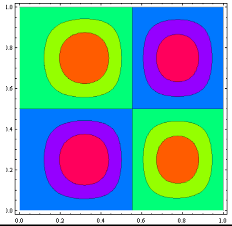
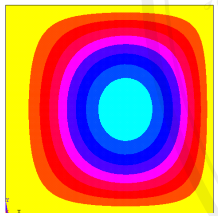
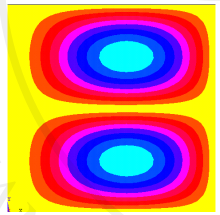
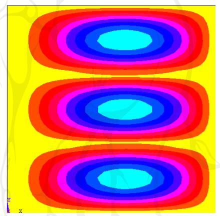
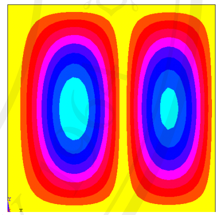
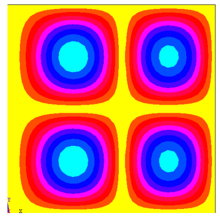
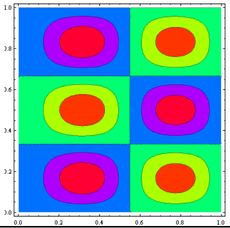
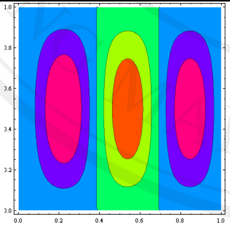
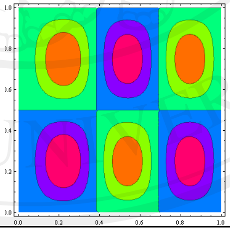
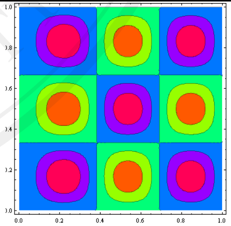
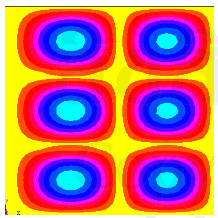
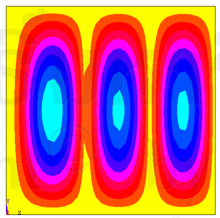
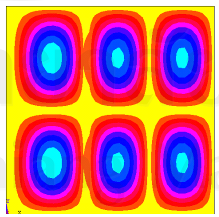
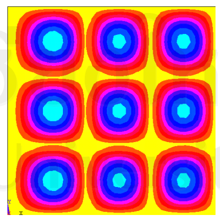
<table>
<tr><th>BCs</th><th>Method</th><th colspan="5">Plate mode</th></tr>
<tr><td rowspan="4">CSSS</td><td>EKM</td><td>(1, 1) </td><td>(1, 2) </td><td>(1, 3) </td><td>(2, 1) </td><td>(2, 2) </td></tr>
<tr><td>FEM</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>EKM</td><td>(2, 3) </td><td>(3, 1) </td><td>(3, 2) </td><td>(3, 3) </td><td></td></tr>
<tr><td>FEM</td><td></td><td></td><td></td><td></td><td></td></tr>
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Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

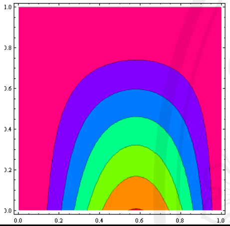
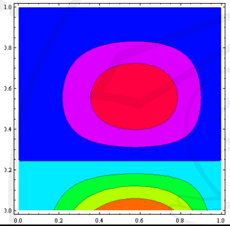
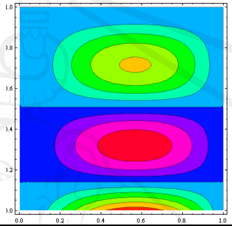
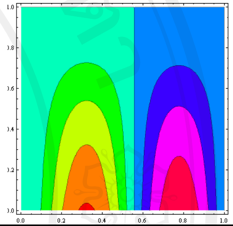
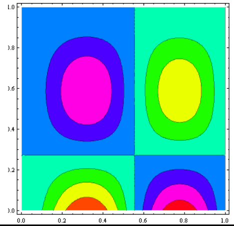
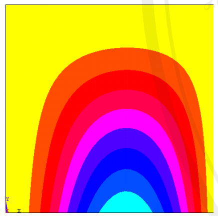
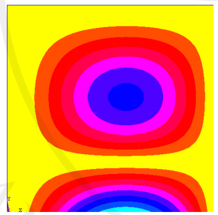
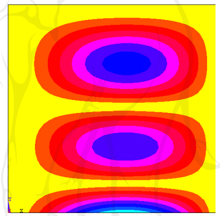
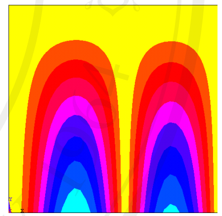
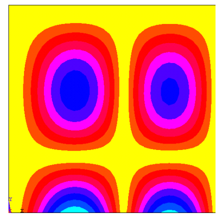
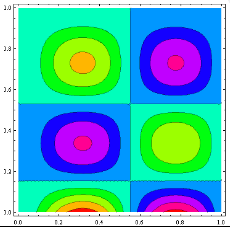
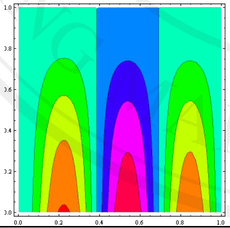
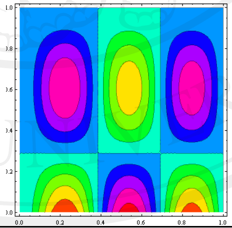
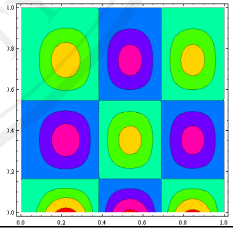
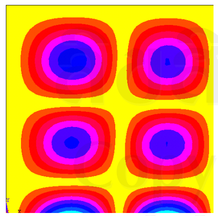
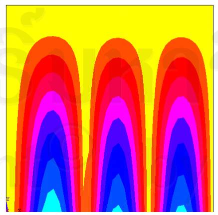
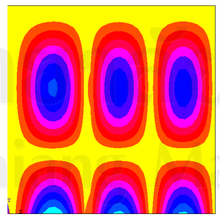
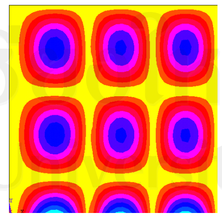
BCs	Method	Plate mode				
CFSC	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

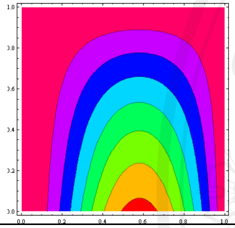
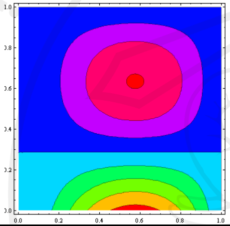
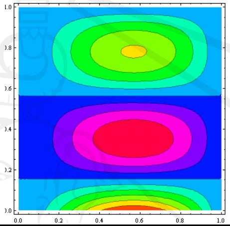
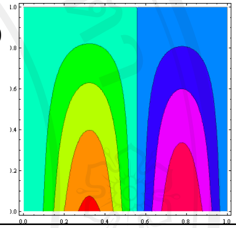
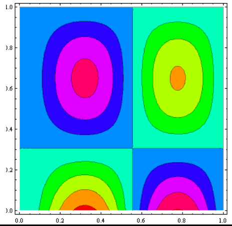
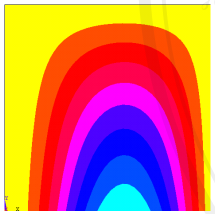
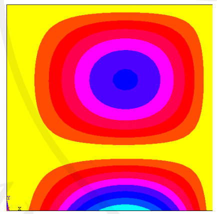
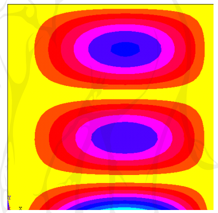
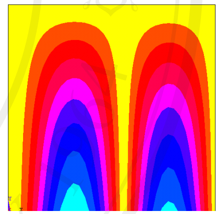
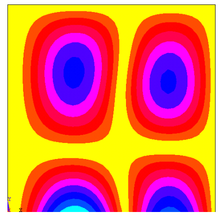
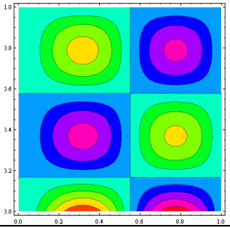
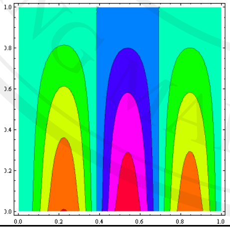
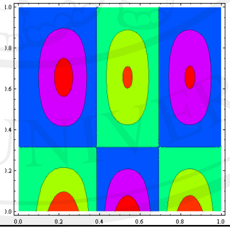
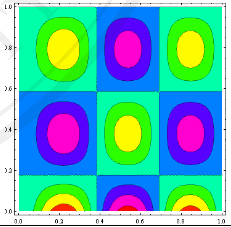
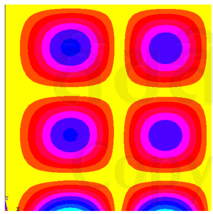
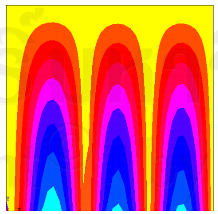
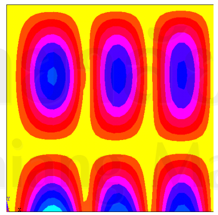
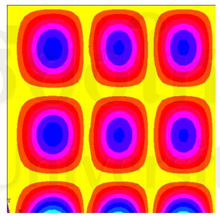
BCs	Method	Plate mode				
CFSS	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

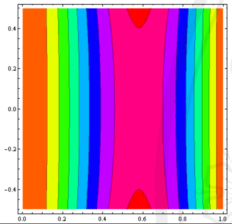
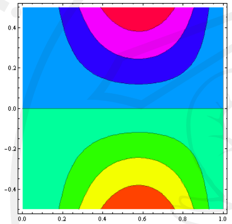
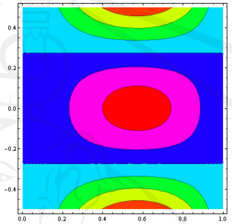
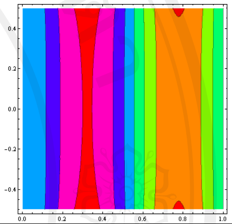
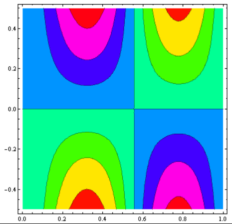
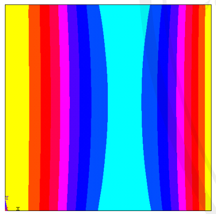
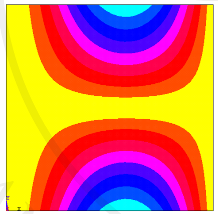
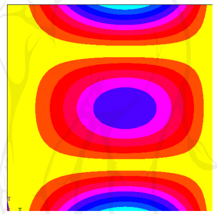
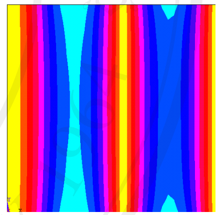
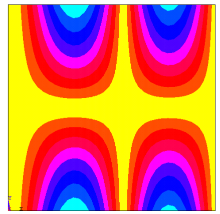
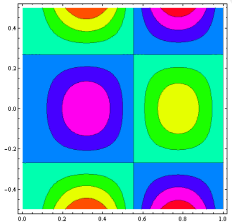
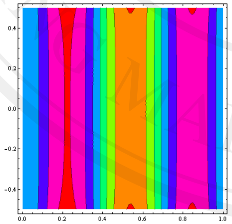
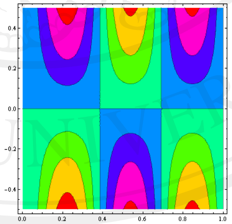
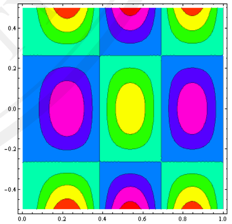
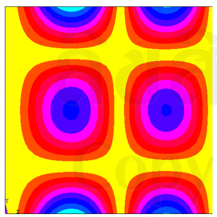
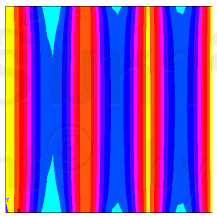
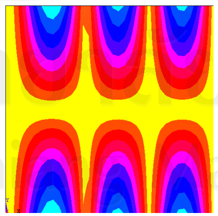
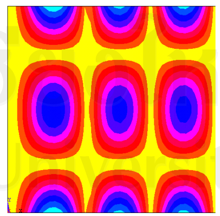
BCs	Method	Plate mode				
CFSF	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

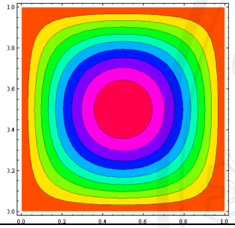
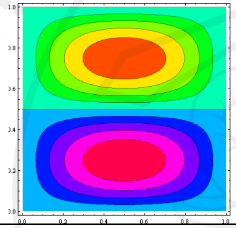
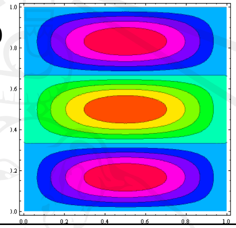
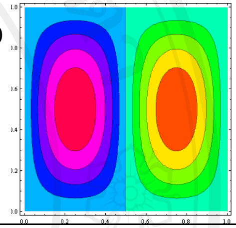
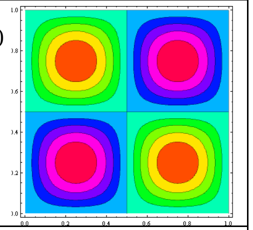
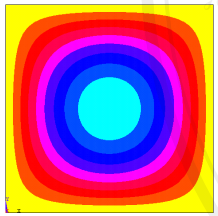
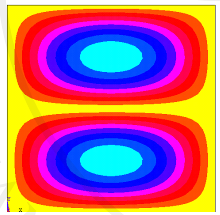
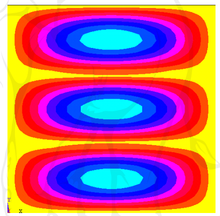
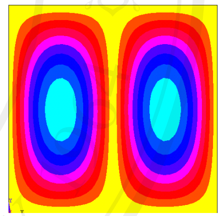
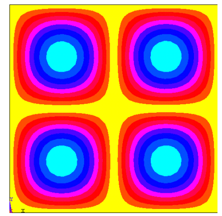
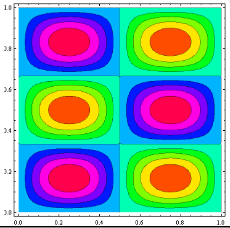
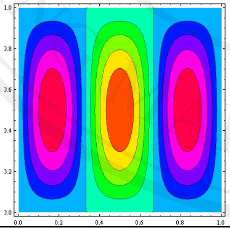
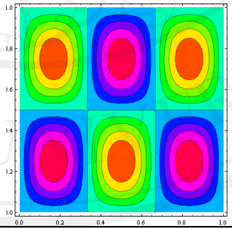
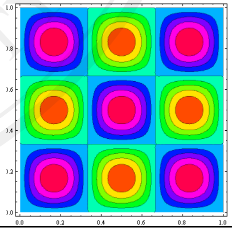
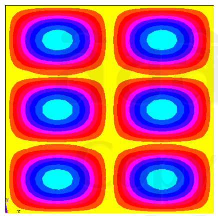
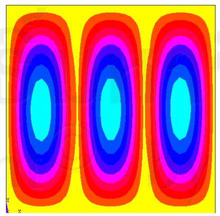
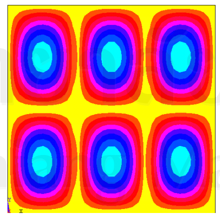
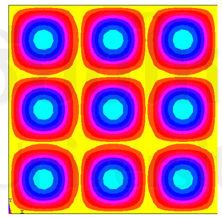
BCs	Method	Plate mode				
SSSS	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

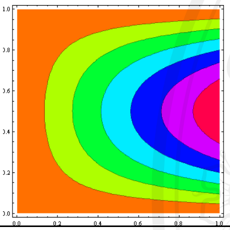
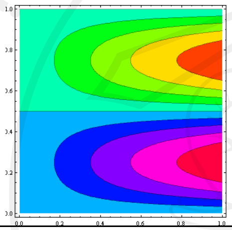
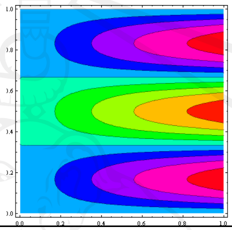
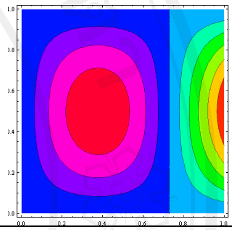
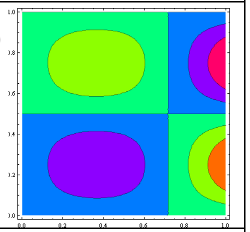
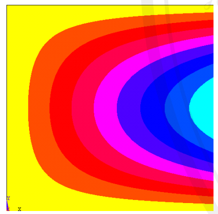
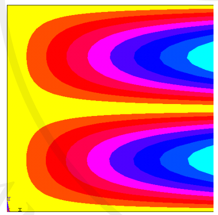
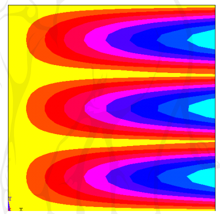
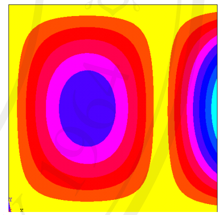
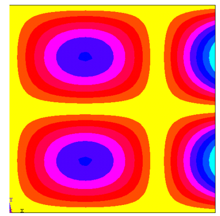
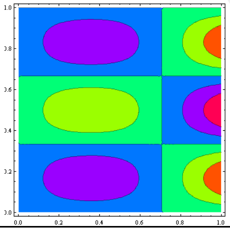
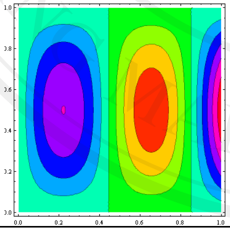
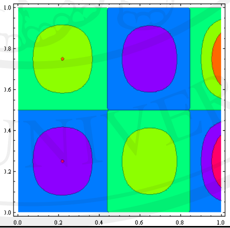
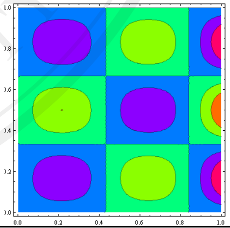
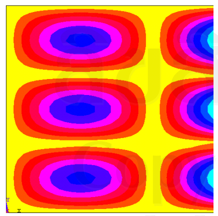
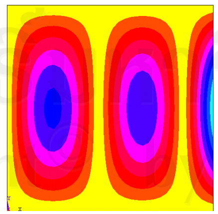
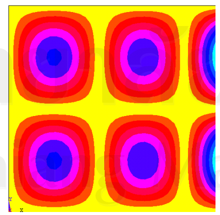
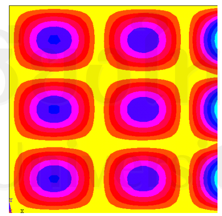
BCs	Method	Plate mode				
SSFS	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

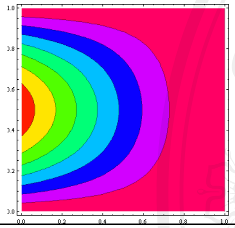
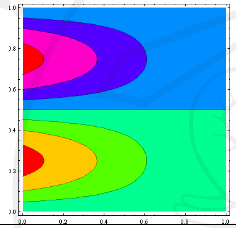
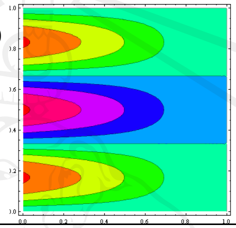
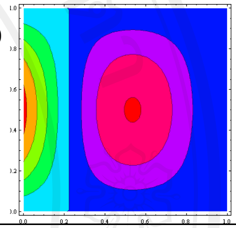
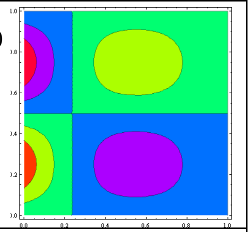
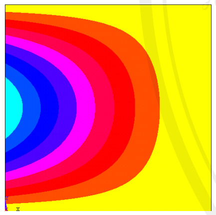
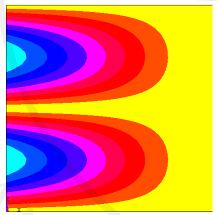
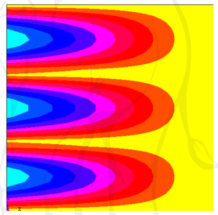
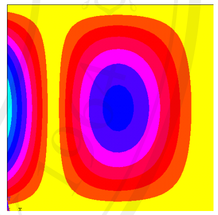
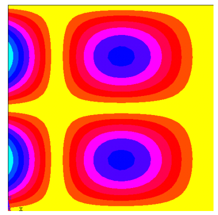
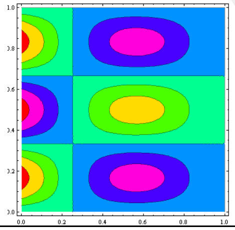
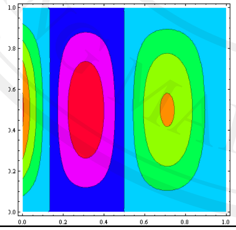
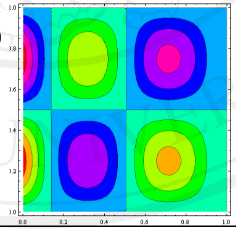
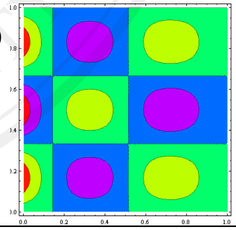
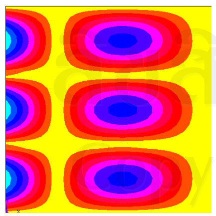
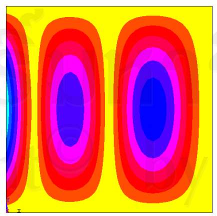
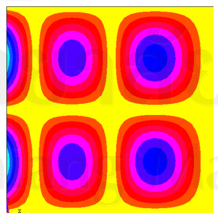
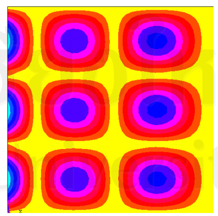
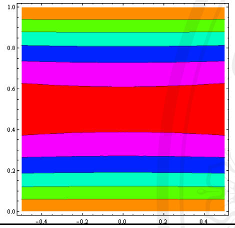
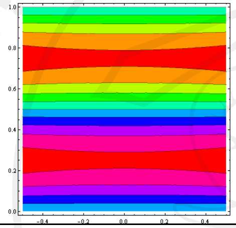
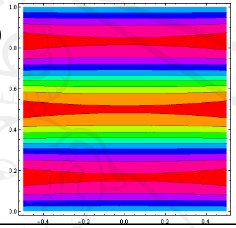
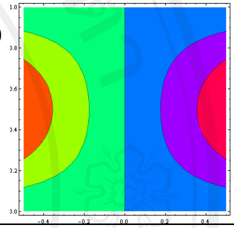
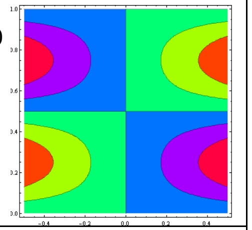
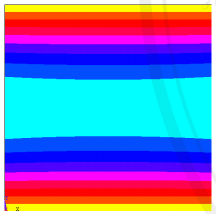
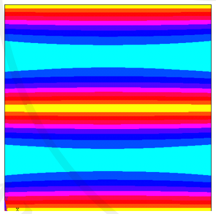
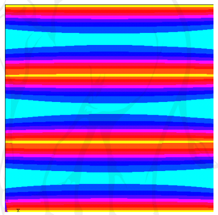
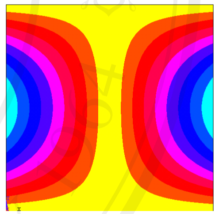
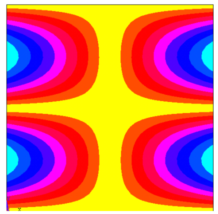
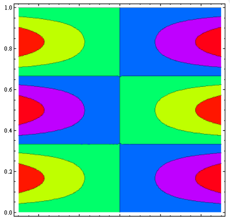
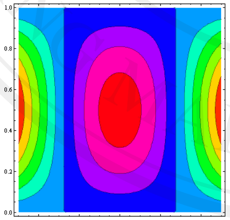
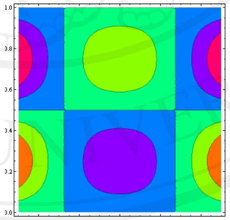
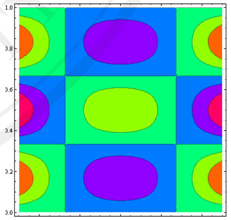
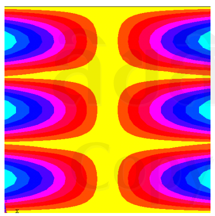
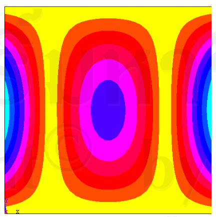
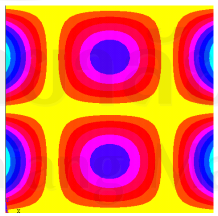
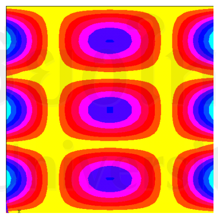
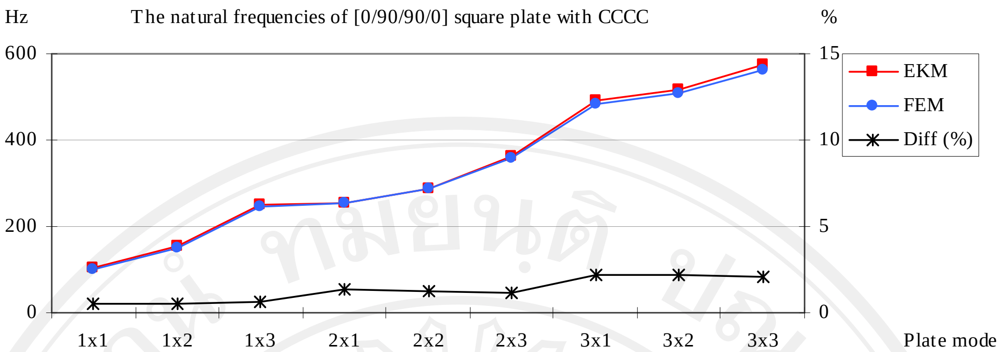
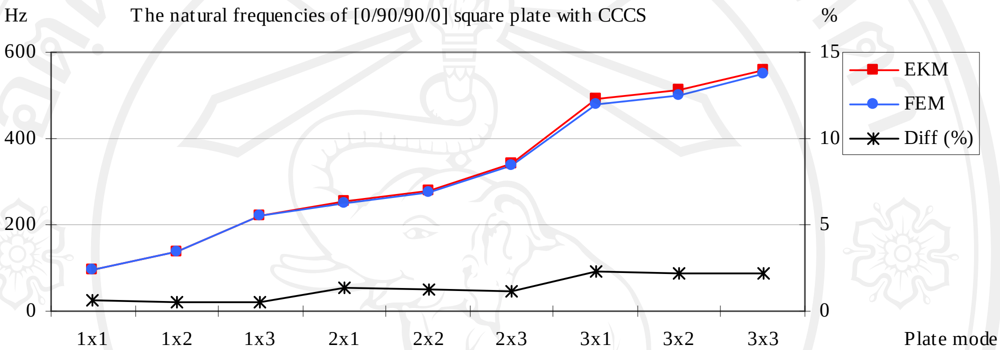
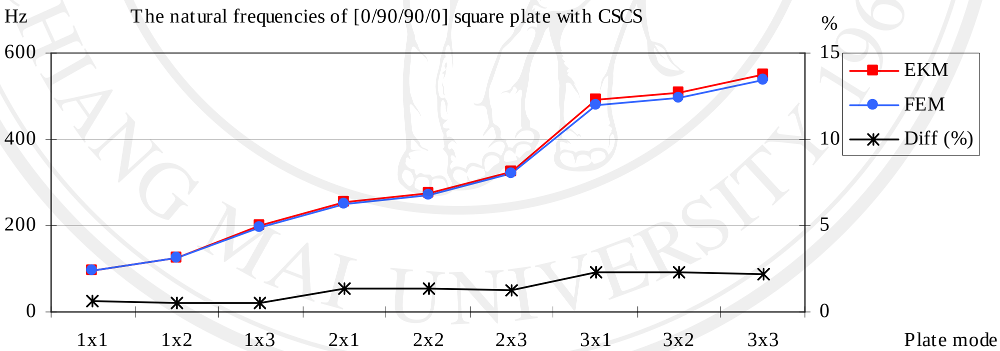
BCs	Method	Plate mode				
FSCS	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					

Table 4.6 The first nine mode shapes of [0/90/90/0] laminated composite square plates (Continued).

BCs	Method	Plate mode				
FSFS	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(2, 1) 	(2, 2) 
	FEM					
	EKM	(2, 3) 	(3, 1) 	(3, 2) 	(3, 3) 	
	FEM					







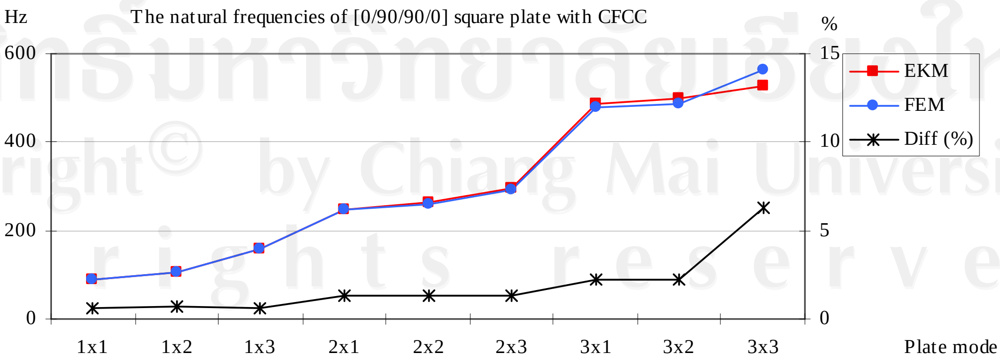


Figure 4.1 The first nine natural frequencies of [0/90/90/0] laminated composite square plates.

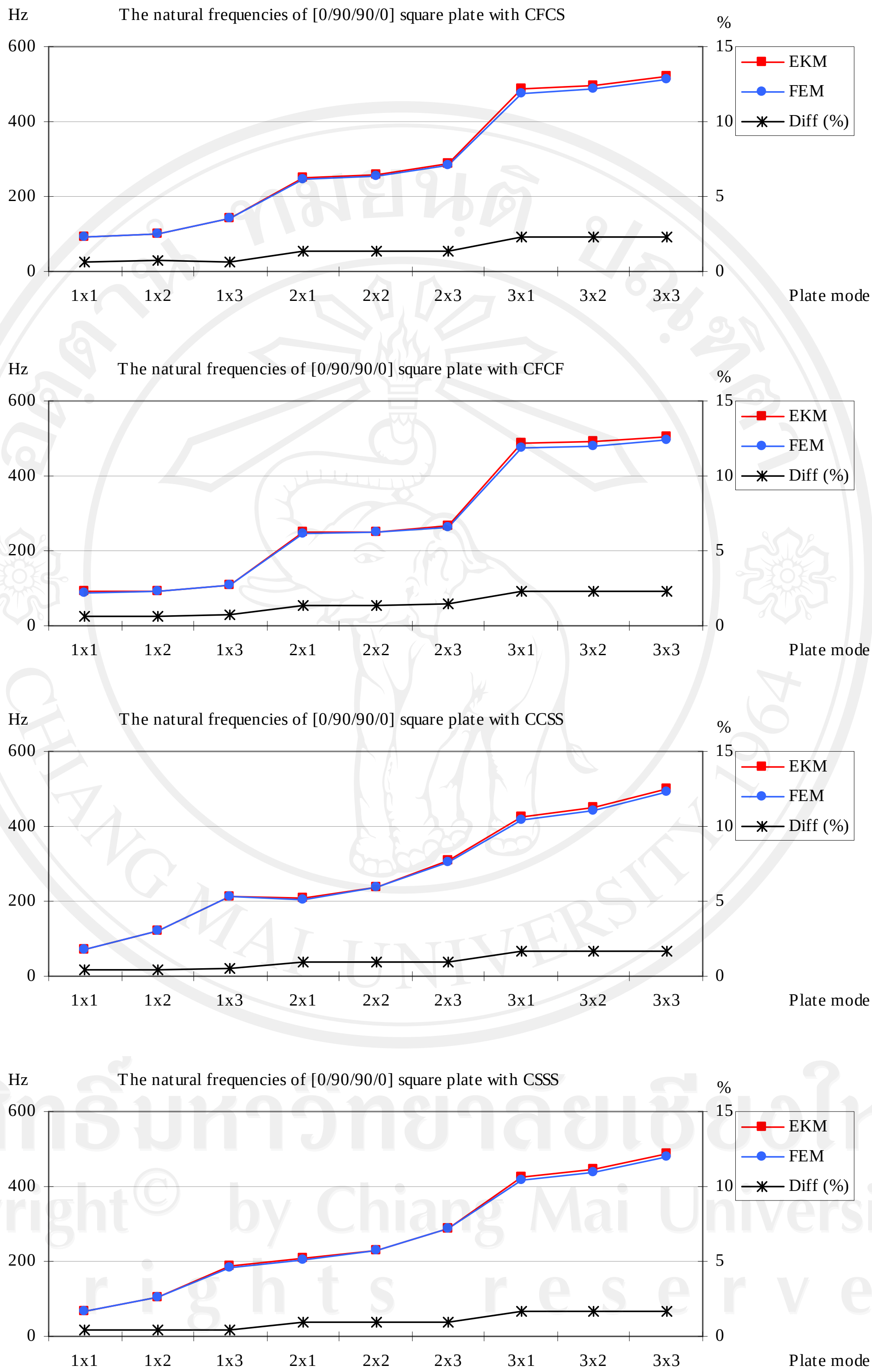
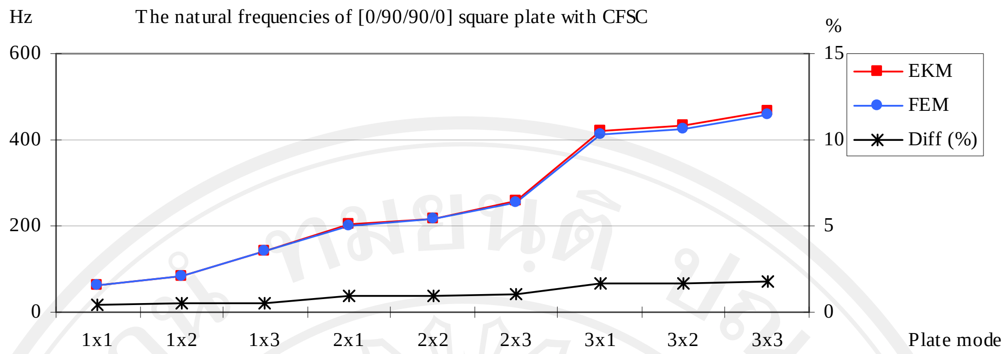
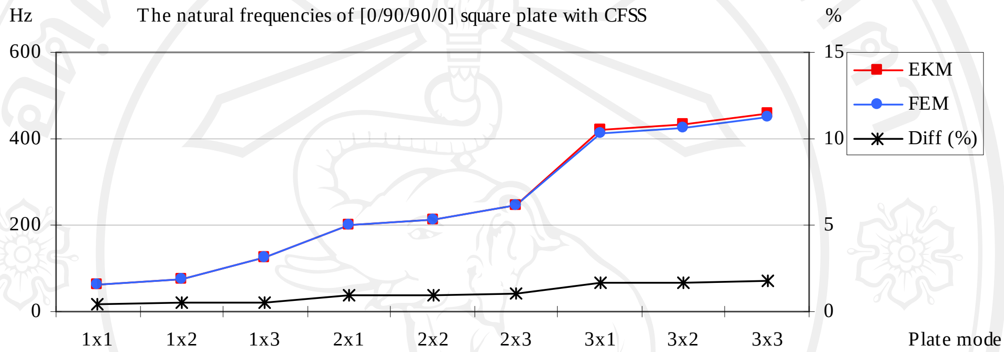
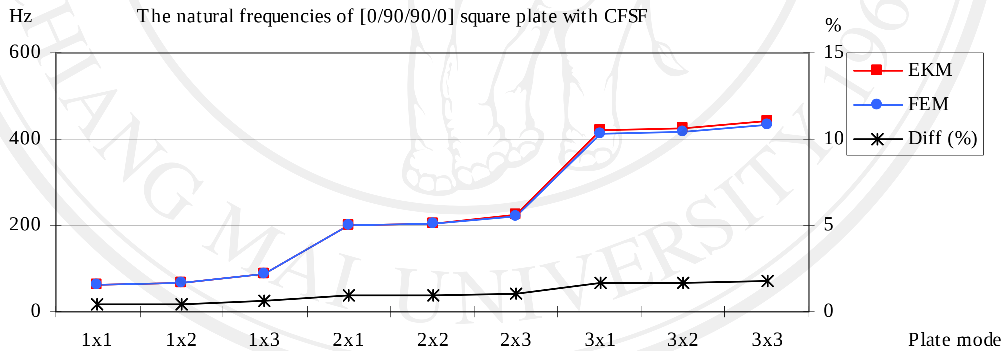


Figure 4.1 The first nine natural frequencies of [0/90/90/0] laminated composite square plates (Continued).







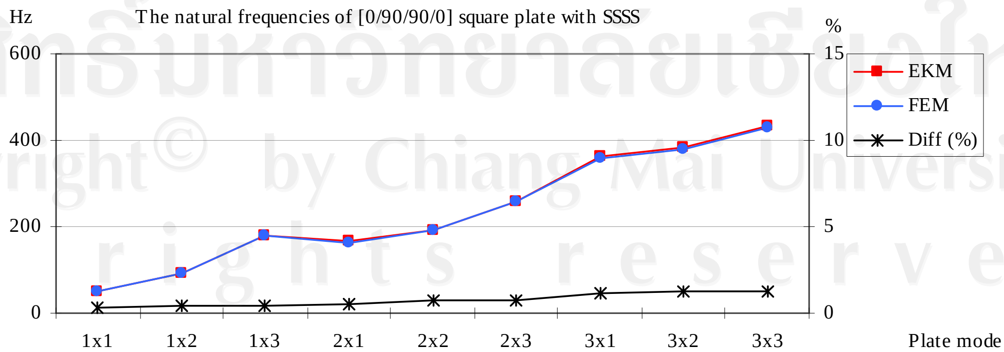
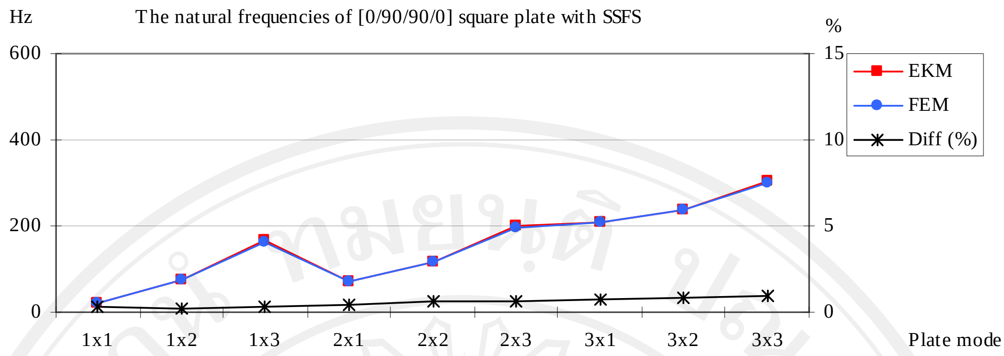
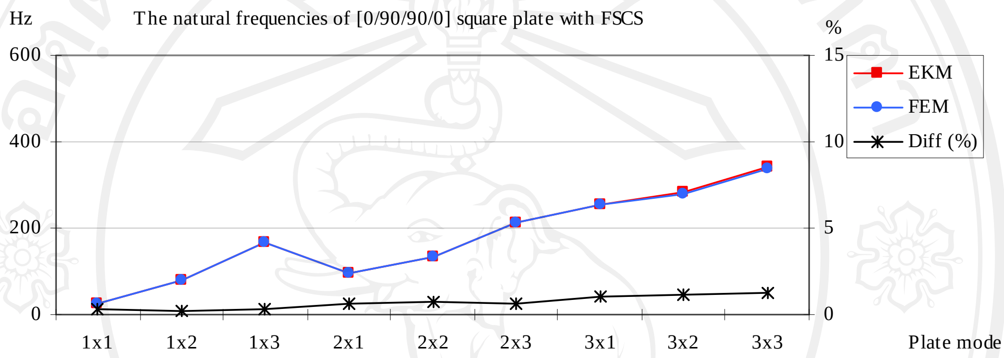


Figure 4.1 The first nine natural frequencies of [0/90/90/0] laminated composite square plates (Continued).





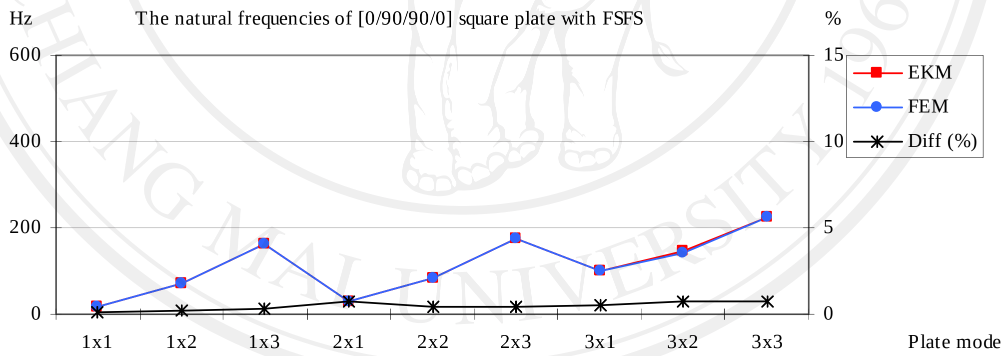


Figure 4.1 The first nine natural frequencies of [0/90/90/0] laminated composite square plates (Continued).

Table 4.7 The first five mode shapes of [45/-45/-45/45] laminated composite square plates.

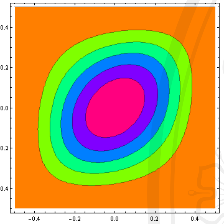
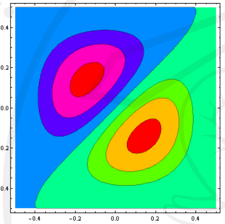
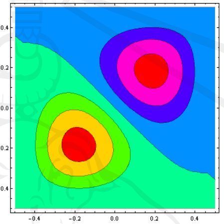
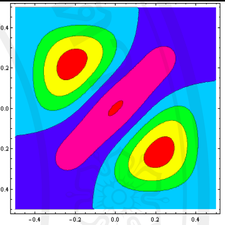
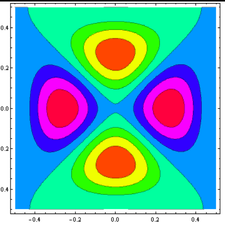
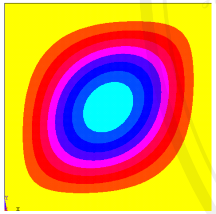
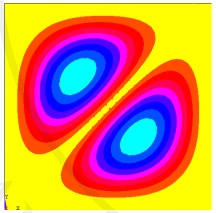
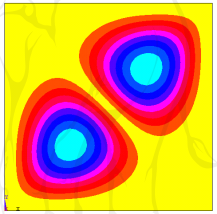
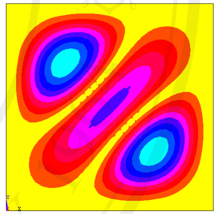
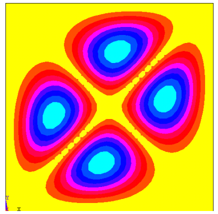
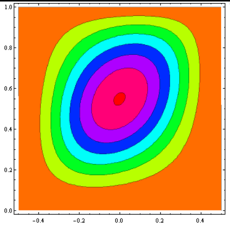
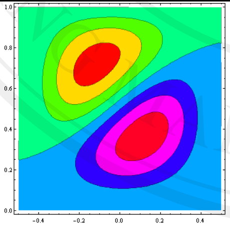
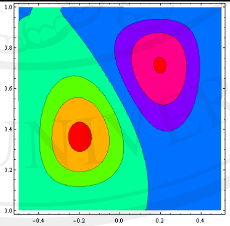
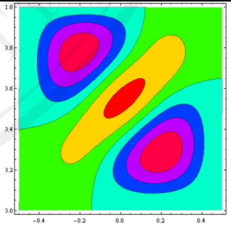
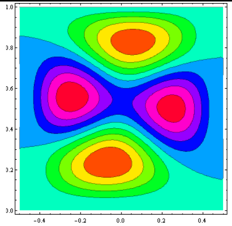
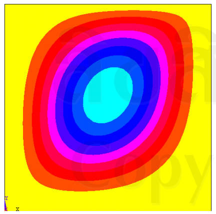
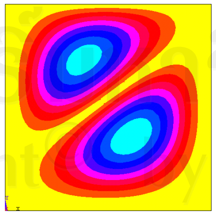
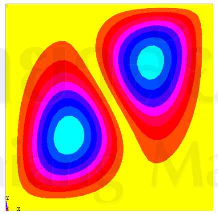
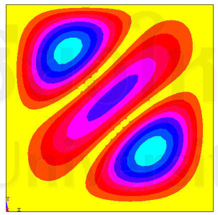
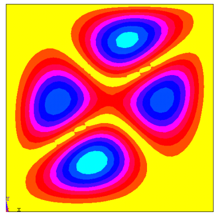
BCs	Method	Plate mode				
CCCC	RRM	1st 	2nd 	3rd 	4th 	5th 
	FEM					
CCCS	RRM	1st 	2nd 	3rd 	4th 	5th 
	FEM					

Table 4.7 The first five mode shapes of [45/-45/-45/45] laminated composite square plates (Continued).

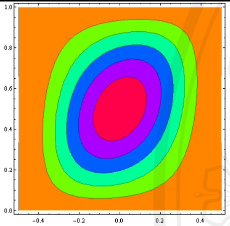
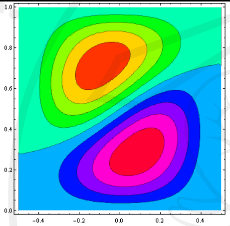
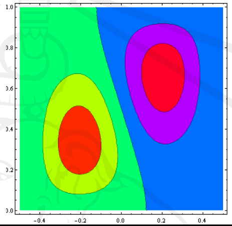
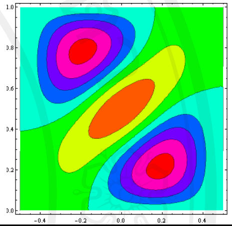
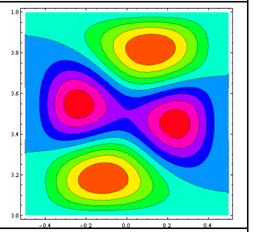
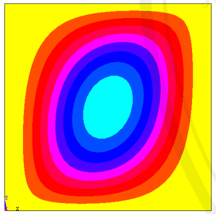
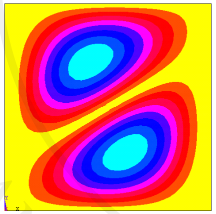
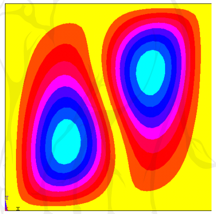
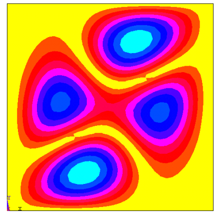
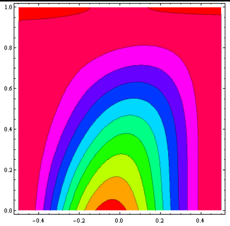
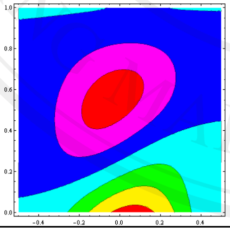
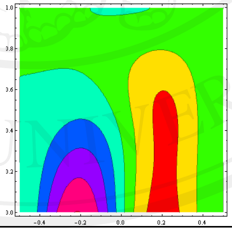
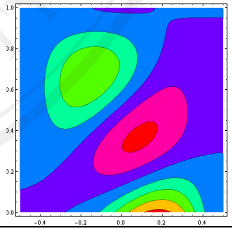
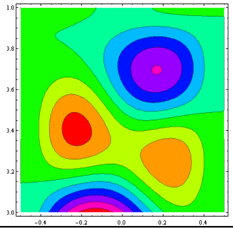
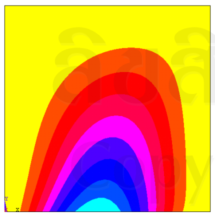
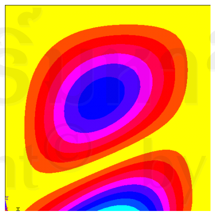
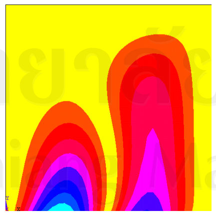
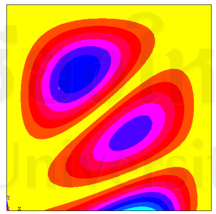
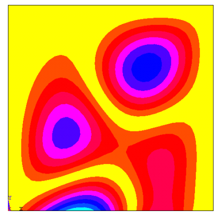
BCs	Method	Plate mode				
CSCS	RRM	1^{st} 	2^{nd} 	3^{rd} 	4^{th} 	5^{th} 
	FEM					
CFCC	RRM	1^{st} 	2^{nd} 	3^{rd} 	4^{th} 	5^{th} 
	FEM					

Table 4.7 The first five mode shapes of [45/-45/-45/45] laminated composite square plates (Continued).

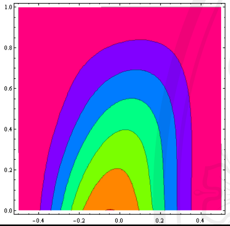
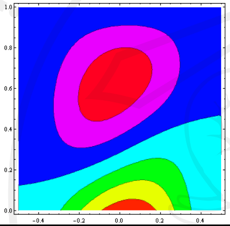
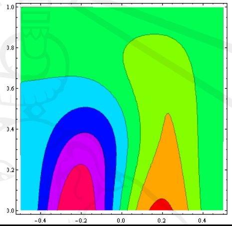
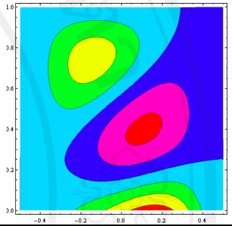
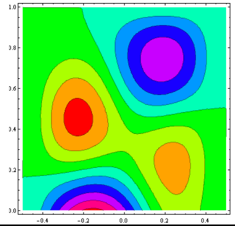
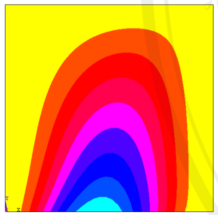
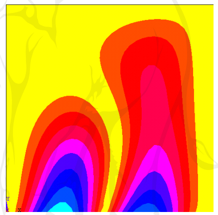
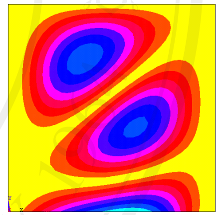
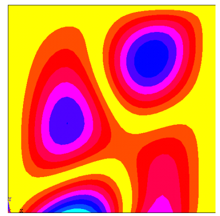
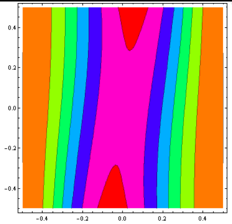
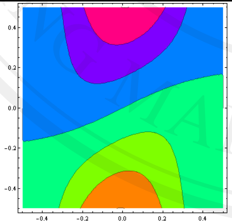
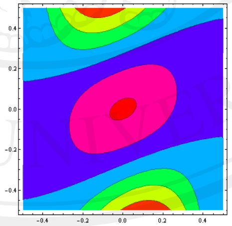
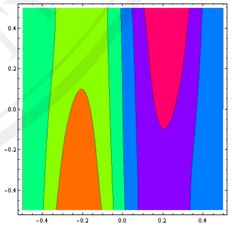
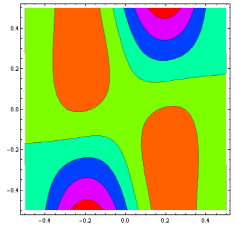
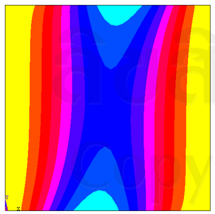
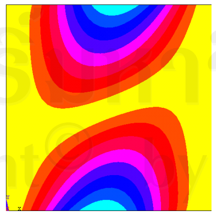
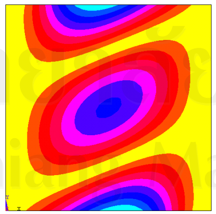
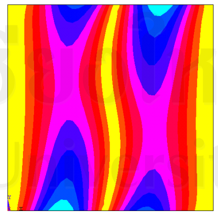
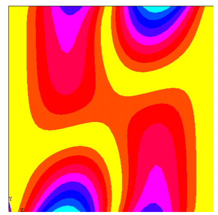
BCs	Method	Plate mode				
CFCS	RRM	1^{st} 	2^{nd} 	3^{rd} 	4^{th} 	5^{th} 
	FEM					
CFCF	RRM	1^{st} 	2^{nd} 	3^{rd} 	4^{th} 	5^{th} 
	FEM					

Table 4.7 The first five mode shapes of [45/-45/-45/45] laminated composite square plates (Continued).

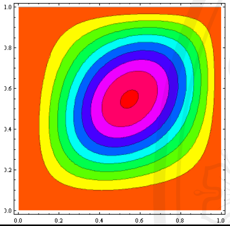
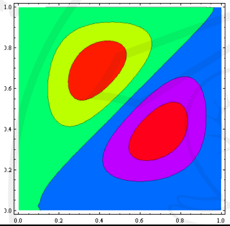
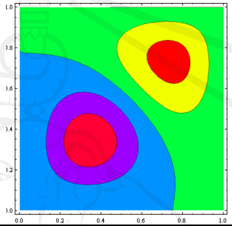
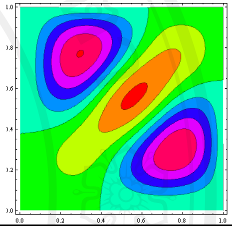
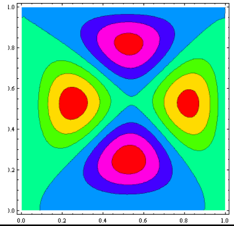
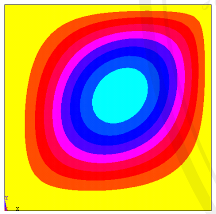
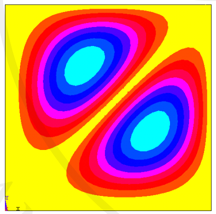
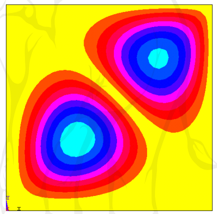
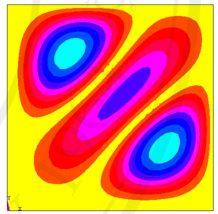
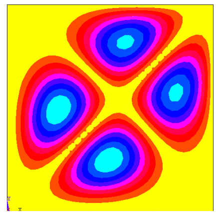
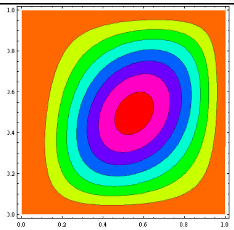
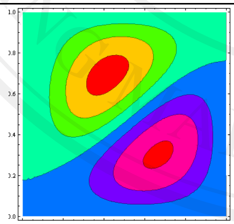
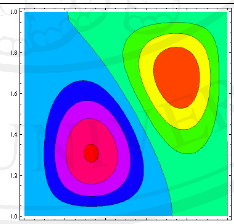
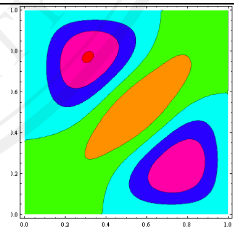
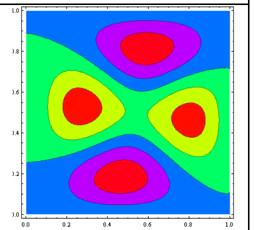
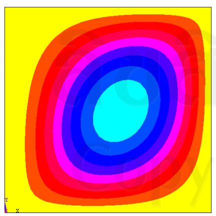
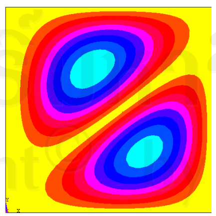
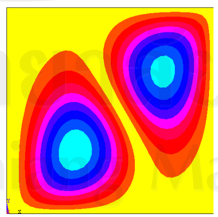
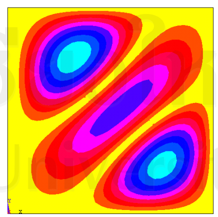
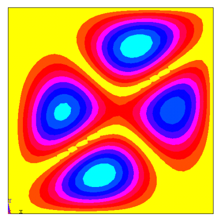
BCs	Method	Plate mode				
CCSS	RRM	1st 	2nd 	3rd 	4th 	5th 
	FEM					
CSSS	RRM	1st 	2nd 	3rd 	4th 	5th 
	FEM					

Table 4.7 The first five mode shapes of [45/-45/-45/45] laminated composite square plates (Continued).

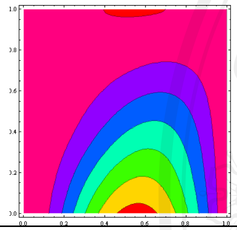
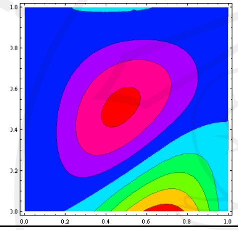
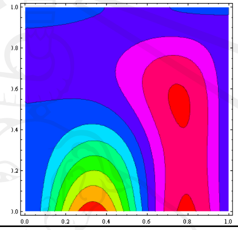
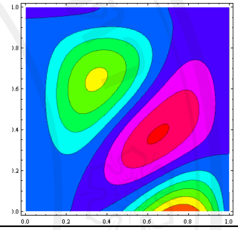
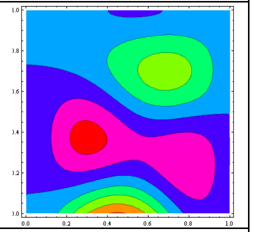
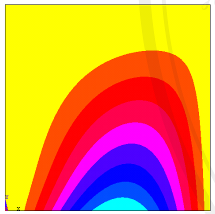
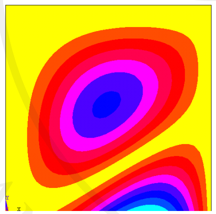
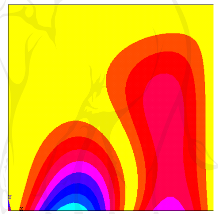
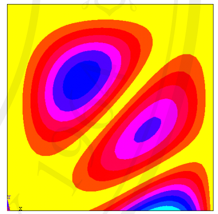
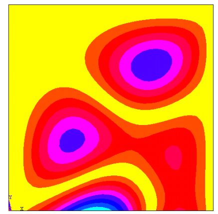
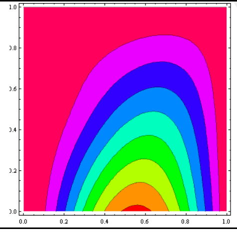
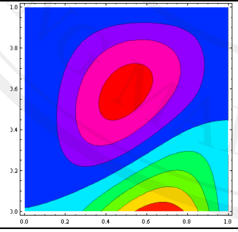
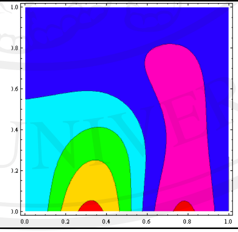
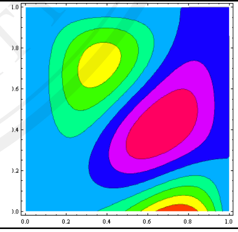
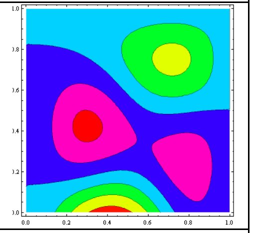
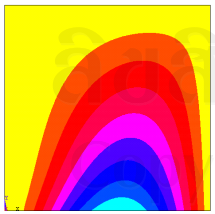
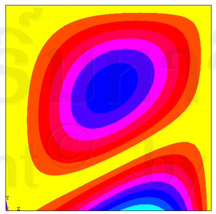
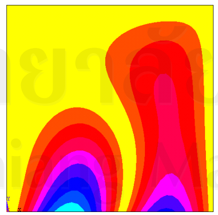
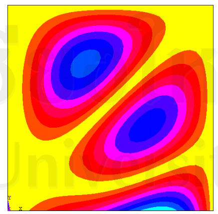
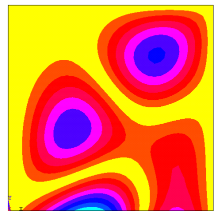
BCs	Method	Plate mode				
CFSC	RRM	1st 	2nd 	3rd 	4th 	5th 
	FEM					
CFSS	RRM	1st 	2nd 	3rd 	4th 	5th 
	FEM					

Table 4.7 The first five mode shapes of [45/-45/-45/45] laminated composite square plates (Continued).

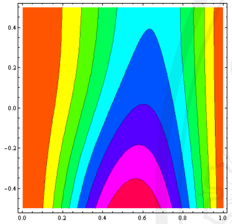
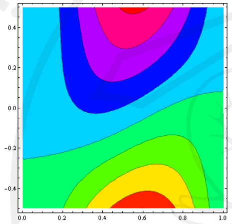
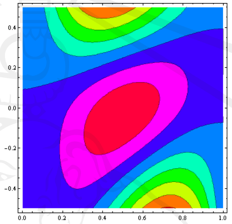
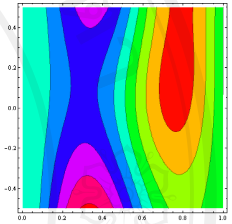
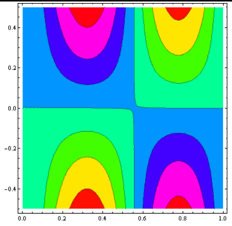
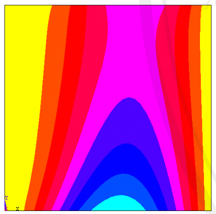
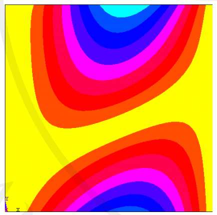
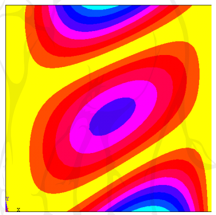
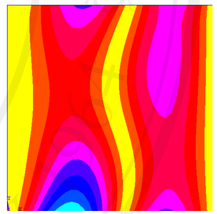
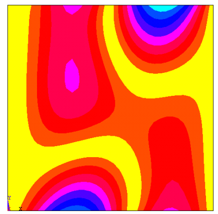
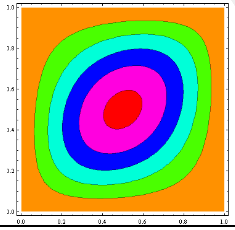
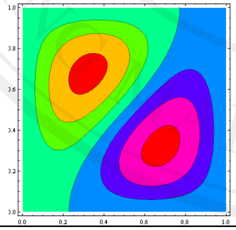
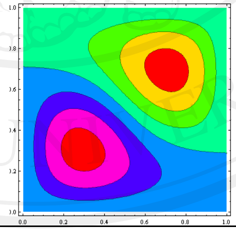
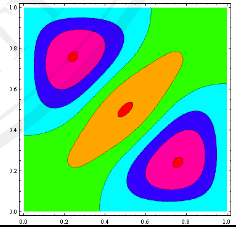
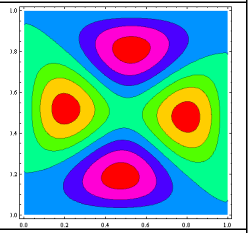
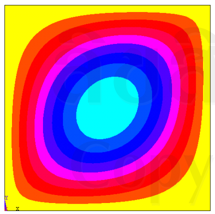
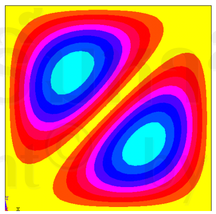
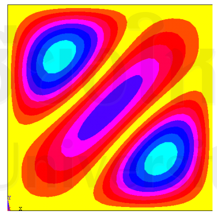
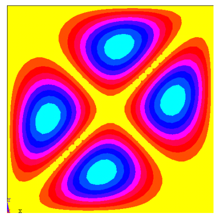
BCs	Method	Plate mode				
CFSF	RRM	1^{st} 	2^{nd} 	3^{rd} 	4^{th} 	5^{th} 
	FEM					
SSSS	RRM	1^{st} 	2^{nd} 	3^{rd} 	4^{th} 	5^{th} 
	FEM					

Table 4.7 The first five mode shapes of [45/-45/-45/45] laminated composite square plates (Continued).

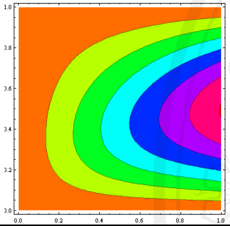
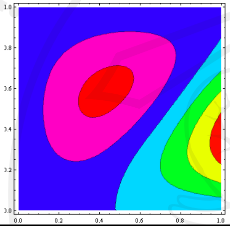
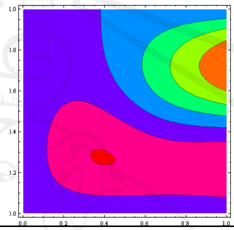
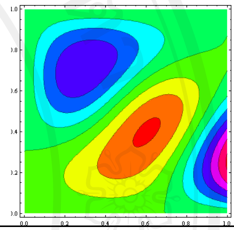
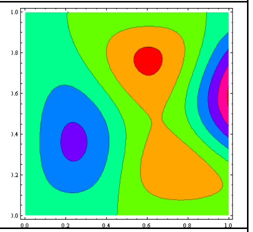
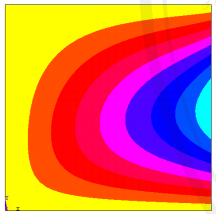
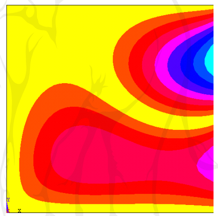
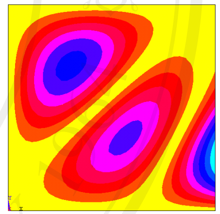
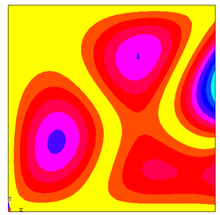
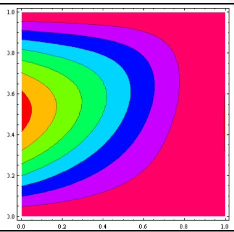
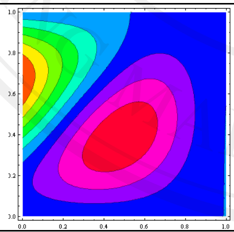
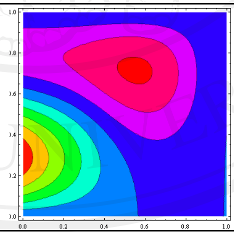
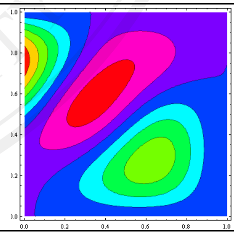
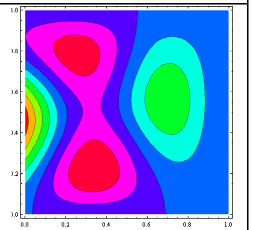
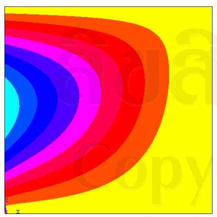
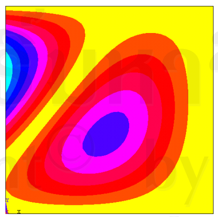
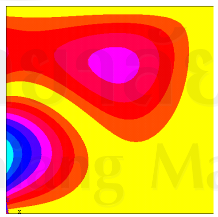
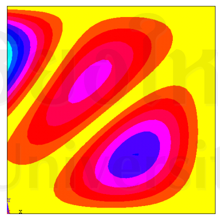
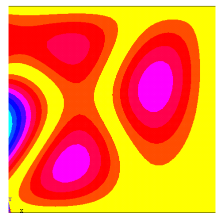
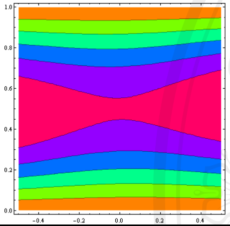
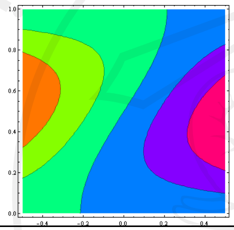
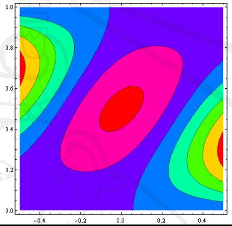
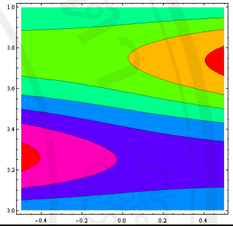
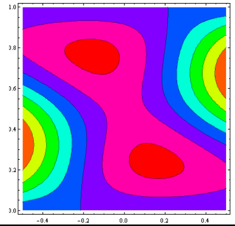
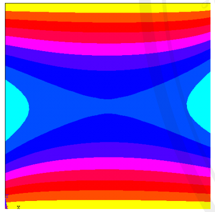
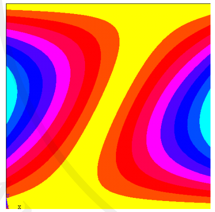
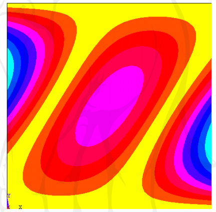
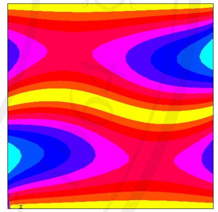
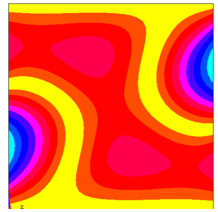
BCs	Method	Plate mode				
SSFS	RRM	1^{st} 	2^{nd} 	3^{rd} 	4^{th} 	5^{th} 
	FEM					
FSCS	RRM	1^{st} 	2^{nd} 	3^{rd} 	4^{th} 	5^{th} 
	FEM					

Table 4.7 The first five mode shapes of [45/-45/-45/45] laminated composite square plates (Continued).

BCs	Method	Plate mode				
FSFS	RRM	1^{st} 	2^{nd} 	3^{rd} 	4^{th} 	5^{th} 
	FEM					

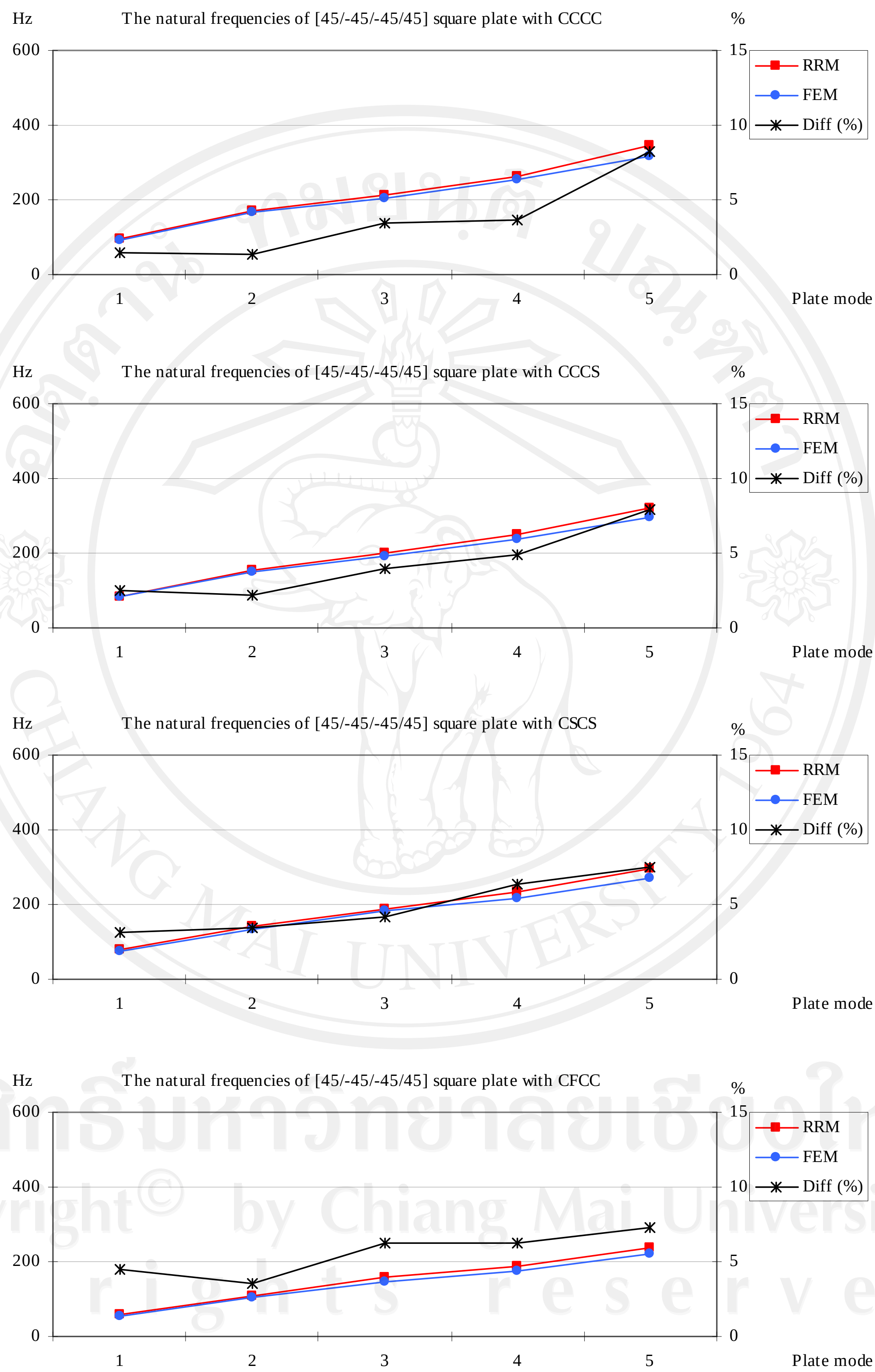


Figure 4.2 The first five natural frequencies of [45/-45/-45/45] laminated composite square plates.

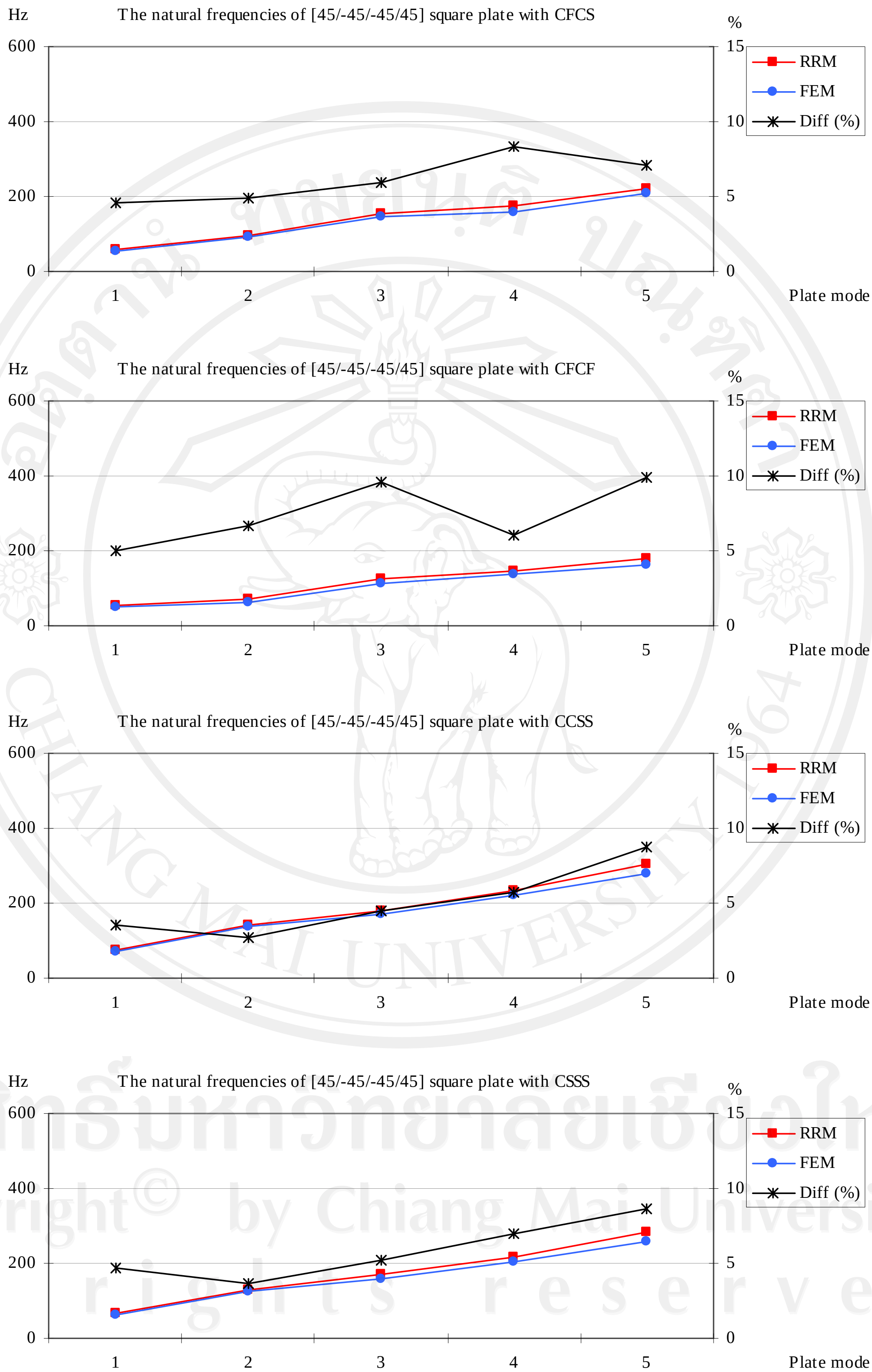


Figure 4.2 The first five natural frequencies of [45/-45/-45/45] laminated composite square plates (Continued).

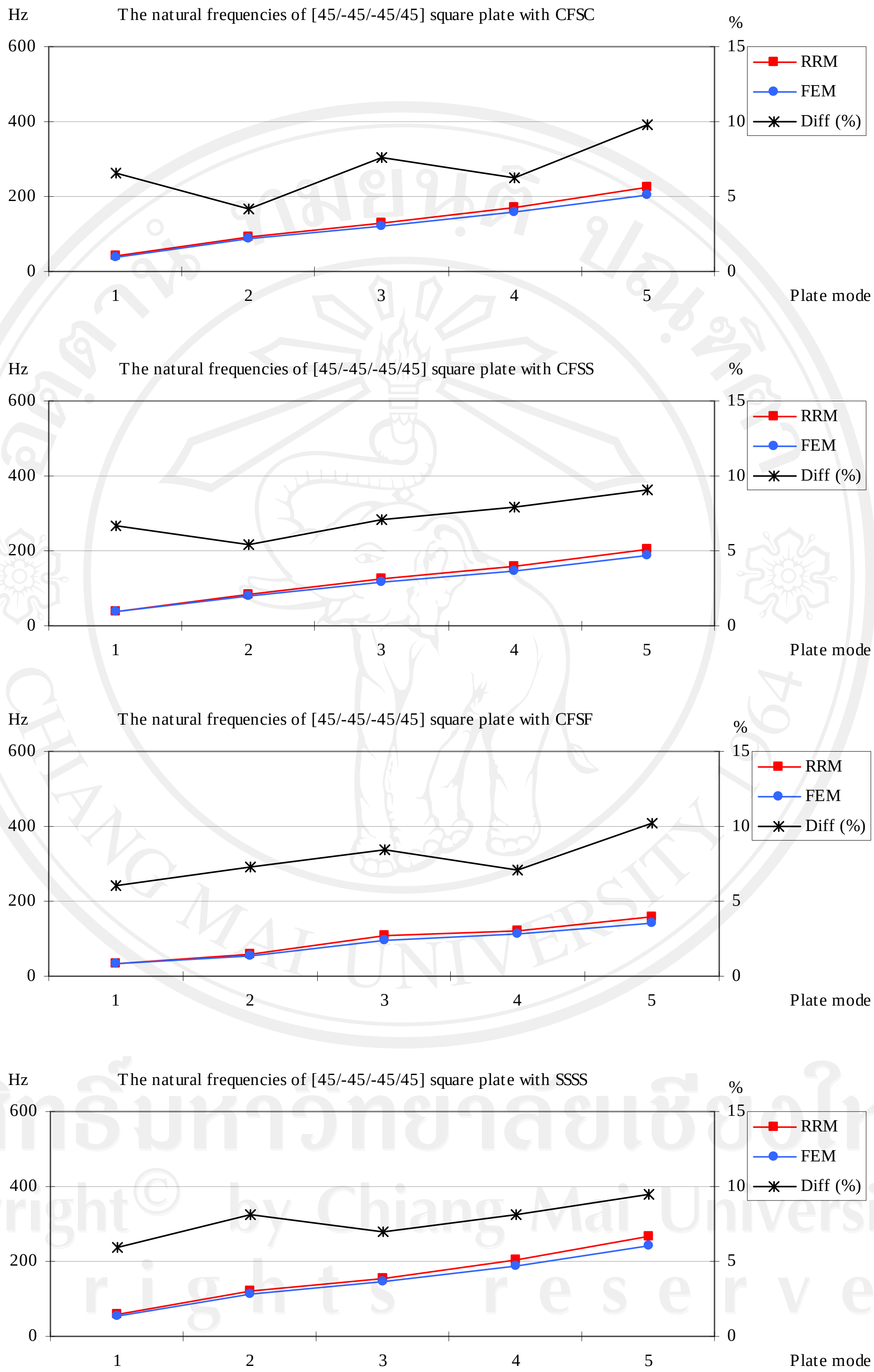


Figure 4.2 The first five natural frequencies of [45/-45/-45/45] laminated composite square plates (Continued).

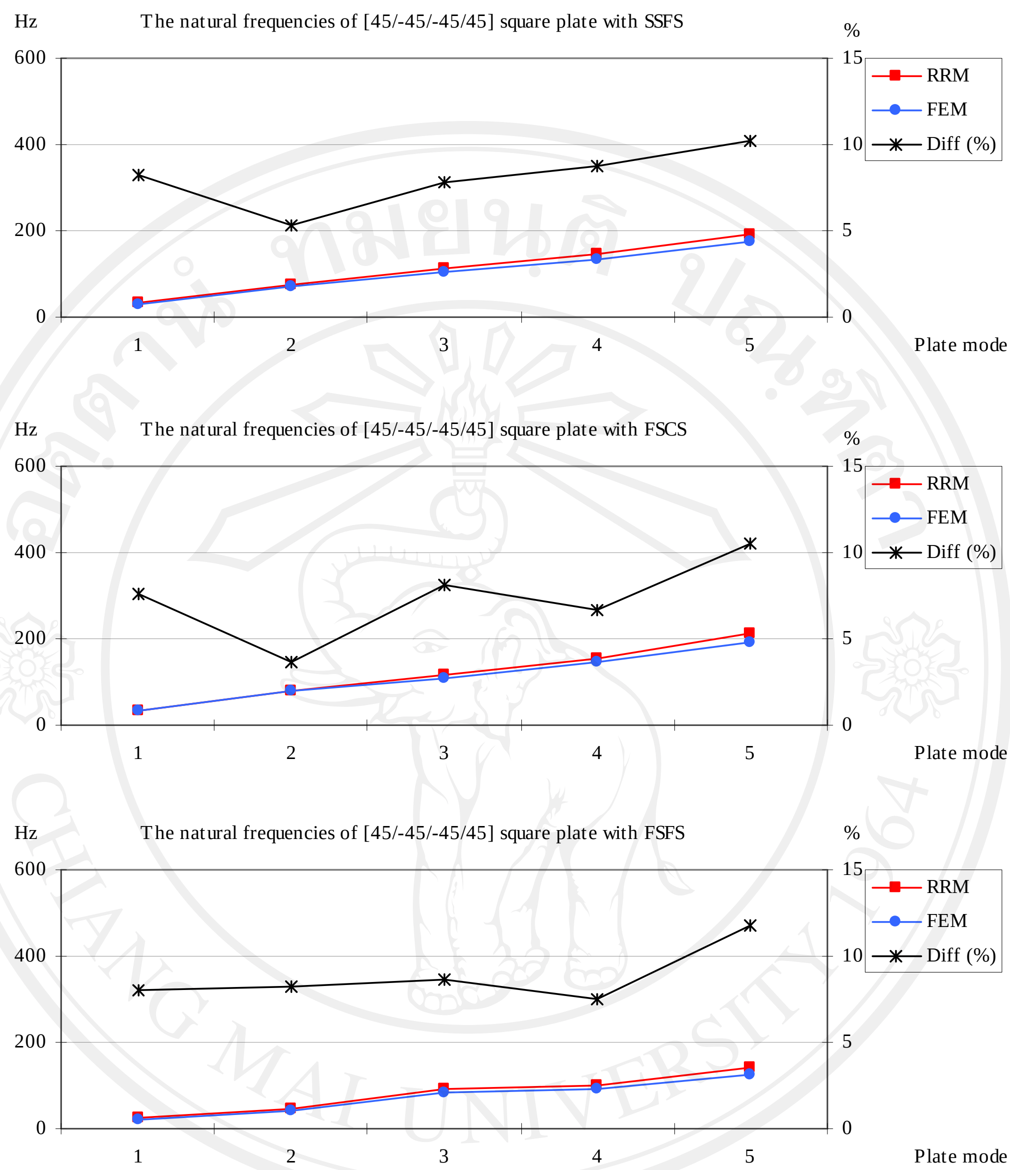


Figure 4.2 The first five natural frequencies of [45/-45/-45/45] laminated composite square plates (Continued).

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate.

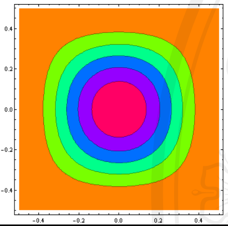
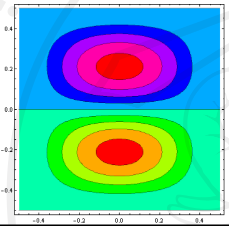
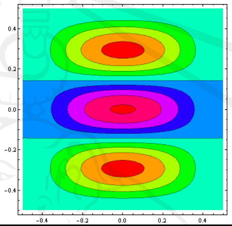
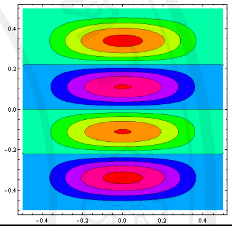
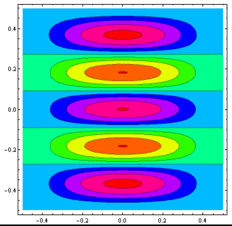
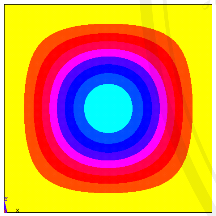
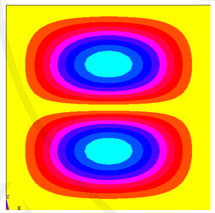
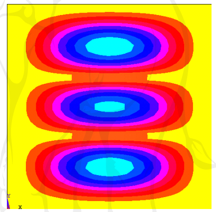
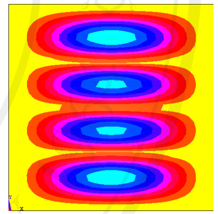
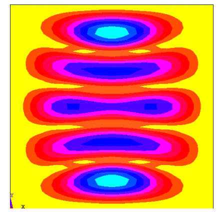
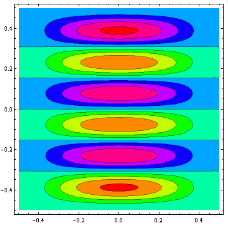
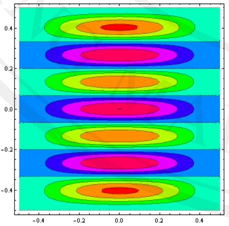
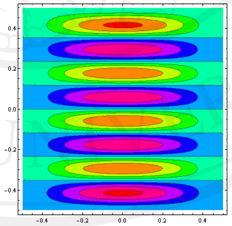
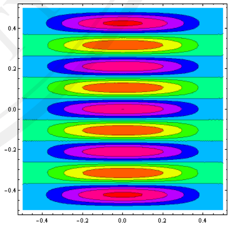
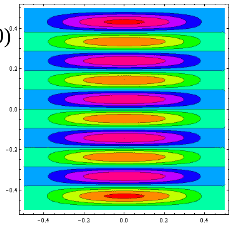
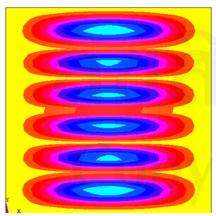
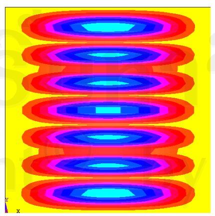
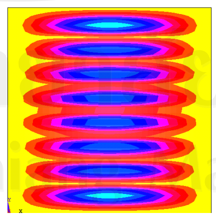
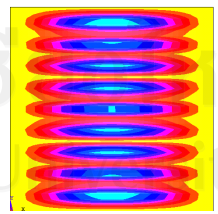
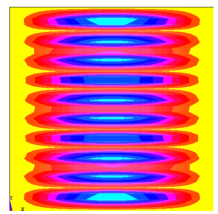
BCs	Method	Plate mode				
CCCC	EKM	(1, 1) 	(1, 2) 	(1, 3) 	(1, 4) 	(1, 5) 
	FEM					
	EKM	(1, 6) 	(1, 7) 	(1, 8) 	(1, 9) 	(1, 10) 
	FEM					

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate (Continued).

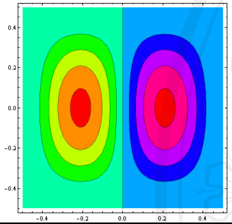
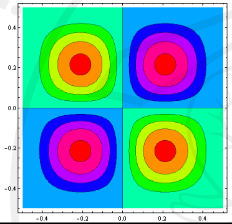
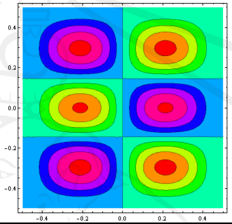
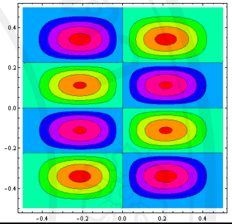
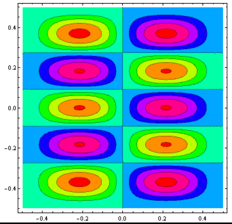
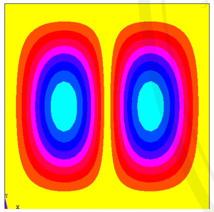
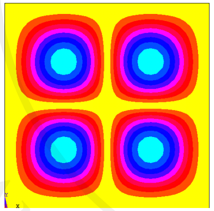
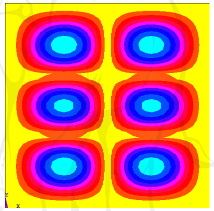
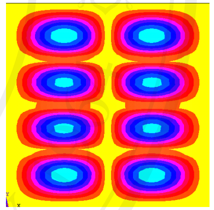
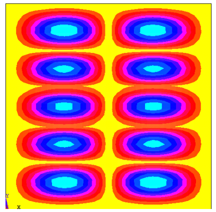
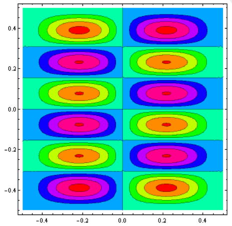
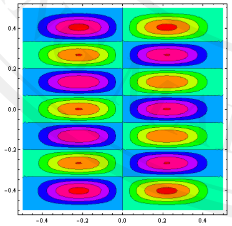
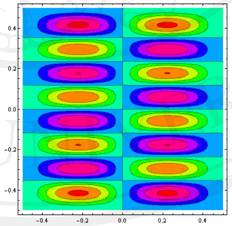
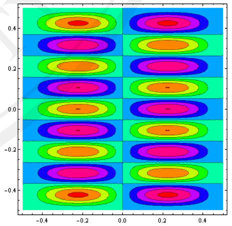
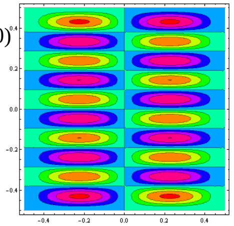
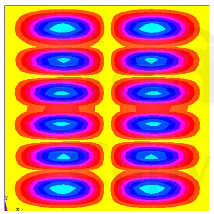
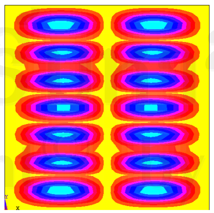
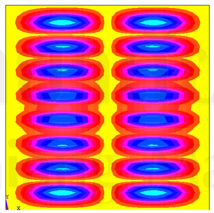
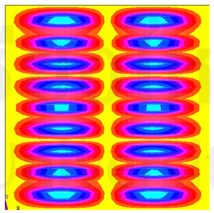
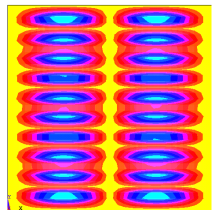
BCs	Method	Plate mode				
CCCC	EKM	(2, 1) 	(2, 2) 	(2, 3) 	(2, 4) 	(2, 5) 
	FEM					
	EKM	(2, 6) 	(2, 7) 	(2, 8) 	(2, 9) 	(2, 10) 
	FEM					

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate (Continued).

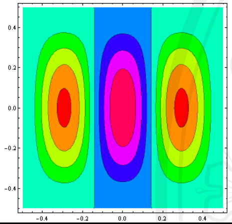
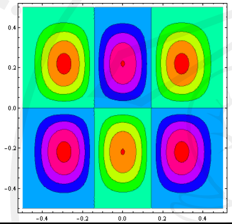
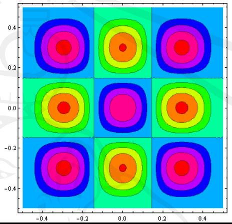
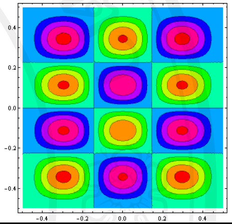
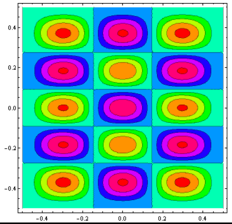
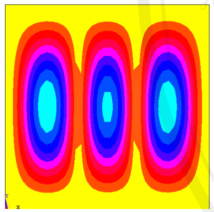
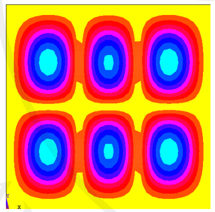
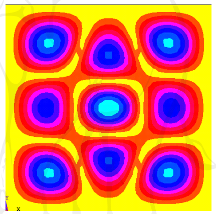
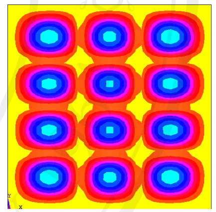
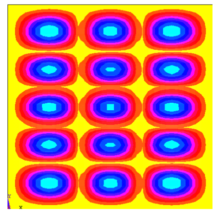
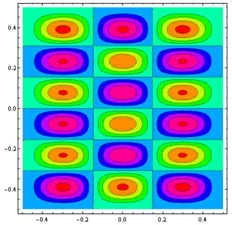
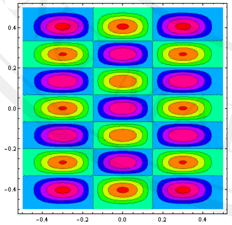
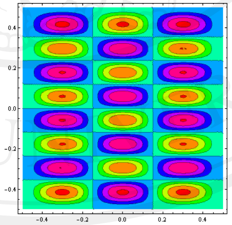
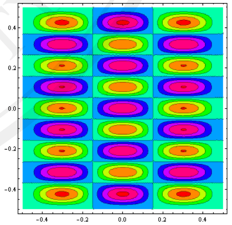
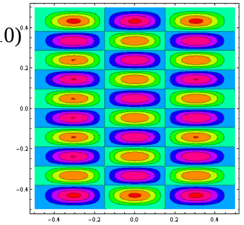
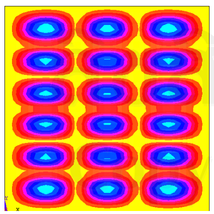
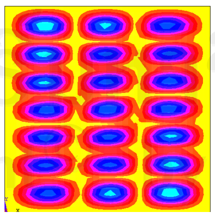
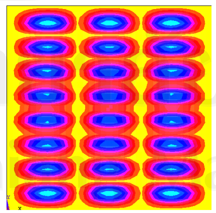
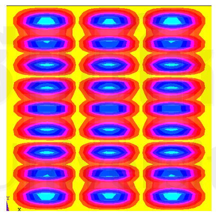
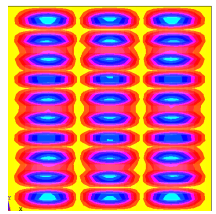
BCs	Method	Plate mode				
CCCC	EKM	(3, 1) 	(3, 2) 	(3, 3) 	(3, 4) 	(3, 5) 
	FEM					
	EKM	(3, 6) 	(3, 7) 	(3, 8) 	(3, 9) 	(3, 10) 
	FEM					

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate (Continued).

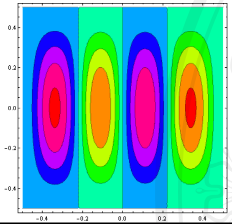
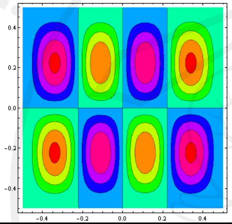
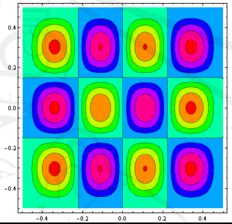
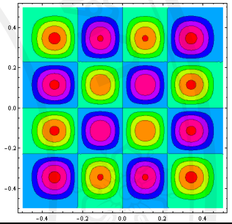
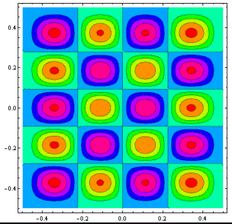
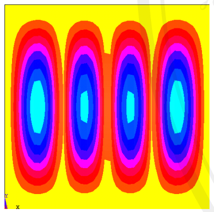
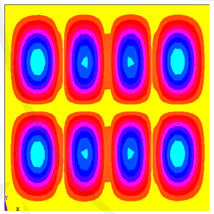
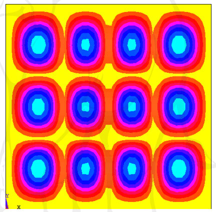
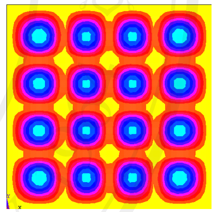
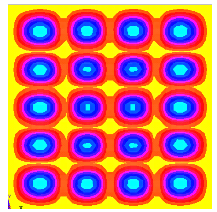
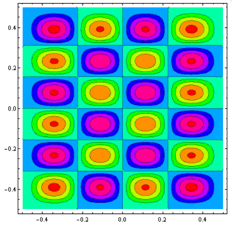
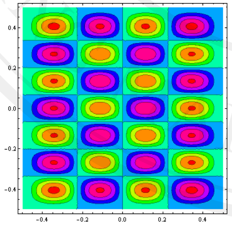
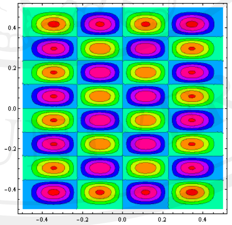
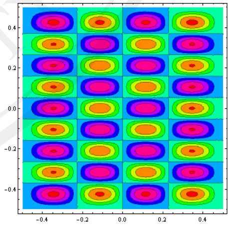
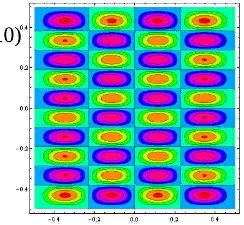
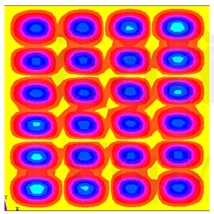
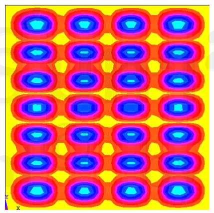
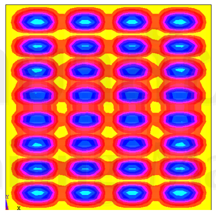
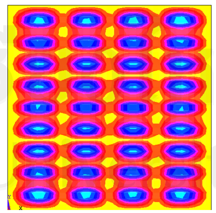
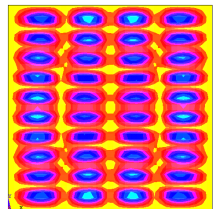
BCs	Method	Plate mode				
CCCC	EKM	(4, 1) 	(4, 2) 	(4, 3) 	(4, 4) 	(4, 5) 
	FEM					
	EKM	(4, 6) 	(4, 7) 	(4, 8) 	(4, 9) 	(4, 10) 
	FEM					

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate (Continued).

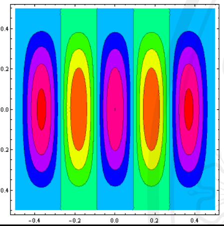
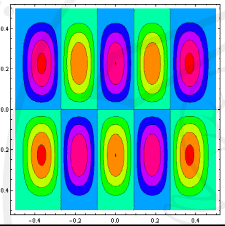
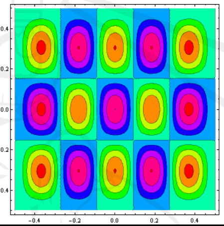
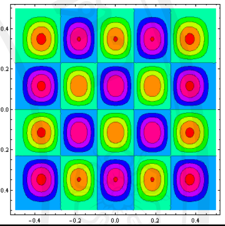
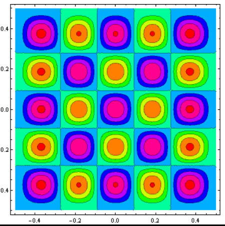
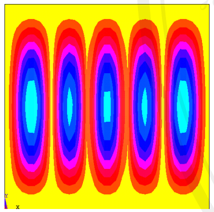
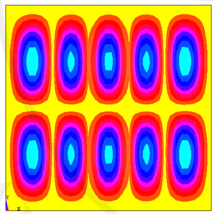
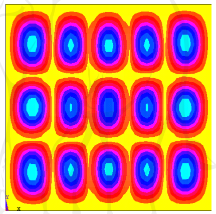
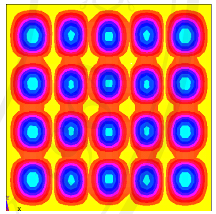
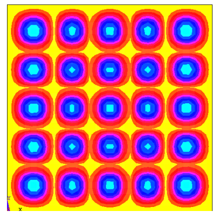
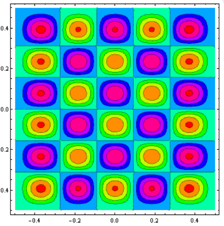
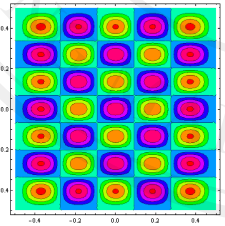
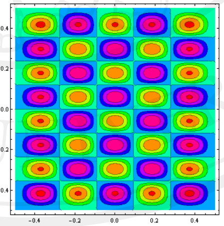
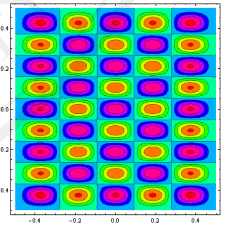
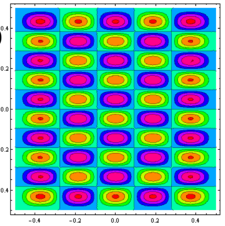
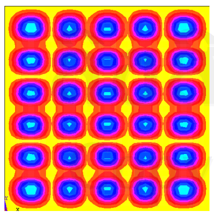
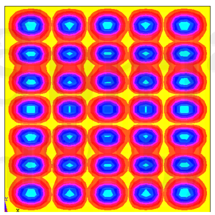
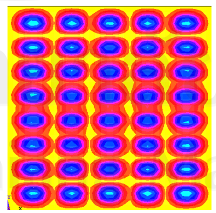
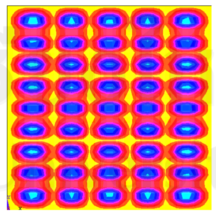
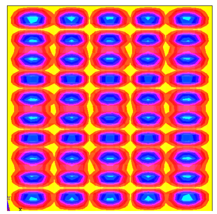
BCs	Method	Plate mode				
CCCC	EKM	(5, 1) 	(5, 2) 	(5, 3) 	(5, 4) 	(5, 5) 
	FEM					
	EKM	(5, 6) 	(5, 7) 	(5, 8) 	(5, 9) 	(5, 10) 
	FEM					

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate (Continued).

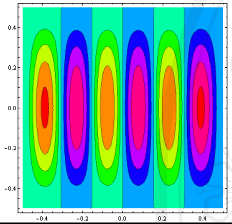
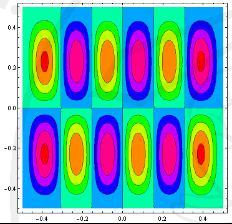
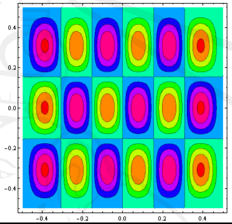
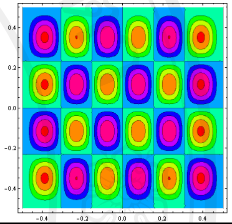
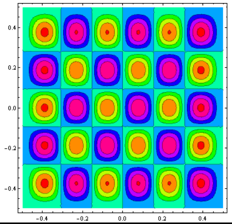
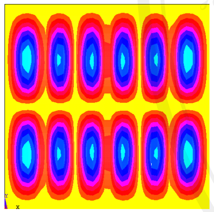
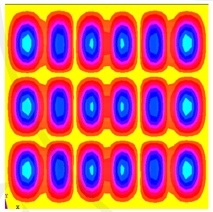
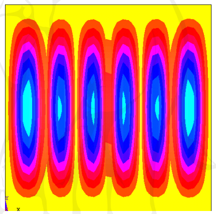
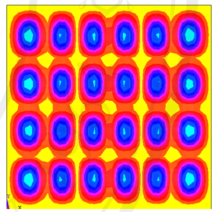
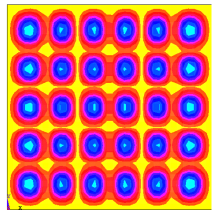
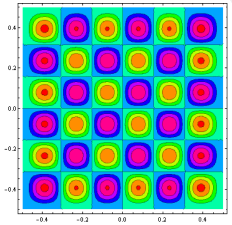
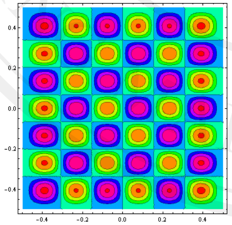
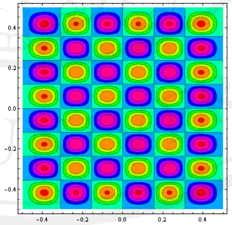
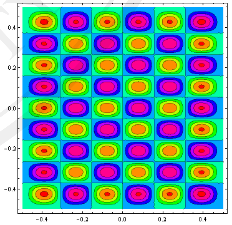
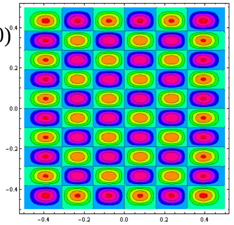
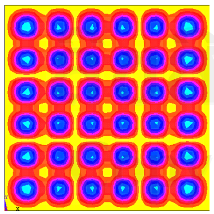
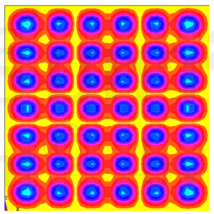
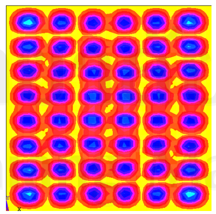
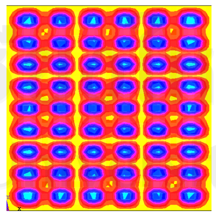
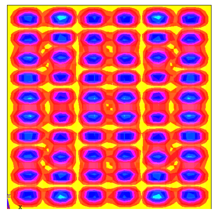
BCs	Method	Plate mode				
CCCC	EKM	(6, 1) 	(6, 2) 	(6, 3) 	(6, 4) 	(6, 5) 
	FEM					
	EKM	(6, 6) 	(6, 7) 	(6, 8) 	(6, 9) 	(6, 10) 
	FEM					

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate (Continued).

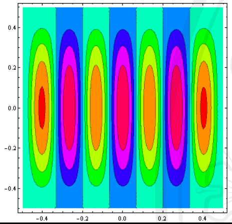
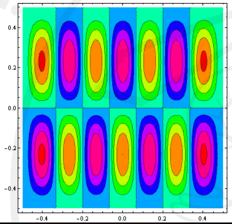
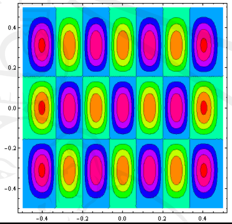
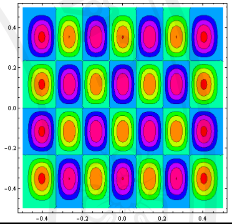
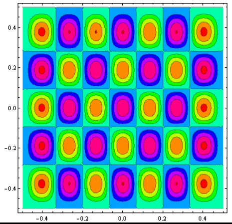
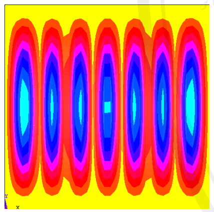
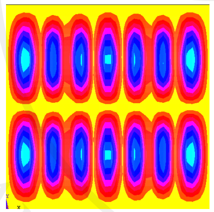
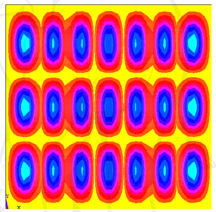
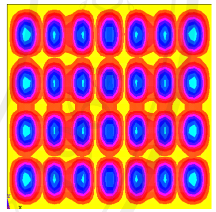
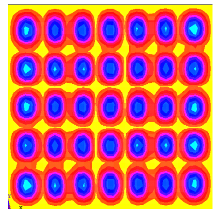
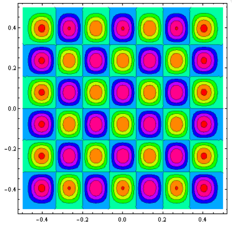
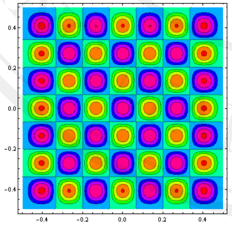
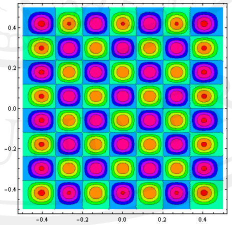
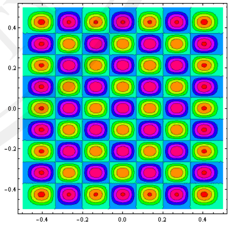
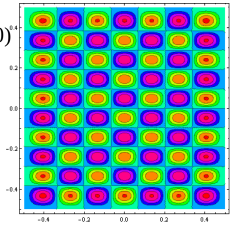
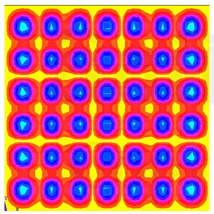
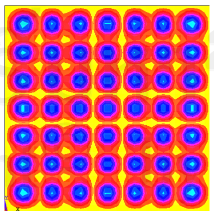
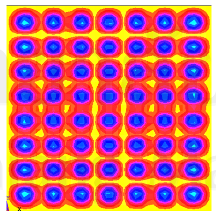
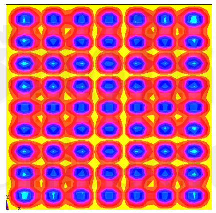
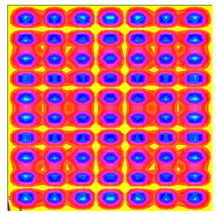
BCs	Method	Plate mode				
CCCC	EKM	(7, 1) 	(7, 2) 	(7, 3) 	(7, 4) 	(7, 5) 
	FEM					
	EKM	(7, 6) 	(7, 7) 	(7, 8) 	(7, 9) 	(7, 10) 
	FEM					

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate (Continued).

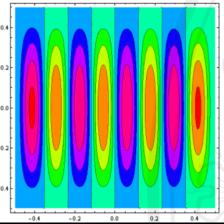
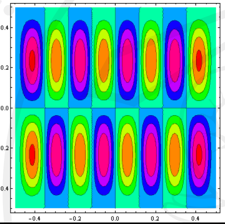
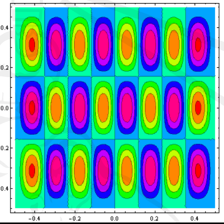
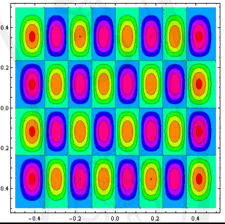
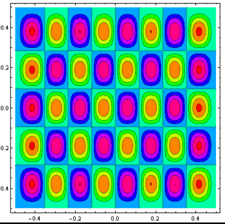
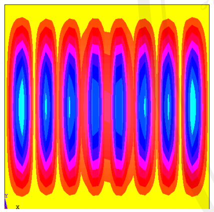
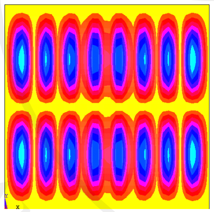
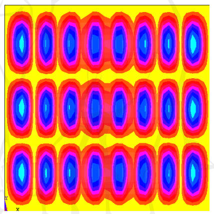
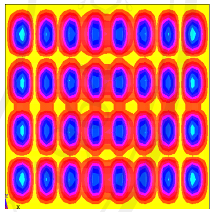
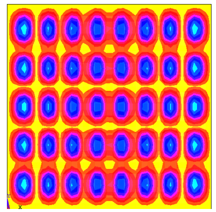
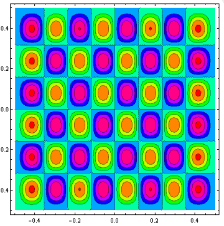
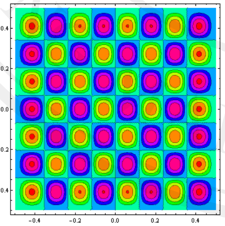
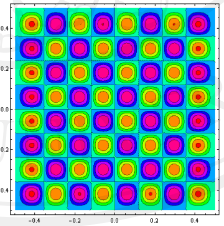
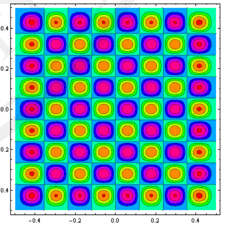
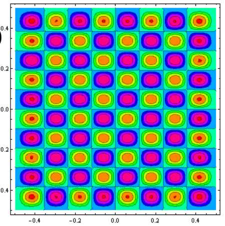
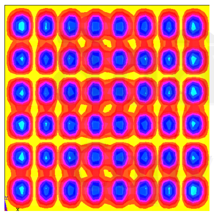
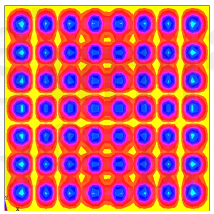
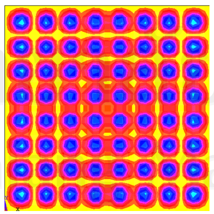
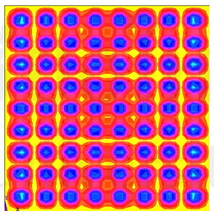
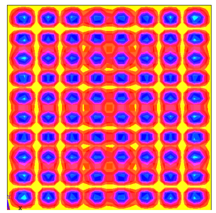
BCs	Method	Plate mode				
CCCC	EKM	(8, 1) 	(8, 2) 	(8, 3) 	(8, 4) 	(8, 5) 
	FEM					
	EKM	(8, 6) 	(8, 7) 	(8, 8) 	(8, 9) 	(8, 10) 
	FEM					

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate (Continued).

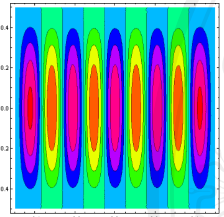
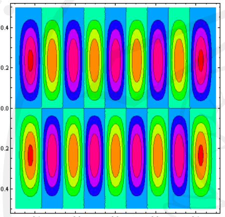
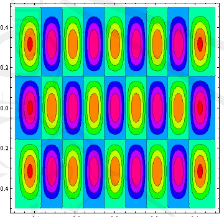
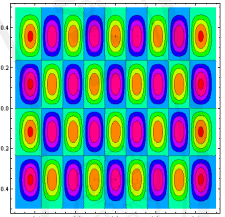
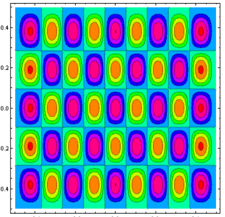
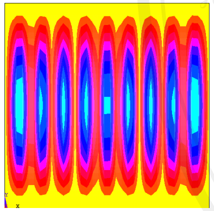
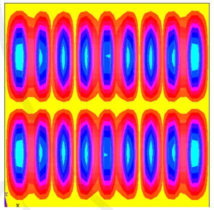
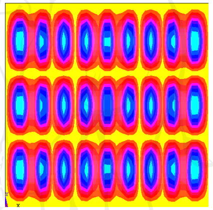
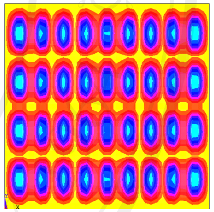
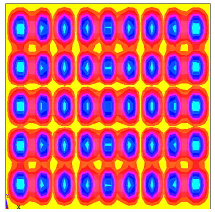
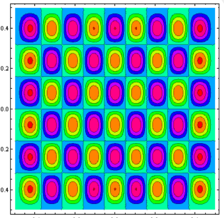
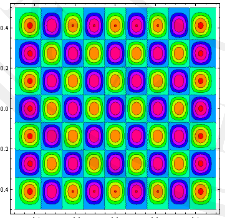
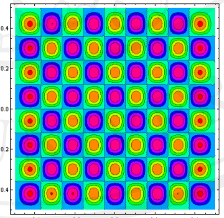
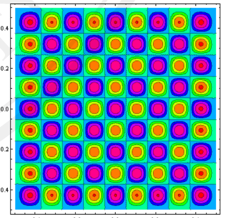
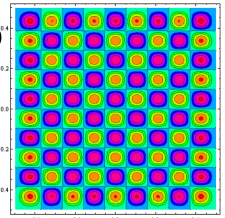
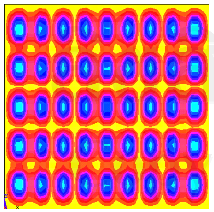
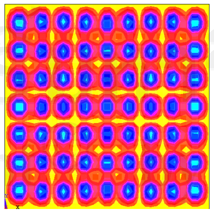
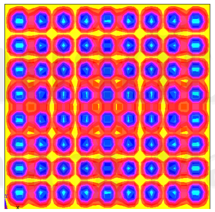
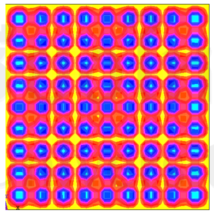
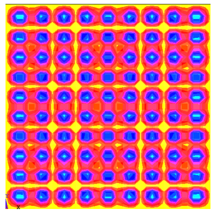
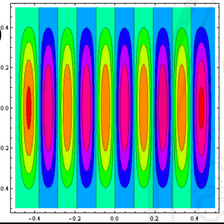
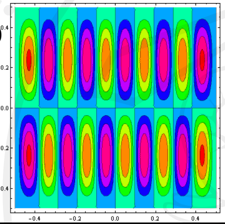
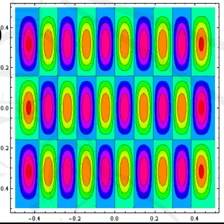
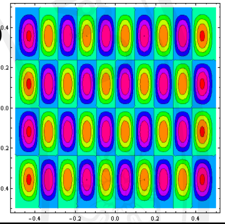
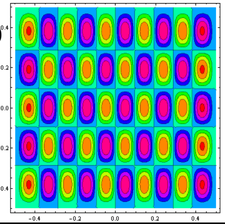
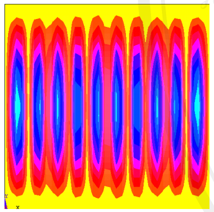
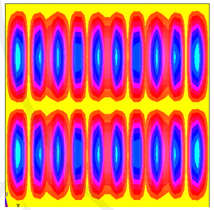
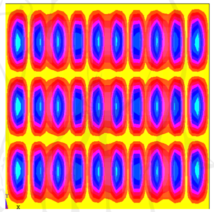
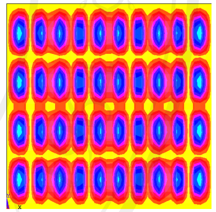
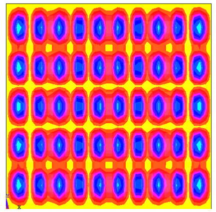
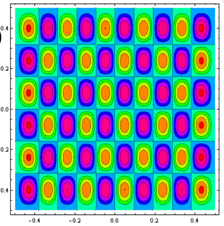
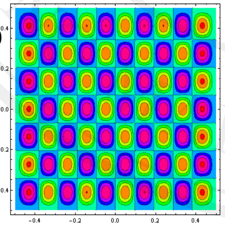
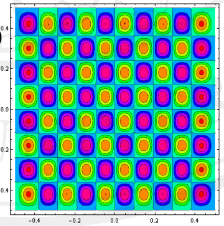
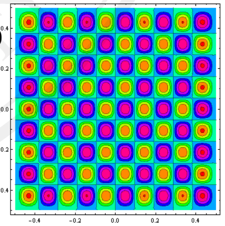
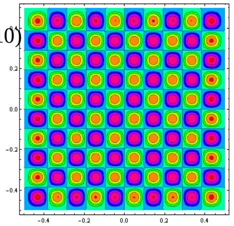
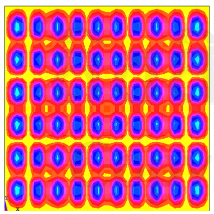
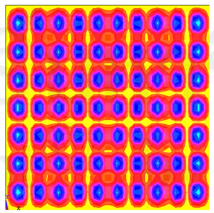
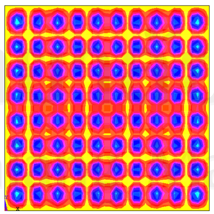
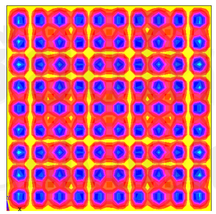
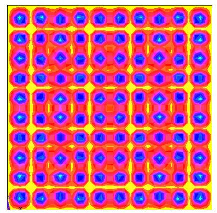
BCs	Method	Plate mode				
CCCC	EKM	(9, 1) 	(9, 2) 	(9, 3) 	(9, 4) 	(9, 5) 
	FEM					
	EKM	(9, 6) 	(9, 7) 	(9, 8) 	(9, 9) 	(9, 10) 
	FEM					

Table 4.8 The first hundred mode shapes of [0/90/90/0] laminated composite square plate (Continued).

BCs	Method	Plate mode				
CCCC	EKM	(10, 1) 	(10, 2) 	(10, 3) 	(10, 4) 	(10, 5) 
	FEM					
	EKM	(10, 6) 	(10, 7) 	(10, 8) 	(10, 9) 	(10, 10) 
	FEM					

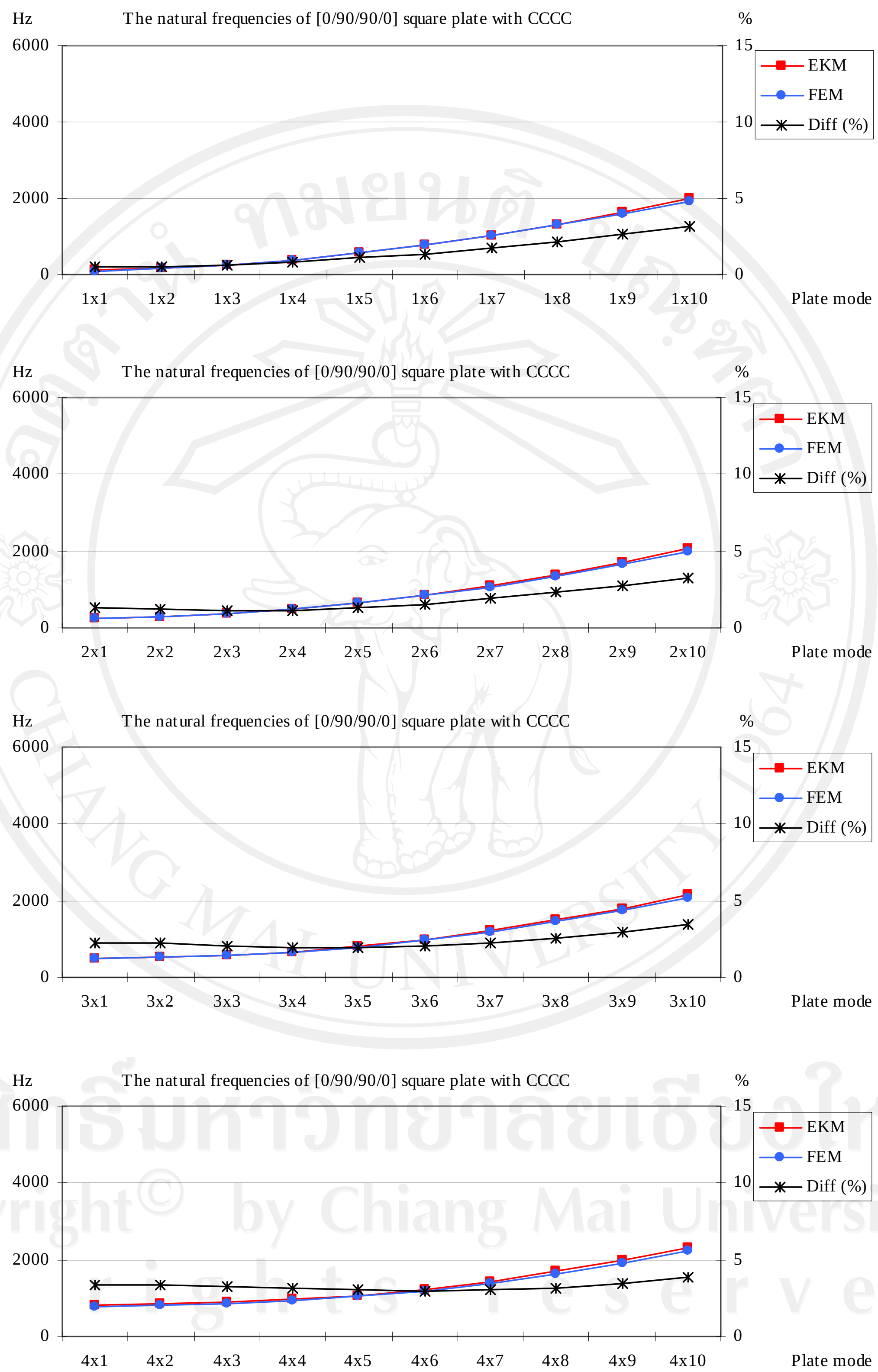
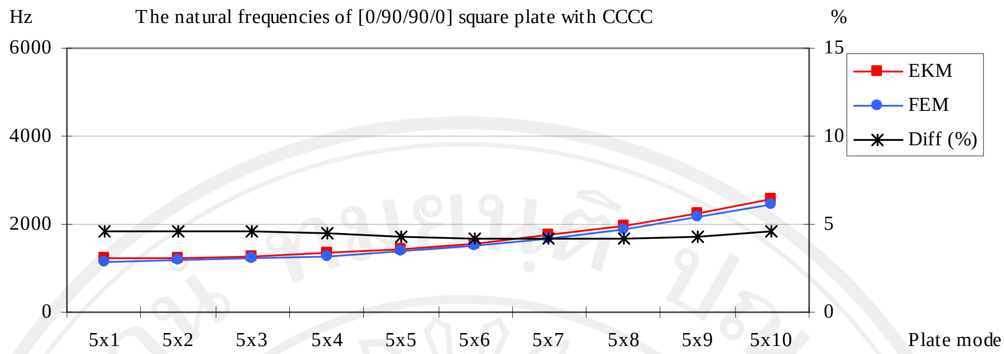
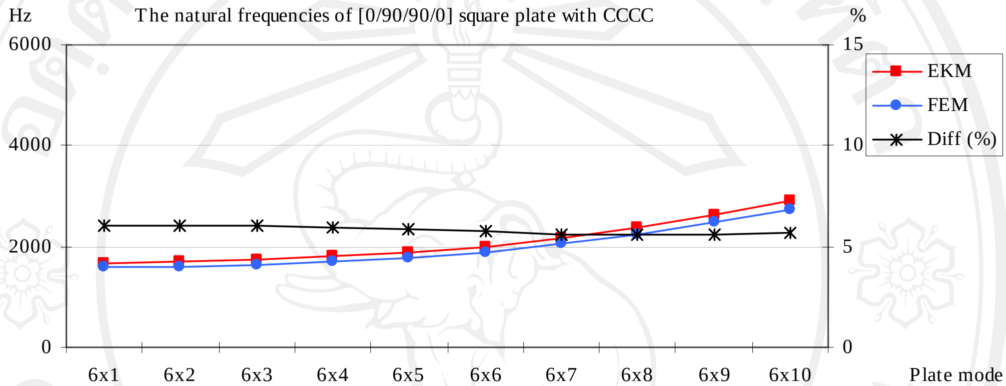
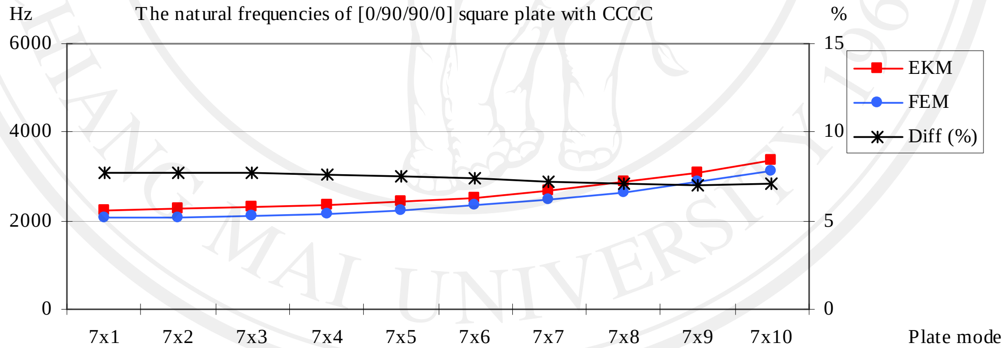


Figure 4.3 The first hundred natural frequencies of [0/90/90/0] laminated composite square plate.







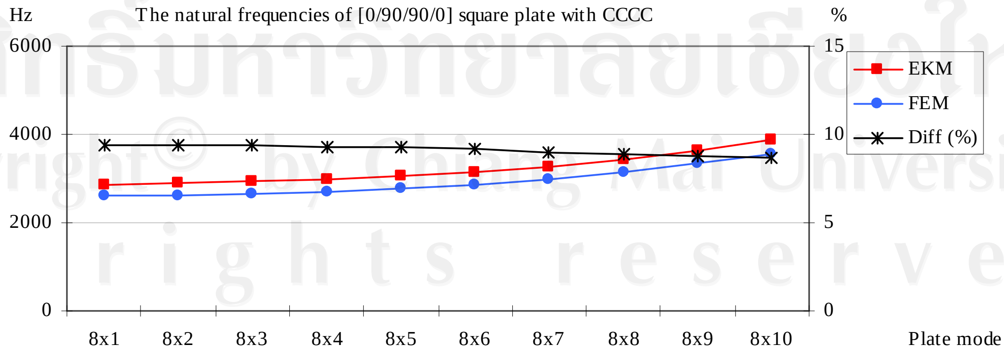
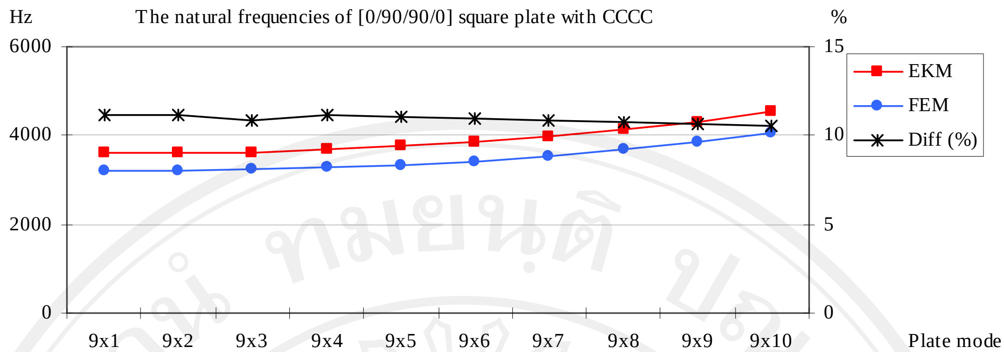


Figure 4.3 The first hundred natural frequencies of [0/90/90/0] laminated composite square plate (Continued).



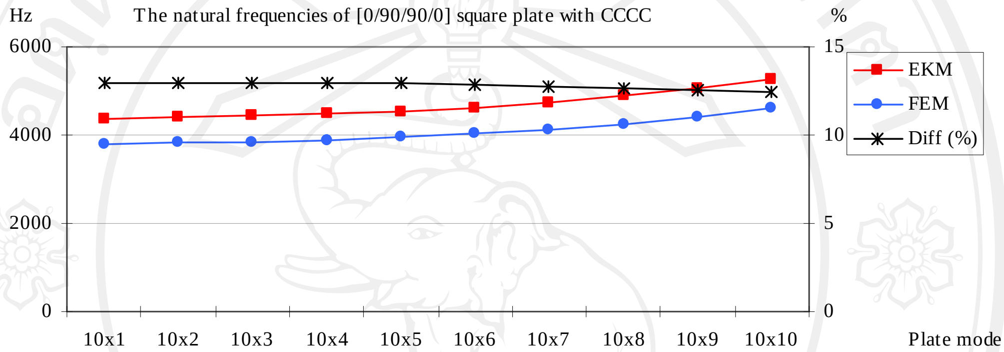


Figure 4.3 The first hundred natural frequencies of [0/90/90/0] laminated composite square plate (Continued).

4.2 Discussion

4.2.1 Dicussion of the comparison of the extended Kantorovich method with the known solutions

The frequency parameters obtained by the extended Kantorovich method and the known solutions are good agreement due to the fact that

(1) All methods derived governing equation of plates from the reliable principle: Newton's second law for Sakata *et al* (1996) and Rajalingham *et al* (1997), the principle of virtual displacements for Reddy (2004) and the extended Kantorovich method.

(2) The comparison range based on the identical assumptions: the classical Kirchhoff thin plate theory.

4.2.2 Discussion of the mode shapes and the natural frequencies of straight nodal lines

The mode shapes and the natural frequencies obtained by the extended Kantorovich method and the finite element method are good agreement due to the fact that

(1) Both methods derived governing equation of plates from the reliable principle: the principle of virtual displacements for the extended Kantorovich method and the finite element method.

(2) The comparison range based on the identical assumptions: the classical Kirchhoff thin plate theory.

The finite element method solves the problem with relative ease in a very short time however it provides no closed-form solution that permits analytical study of the effect of changing various parameters such as plate dimensions, material properties of plates, or boundary conditions of plates.

The extended Kantorovich method which provides a closed-form solution is modified to evaluate the mode shapes and the natural frequencies of plate which described by the partial differential equation. In the modification the extended Kantorovich method assumed the solution as $w_0(x, y) = X(x)Y(y)$ and used the variational method in order to reduce the partial differential equations to ordinary differential equations in *x* and *y* coordinate directions with a constant coefficient as equation (2.41) and (2.38), respectively. The ordinary differential equations can be solved exactly in terms of four unknown modal parameters, equation (2.45) and (2.48). The iterative calculation is applied to the extended Kantorovich method to evaluate the modal parameters and force the final solution needed to satisfy the boundary conditions. By the numerical example of topic 3.1.2, it is demonstrated that the final solution is obtained from the fourth iteration.

The arbitrary function can be used as a basis function in the iterative calculation, however, the convergence of the final solution of the basis function which satisfied the boundary conditions is faster than the other. In this study the beam functions is used as a basis function due to the fact that the beam function is satisfied the boundary conditions and the natural frequencies of the beam modes is closed to the plate modes which bring the evaluation of the particular mode shape and natural frequency of plates.

4.2.3 Discussion of the mode shapes and the natural frequencies of curved nodal lines

The extended Kantorovich method alone can not be provided for the mode shapes and the natural frequencies of curved nodal lines of plates, because the governing differential equations (2.38) and (2.41) do not contain the stiffness terms D_{16} and D_{26} due to the assuming a closed-form approximate solution as $w_0(x, y) = X(x)Y(y)$. The stiffness terms D_{16} and D_{26} are the bending-twist (Table 2.1) of the plates which provided a curved nodal lines of plates. The Rayleigh-Ritz method can be applied to find the mode shapes and the natural frequencies for more general plates however the disadvantages of the Rayleigh-Ritz method are as follows:

(1) The basis functions are required to satisfy the boundary conditions. In general, a large number of basis functions provide a better approximation of the solution.

(2) The numerical solution of a matrix eigenvalue problem as equation (2.54) yields an arrangement in ascending frequency order therefore the particular natural frequency can not be obtained separately. Moreover, a higher natural frequency involves a large number of basis functions which yields a higher order matrix eigenvalue problem.

Note that the difference percentage obtained by the combination of the extended Kantorovich method and the Rayleigh-Ritz method and the finite element method increases as the natural frequency increases due to the fact that the Rayleigh-Ritz method, larger numbers of basis functions are used if a higher natural frequency is required. In this study, nine basis functions in each coordinate direction obtained by the extended Kantorovich method are used in the Rayleigh-Ritz method. To remove this discrepancy, more basis functions in the Rayleigh-Ritz method has to be considered.

4.2.4 Discussion of the higher mode shapes and the higher natural frequencies of straight nodal lines

The difference percentage obtained by the extended Kantorovich method and the finite element method increases as the natural frequency increases may be results of

(1) In the extended Kantorovich method, during vibration a plate (Figure 2.7) performs not only a translatory motion but also rotates. The angle of rotations, which are equal to the slope of the displacement curve, are expressed by $-z(\partial w_0 / \partial x)$ and $-z(\partial w_0 / \partial y)$. The corresponding kinetic energy will be given by

$$\frac{1}{2}\int_0^a\int_0^b m\left(-z\frac{\partial^2 w_0}{\partial x \partial t}\right)^2 dxdy + \frac{1}{2}\int_0^a\int_0^b m\left(-z\frac{\partial^2 w_0}{\partial y \partial t}\right)^2 dxdy$$

In the case of composite plates, these terms become more and more important with higher-order vibration which cause the natural frequencies become over predicted in comparing to the solutions obtained with the rotate motion.

(2) In the finite element method, the actual structure are divided into elements, these elements are not infinitely small as in the case of derivation of system equations. This phenomenon is obvious in the higher natural frequency.