CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The conclusions can be divided into three parts (Topic 5.1.2-5.1.4) to follow the research objective:

(1) Derive a closed-form approximate solution of the mode shapes and the natural frequencies of straight nodal lines.

(2) Derive a closed-form approximate solution of the mode shapes and the natural frequencies of curved nodal lines.

(3) Derive a closed-form approximate solution of the higher mode shapes and the higher natural frequencies of straight nodal lines.

Additional conclusion of the comparison of the extended Kantorovich method with the known solutions is presented in Topic 5.1.1.

5.1.1 The extended Kantorovich method is employed to evaluate the frequency parameters of isotropic, orthotropic, and [0/90/90/0] laminated composite rectangular plates. A good agreement comparing with Sakata *et al* (1996), Rajalingham *et at* (1997) and Reddy (2004) verifies the accuracy of the extended Kantorovich method.

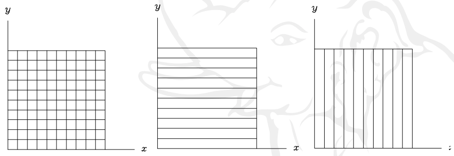
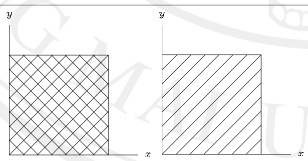
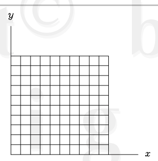
5.1.2 The extended Kantorovich method is employed to generate a closed-form approximate solution for the mode shapes and the natural frequencies of straight nodal lines of symmetrically laminated composite rectangular plates with various boundary conditions. A closed-form approximate solution is written in terms of $w(x,y) = X(x)Y(y)$. The mode shapes and the natural frequencies of symmetrically unidirectional 0°, unidirectional 90° and cross-ply laminated composite rectangular plates with various boundary conditions are evaluated by the extended Kantorovich method and the finite element method. A good agreement comparing with the finite element method, the maximum difference percentage is 6.244% (Table 5.1), verifies that the extended Kantorovich method is validated for the mode shapes and the natural frequencies of straight nodal lines of symmetrically laminated composite rectangular plates with various boundary conditions. The advantages of the extended Kantorovich method are as follows:

(1) The rectangular plate vibration problem can be resolved by the use of ordinary differential equations.
(2) Any arbitrary function can be used as a basis function in the iterative calculation. A basis function which satisfies boundary conditions will make the convergence of the final solution rapid.
(3) The final solution converges rapidly; therefore, the final solution can be obtained from the fourth iteration.
(4) The particular natural frequency can be obtained separately, with good accuracy.

5.1.3 The combination of the extended Kantorovich method and the Rayleigh-Ritz method is employed to generate a closed-form approximate solution for the mode shapes and the natural frequencies of curved nodal lines of symmetrically laminated composite rectangular plates with various boundary conditions. A closed-form approximate solution is written in terms of $w(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}A_{ij}f_i(x)g_j(y)$. The mode shapes and the natural frequencies of symmetrically unidirectional 45° and angle-ply 45° laminated composite square plates with various boundary conditions are evaluated by the combination of the extended Kantorovich method and the Rayleigh-Ritz method and the finite element method. A good agreement comparing with the finite element method, the maximum difference percentage is 11.727% (Table 5.1), verifies that the combination of the extended Kantorovich method and the Rayleigh-Ritz method is validated for the mode shapes and the natural frequencies of curved nodal lines of symmetrically laminated composite rectangular plates with various boundary conditions; larger numbers of basis functions are used if a higher accuracy is required.

5.1.4 The extended Kantorovich method is employed to generate a closed-form approximate solution for the higher mode shapes and the higher natural frequencies of straight nodal lines of symmetrically laminated composite rectangular plates with CCCC boundary conditions. A closed-form approximate solution is written in terms of $w(x,y)=X(x)Y(y)$. The first hundred mode shapes and the first hundred natural frequencies of symmetrically cross-ply laminated composite rectangular plates with CCCC boundary conditions are evaluated by the extended Kantorovich method and the finite element method. A good agreement comparing with the finite element method, the maximum difference percentage is 13.002% (Table 5.1), verifies that the extended Kantorovich method is validated for the first hundred mode shapes and the natural frequencies of straight nodal lines of symmetrically laminated composite rectangular plates with CCCC boundary conditions.

Table 5.1 The conclusion of research.

Study No.	Detail		Method: Presented	Method: Verification	%Maximum difference
1	Lamination scheme	 Cross-ply, Unidirectional 0°, Unidirectional 90°	EKM	FEM	6.244%
	Boundary conditions	CCCC, CCCS, CCSS, CFCC, CFCF, CFCS, CFSC, CFSF, CFSS, CSCS, CSSS, FSCS, FSFS, SSFS, SSSS			
	Number of Frequency	9			
2	Lamination scheme	 Angle-ply 45°, Unidirectional 45°	EKM+RRM (9 basis functions)	FEM	11.727%
	Boundary conditions	CCCC, CCCS, CCSS, CFCC, CFCF, CFCS, CFSC, CFSF, CFSS, CSCS, CSSS, FSCS, FSFS, SSFS, SSSS			
	Number of Frequency	5			
3	Lamination scheme	 Cross-ply	EKM	FEM	13.002%
	Boundary conditions	CCCC			
	Number of Frequency	100			

5.2 Recommendations

5.2.1 For the mode shapes and the natural frequencies with curved nodal lines of symmetrically laminated composite rectangular plates with various boundary conditions, the multi-term of the extended Kantorovich method should be considered. Due to the fact that the multi-term of the extended Kantorovich method provides the bending-twist coupling (D_{16} and D_{26}) in the governing differential equation which cause twist of the laminate.

5.2.2 For the higher mode shapes and the higher natural frequencies with straight nodal lines of symmetrically laminated composite rectangular plates with CCCC boundary conditions, the effects of rotate motion should be considered in the kinetic energy equation.