

Chapter 1

Introduction

In recent years, research on stability for neural networks (NNs) have been interesting studies and applied to application information processing problems and received much due to their potential applications in associative memory, parallel computation, pattern recognition, signal processing and optimization problems. It is well known that delays are often the sources of instability and oscillation in system. Therefore, the stability analysis of discrete-time neural networks (DNNs) has become an important topic of theoretical studies in neural networks; for examples, asymptotic stability, robust stability, and exponential stability of neural networks have been studied by many researchers, see [3, 12, 13, 15, 20, 22, 23, 24, 33, 34, 35].

The study of stability for systems with uncertainties is called robust stability. Important types of uncertainties are called parametric uncertainties and norm-bounded uncertainties, see [7, 12, 22, 26, 28, 29, 30, 33]. In [12], authors have presented global robust stability for delayed neural networks with polytopic type uncertainty, a stability condition for delayed neural networks is derived by using the S-procedure. Y. Liu, Z. Wang, A. Serrano and X. Liu [20], have studied discrete-time recurrent neural networks with time-varying delays: Exponential stability analysis. Y. Liu, X. Liu [22], have considered the problem of robust stability of discrete-time stochastic neural networks with time-varying delays by using a Lyapunov-Krasovskii functional criteria of robust stability is proposed in terms of LMIs. The problem of the robust stability analysis of neural networks with time-varying delay with norm-bounded uncertainties are proposed in [33]. The stability condition is given in terms of a LMI.

1. Uncertainty

1.1 Polytopic Uncertainty

In [29], Domingos C. W. Ramos and Pedro L. D. Peres have studied the robust stability of discrete-time linear parameter dependent (LPD) system of the form

$$x(k+1) = A(\alpha)x(k),$$

where $x(k) \in \mathbb{R}^n$ and $A(\alpha) \in \mathbb{R}^{n \times n}$ belongs to a convex bounded (polytope type) uncertain domain Ω , where

$$A(\alpha) \in \Omega, \quad \Omega = \left\{ A(\alpha) = \sum_{i=1}^N \alpha_i A_i, \quad \sum_{i=1}^N \alpha_i = 1, \quad \alpha_i \geq 0 \right\}.$$

In [28], M. C. de Oliveira, J. Bernussou and J. C. Geromel have studied the robust stability of the linear discrete-time uncertain system

$$x_{k+1} = A(\alpha)x_k,$$

where the dynamic matrix $A(\alpha)$ belongs to a convex polytopic set defined as

$$\mathcal{A} := \left\{ A(\alpha) : A(\alpha) = \sum_{i=1}^N \alpha_i A_i, \quad \sum_{i=1}^N \alpha_i = 1, \quad \alpha_i \geq 0 \right\},$$

and they considered the linear discrete-time system

$$x_{k+1} = A(\alpha)x_k + B(\beta)u_k,$$

where the dynamic matrix $A(\alpha)$ belonging to $\mathcal{A}$ and $B(\beta)$ is in convex polytope defined by

$$\mathcal{B} := \left\{ B(\beta) : B(\beta) = \sum_{i=1}^M \beta_i B_i, \quad \sum_{i=1}^M \beta_i = 1, \quad \beta_i \geq 0 \right\}.$$

Furthermore, Y. He, Q.-G. Wang and W.-X. Zheng [12], have studied the global robust stability for delayed neural networks with polytopic type uncertainties.

$$\dot{x}(t) = -Ax(t) + W_0g(x(t)) + W_1g(x(t - \tau(t))),$$

where $x(\cdot) = [x_1(\cdot), x_2(\cdot), \dots, x_n(\cdot)]^T$ is the neuron state vector, $g(x(\cdot)) = [g_1(x_1(\cdot)), g_2(x_2(\cdot)), \dots, g_n(x(\cdot))]^T$ is the neuron activation function, the matrices A , W_0 and W_1 are uncertainties and satisfy real convex polytopic :

$$\begin{bmatrix} A & W_0 & W_1 \end{bmatrix} \in \Omega,$$

$$\Omega := \left\{ \begin{bmatrix} A(\xi) & W_0(\xi) & W_1(\xi) \end{bmatrix} = \sum_{i=1}^N \xi_i \begin{bmatrix} A_i & W_{0i} & W_{1i} \end{bmatrix}, \quad \sum_{i=1}^N \xi_i = 1, \quad \xi_i \geq 0 \right\},$$

where $A_i = \text{diag}\{a_{1i}, \dots, a_{ni}\}$, $i = 1, \dots, N$ are diagonal matrices, W_{0i}, W_{1i} , $i = 1, \dots, N$ are constant matrices, the delay $\tau(t)$ satisfies

$$\dot{\tau}(t) \leq d < 1,$$

where d is a constant, $g_j(\cdot)$, $j = 1, \dots, n$, satisfies the following condition:

$$0 \leq \frac{g_j(x_j)}{x_j} \leq \mu_j, \quad g_j(0) = 0, \quad \forall x_j \neq 0, \quad j = 1, \dots, n.$$

1.2 Norm-bounded Uncertainties

In [33], H. Zhang and X. Liao have studied the robust stability analysis of neural networks with time-varying delay. They considered the following discrete-time neural networks model

$$\dot{x}(t) = -(A + \Delta A)x(t) + (W + \Delta W)g(x(t)) + (W_1 + \Delta W_1)g(x(t - \tau(t))),$$

where $x(t) = [x_1(t), \dots, x_n(t)]^T$ is the neuron state vector, $A = \text{diag}(a_1, \dots, a_n)$ is a positive diagonal matrix, W and W_1 are interconnection weight matrices and $\tau(t)$ denotes the time-varying delay satisfying $\dot{\tau}(t) \leq d < 1$ where d is constant, $\Delta A, \Delta W$ and ΔW_1 are parametric uncertainties and $g(x) = [g_1(x_1), \dots, g_n(x_n)]$ denotes the neuron activation function and satisfies $0 \leq \frac{g_j(x_j)}{x_j} \leq k$, $g_j(0) = 0$, $\forall x_j \neq 0$, $j = 1, \dots, n$. The uncertainties $\Delta A, \Delta W$ and ΔW_1 are defined by

$$\Delta A = H_0 F_0 E_0, \quad \Delta W = H F E \quad \text{and} \quad \Delta W_1 = H_1 F_1 E_1,$$

where H_0, H, H_1, E_0, E and E_1 are known constant matrices of appropriate dimensions, and F_0, F, F_1 are unknown matrices (the parameter uncertainty) satisfying

$$F_0^T F_0 < I, \quad F^T F < I, \quad F_1^T F_1 < I,$$

where I is the identity matrix.

In [22], Y. Liu, Z. Wang, and X. Liu have studied the robust stability of discrete-time stochastic neural networks with time-varying delays. They considered, on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, the following discrete-time stochastic neural networks (DSNNs) with time-varying delays of the form

$$\begin{aligned} x(k+1) = & (A + \Delta A(k))x(k) + (B + \Delta B(k))G(x(k)) + (D + \Delta D(k))H(x(k - \tau(k))) \\ & + \sigma(x(k), x(k - \tau(k)), k)w(k), \end{aligned} \quad (1.1)$$

where $x(k) = (x_1(k), x_2(k), \dots, x_n(k))^T \in \mathbb{R}^n$ is the neural state vector, the positive integer $\tau(k)$ denotes the time-varying delay satisfying

$$\tau_m \leq \tau(k) \leq \tau_M, \quad k \in \mathbb{N},$$

where τ_m and τ_M are known positive integers. The diagonal matrix $A = \text{diag}(a_1, a_2, \dots, a_n)$ is real constant diagonal with entries $|a_i| < 1$, $B = [b_{ij}]_{n \times n}$ and $D = [d_{ij}]_{n \times n}$ are connection weight matrices and the discretely delayed connection

weight matrix, respectively; Whereas the time-varying matrices $\Delta A(k)$, $\Delta B(k)$ and $\Delta D(k)$ represent the time-varying parameter uncertainties satisfying

$$[\Delta A(k) \ \Delta B(k) \ \Delta D(k)] = MF(k)[N_1 \ N_2 \ N_3],$$

where M and N_i ($i = 1, 2, 3$) are known real constant matrices, and $F(k)$ is the unknown time-varying matrix-valued function subject to the following condition:

$$F^T(k)F(k) \leq I \quad \forall k \in \mathbb{N}^+.$$

In DSNNs (1.1), $w(k)$ is a scalar Wiener process (Brownian Motion) on $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$\mathbb{E}[w(k)] = 0, \quad \mathbb{E}[w^2(k)] = 1, \quad \mathbb{E}[w(i)w(j)] = 0 (i \neq j),$$

and $\sigma : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$ is a continuous function satisfying

$$\sigma^T(x, y, k)\sigma(x, y, k) \leq \rho_1 x^T x + \rho_2 y^T y, \quad x, y \in \mathbb{R}^n,$$

where $\rho_1 > 0$ and $\rho_2 > 0$ are known constant scalars, and $G(x(k)) = [g_1(x_1(k)), g_2(x_2(k)), \dots, g_n(x_n(k))]^T$ and $H(x(k)) = [h_1(x_1(k)), h_2(x_2(k)), \dots, h_n(x_n(k))]^T$ denote the neuron activation functions.

Motivated by these results, we are studied the robust stability problem of discrete-time and discrete-time stochastic neural networks with time-varying delay. The advantages of our study are

- (i) There are norm-bounded uncertainties, polytopic type uncertainties and stochastic perturbation in our study.
- (ii) If we consider discrete-time neural networks with time-varying delay without norm-bounded uncertainties and without polytopic uncertainties, then we obtain a result of [20] as our corollary.

In the past few years, many researchers have studied on the asymptotic stability of the generalized difference equations, see [4, 6, 18, 19, 31, 35, 37]. In [4], L. Berezansky, E. Broverman and E. Liz have derived sufficient conditions for the global stability by using discrete type inequalities. N. S. Bay and V. N. Phat [6], have studied new stability conditions are given for nonlinear difference equations on the assumption that the nonlinear part satisfies certain discrete type inequalities. In [37], authors have considered the problem of asymptotic stability of discrete-time interval system with delay. A sufficient condition is given by means of the inequality techniques.

2. Discrete type Inequalities

In [18], E. Liz and J. B. Ferreira have studied the global stability of generalized difference equation

$$\Delta x_n = f(n, x_n, x_{n-1}, \dots, x_{n-r}), \quad n \in \mathbb{N},$$

where $\Delta x_n = x_{n+1} - x_n$, and $f : \mathbb{N} \times \mathbb{R}^{r+1}$, and they considered the following generalized difference equation :

$$\Delta x_n = -ax_n + f(n, x_n, x_{n-1}, \dots, x_{n-r}), \quad a > 0.$$

In [19], E. Liz, A. F. Ivanov and J. B. Ferreira have studied discrete halanay-type inequalities and applications. They considered the following inequalities:

$$\begin{aligned} \Delta u_n &\leq -Au_n + B\widetilde{u}_n + Cv_n + D\widehat{v}_n, \quad n \geq 0 \\ u_n &\leq (1-A)^n u_0 + \sum_{i=0}^{n-1} (1-A)^{n-i-1} [B\widetilde{u}_i + Cv_i + D\widehat{v}_i], \quad n \geq 0, \\ v_n &\leq Ev_n + F\widetilde{u}_n, \quad n \geq 0, \end{aligned}$$

where $\Delta u_n = u_{n+1} - u_n$, $\widetilde{u}_n = \max\{u_n, \dots, u_{n-r}\}$, $\widehat{v}_n = \max\{v_{n-1}, \dots, v_{n-r}\}$, $r \geq 1$, and A, B, C, D, E, F are real constants.

In [35], Q. Zhang, X. Wei and J. Xu have studied the global exponential stability of discrete-time hopfield neural networks with time-varying of the following form

$$y(n+1) = Cy(n) + Ag(y(n)) + Bg(y(n - \kappa(n))),$$

where $y(n) = [y_1(n), \dots, y_m(n)]^T \in \mathbb{R}^m$ is the state vector, $\kappa(n)$ is the transmission delay satisfying $0 \leq \kappa(n) \leq \kappa$ (κ is a constant), $g(x(n)) = [g_1(x_1(n)), \dots, g_m(x_m(n))]^T \in \mathbb{R}^m$ is the activation function of the neuron, where $g_j(\cdot)$, $j = 1, \dots, m$ satisfies

$$|g_j(\xi_j)| \leq L_j |\xi_j|.$$

Motivated by these results, we are investigated the discrete type inequalities and global stability of discrete-time with time-varying delays. The advantages of our results are

- (i) There are multiple time-varying delays in our study. In the case that there is only one time-varying delay, we obtain a result in [35] as our corollary.
- (ii) In our results, the discrete type inequality is less conservative than the discrete Halanay type inequality in [18] for the case of positive sequences.

In the past decades, several authors have studied the robust H_∞ filtering problem for uncertain discrete-time system with time-varying delay, see [10, 14, 17, 21, 36]. In [17], authors have investigated the delay-dependent robust H_∞ filtering for uncertain discrete-time singular systems with interval time-varying delay in which the uncertainty is of polytopic type. They have considered the design of a linear robust H_∞ filter such that the resulting filtering error singular system is regular, causal, and asymptotically stable with a H_∞ norm-bounded.

Y. Liu, Z. Wang, and X. Liu [21], have studied robust H_∞ filtering for discrete nonlinear stochastic systems with time-varying delay. By using Lyapunov stability theory, they have obtained conditions for H_∞ filter in terms of the solution to a linear matrix inequality. In [8], authors have derived H_∞ filter for discrete-time switched systems with time-varying delays. By using switched Lyapunov functionals, a sufficient condition for the solvability of this problem is obtained in terms of LMIs. In [25], authors have proposed delay-dependent H_∞ filtering of a class of switched discrete-time state delay systems. By using switched Lyapunov functionals, sufficient conditions for the solvability of this problem are obtained in terms of LMIs.

3. H_∞ filtering problem

In [10], X.-M. Zhang and Q.-L. Han have studied H_∞ filter design for discrete-time systems with sector-bounded nonlinearities described as

$$\begin{cases} x(k+1) = Ax(k) + Ff(x(k)) + B\omega(k) \\ y(k) = Cx(k) + Hh(x(k)) + D\omega(k) \\ z(k) = Lx(k), \end{cases} \quad (1.2)$$

where $x(k) \in \mathbb{R}^n$ is state vector, $\omega(k) \in \mathbb{R}^r$ is the disturbance input, which is assumed to belong to $L_2[0, \infty]$, $z(k) \in \mathbb{R}^q$ is the regulated output, and $y(k) \in \mathbb{R}^p$ is the measured output. The system matrices A, F, B, C, H, D and L are known constant matrices of appropriate dimensions. The known functions $f(x(k))$ and $h(x(k))$ are the vector-valued nonlinear functions.

Assumption 1. The vector-valued nonlinear functions $f(\cdot)$ and $h(\cdot)$ are assumed to satisfy the following sector-bounded conditions:

$$\begin{cases} [f(x) - f(y) - M_1\eta]^T [f(x) - f(y) - M_2\eta] \leq 0, \\ [h(x) - h(y) - M_1\eta]^T [h(x) - h(y) - M_2\eta] \leq 0, \end{cases}$$

where $\eta = x - y$, $\forall x, y \in \mathbb{R}^n$, $M_1, M_2 \in \mathbb{R}^{n \times n}$, $N_1, N_2 \in \mathbb{R}^{n \times n}$ are known constant matrices, and

$$f(0) = 0, \quad g(0) = 0.$$

The sensor outage cases are considered as follows

$$y_{ij}^F(k) = (1 - \rho_i^j) y_i(k), \quad 0 \leq \underline{\rho}_i^j \leq \rho_i^j \leq \overline{\rho}_i^j \leq 1 \\ i = 1, \dots, p, \quad j = 1, \dots, L,$$

where ρ_i^j is an unknown constant, $\underline{\rho}_i^j$ and $\overline{\rho}_i^j$ represent the lower and upper bounds of ρ_i^j , respectively. Denote

$$y_j^F(k) = [y_{1j}^F(k), y_{2j}^F(k), \dots, y_{pj}^F(k)]^T = (I - \rho_j) y(k),$$

where $\rho^j = \text{diag}\{\rho_1^j, \rho_2^j, \dots, \rho_p^j\}$ and $j = 1, \dots, L$. The scaling factors ρ^j satisfy

$$\begin{aligned} N_{\rho^j} &= \{\rho^j | \rho^j = \text{diag}\{\rho_1^j, \rho_2^j, \dots, \rho_p^j\} \in \mathbb{R}^p, \\ 0 &\leq \underline{\rho}_i^j \leq \rho_i^j \leq \overline{\rho}_i^j \leq 1 \quad i = 1, 2, \dots, p\}. \end{aligned}$$

A uniform sensor failure model is given by

$$y^F(k) = (I - \rho)y(k), \quad \rho \in \{\rho^1, \rho^2, \dots, \rho^L\}, \quad (1.3)$$

where ρ can be described by $\rho = \text{diag}\{\rho_1, \rho_2, \dots, \rho_p\}$.

Then, system (1.2) with sensor failure (1.3) is described by

$$\begin{cases} x(k+1) = Ax(k) + Ff(x(K)) + B\omega(k) \\ y^F(k) = (1 - \rho)(Cx(k) + Hh(x(k)) + D\omega(k)) \\ z(k) = Lx(k). \end{cases} \quad (1.4)$$

The reliable filter is of the form

$$\begin{cases} \hat{x}(k+1) = A_f \hat{x}(k) + B_f y^F(k) + F_f f(\hat{x}(k)) \\ \hat{z}(k) = C_f \hat{x}(k), \end{cases} \quad (1.5)$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the filter state, $\hat{z} \in \mathbb{R}^q$ is the estimation of $z(k)$, A_f, B_f, C_f and F_f are the filter parameter matrices to be designed.

Applying filter (1.5) to system (1.4), we obtain the filtering error system:

$$\begin{cases} \xi(k+1) = \bar{A}\xi(k) + \bar{A}_{f1}f(K_1\xi(k)) + \bar{A}_{f2}f(K_2\xi(k)) + \bar{A}_h h(K_1\xi(k)) + \bar{B}\omega(k) \\ e(k) = \bar{C}\xi(k), \end{cases} \quad (1.6)$$

where $\xi(k) = \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix}$, $K_1 = \begin{bmatrix} I & 0 \end{bmatrix}$, $K_2 = \begin{bmatrix} 0 & I \end{bmatrix}$, $e(k) = z(k) - \hat{z}(k)$ is estimation error, and

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A & 0 \\ B_f(I - \rho)C & A_f \end{bmatrix}, \quad \bar{A}_{f1} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad \bar{A}_{f2} = \begin{bmatrix} 0 \\ F_f \end{bmatrix}, \quad \bar{A}_h = \begin{bmatrix} 0 \\ B_f(I - \rho)H \end{bmatrix}, \\ \bar{B} &= \begin{bmatrix} B \\ B_f(I - \rho)D \end{bmatrix} \text{ and } \bar{C} = \begin{bmatrix} L & -C_f \end{bmatrix}. \end{aligned}$$

We denote the filtering error system without sensor failures, i.e., $\rho = 0$, as follows:

$$\begin{cases} \xi(k+1) = \bar{A}\xi(k) + \bar{A}_{f1}f(K_1\xi(k)) + \bar{A}_{f2}f(K_2\xi(k)) + \bar{A}_h h(K_1\xi(k)) + \bar{B}\omega(k) \\ e(k) = \bar{C}\xi(k), \end{cases} \quad (1.7)$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix}, \quad \bar{A}_{f1} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad \bar{A}_{f2} = \begin{bmatrix} 0 \\ F_f \end{bmatrix}, \quad \bar{A}_h = \begin{bmatrix} 0 \\ B_f H \end{bmatrix}, \\ \bar{B} &= \begin{bmatrix} B \\ B_f D \end{bmatrix} \text{ and } \bar{C} = \begin{bmatrix} L & -C_f \end{bmatrix}. \end{aligned}$$

The objective is to develop a filter of the form (1.5) such that the filtering error systems (1.6) and (1.7) satisfy the following requirements:

- (1) While there is no exogenous disturbance, i.e., $\omega(k) = 0$, the filtering error system (1.6) and (1.7) are asymptotically stable.
- (2) For given constants $\gamma_f > \gamma_\eta > 0$, find filter (1.5) such that
 - (a) The filtering error system (1.6) in the normal case, i.e., $\rho = 0$, (1.7) has a prescribed level γ_η of H_∞ performance, In other words, under the zero initial condition, $\|e\|_2 < \gamma_\eta \|\omega\|_2$ is satisfied for any nonzero $\omega \in l_2$,
 - (b) The filtering error system (1.6) in the sensor failure case, i.e., $\rho \in \{\rho_1, \rho_2, \dots, \rho_p\}$ with $\rho^j \in N_{\rho^j}$, $j = 1, \dots, L$, has a prescribed level γ_f of H_∞ performance, In other words, under the zero initial condition, $\|e\|_2 < \gamma_f \|\omega\|_2$ is satisfied for any nonzero $\omega \in l_2$.

In [36], X.-M. Zhang and Q.-L. Han have studied robust H_∞ filtering for uncertain discrete-time systems with time-varying delay of the following form

$$\begin{cases} x(k+1) = A_0x(k) + A_1x(k-h(k)) + B_1\omega(k) \\ y(k) = C_0x(k) + C_1x(k-h(k)) + B_2\omega(k) \\ z(k) = L_0x(k) + L_1x(k-h(k)) + B_3\omega(k) \\ x(k) = \phi(k), \quad k = -\hat{h}, -\hat{h}+1, \dots, 0, \end{cases} \quad (1.8)$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^m$ is the measured output, $z(k) \in \mathbb{R}^p$ is signal to be estimated, $\omega(k) \in \mathbb{R}^q$ is assumed to be an arbitrary noise signal belonging to l_2 , $\phi(k)$, $k = -\hat{h}, -\hat{h}+1, \dots, 0$ is a known given initial condition sequence, and $h(k)$ is time-varying delay satisfying

$$0 < \underline{h} \leq h(k) \leq \bar{h} < \infty, \quad k = 1, 2, \dots$$

where $\bar{h}, \underline{h}$ are known constants. The system matrices belong to a convex set

$$\chi := (A_0, A_1, B_1, C_0, C_1, B_2, L_0, L_1, B_3) \in \Omega, \quad (1.9)$$

where

$$\Omega := \left\{ \chi \mid \chi = \sum_{i=1}^r \rho_i \chi_i, \quad \sum_{i=1}^r \rho_i = 1, \quad \rho_i \geq 0 \right\},$$

with $\chi_i := (A_{0i}, A_{1i}, B_{1i}, C_{0i}, C_{1i}, B_{2i}, L_{0i}, L_{1i}, B_{3i})$. They designed a full order filter with state-space of the following form:

$$\begin{cases} \hat{x}(k+1) = A_f \hat{x}(k) + B_f y(k), & \hat{x}(0) = 0 \\ \hat{z}(k) = C_f \hat{x}(k) + D_f y(k), \end{cases} \quad (1.10)$$

where constant matrices $A_f \in \mathbb{R}^{n \times n}$, $B_f \in \mathbb{R}^{n \times m}$, $C_f \in \mathbb{R}^{p \times n}$ and $D_f \in \mathbb{R}^{p \times m}$ are filter parameters to be determined. Defining the state vector $\bar{x}(k) := [x^T(k), \hat{x}^T(k)]^T$ and the estimation error $\bar{z}(k) := z(k) - \hat{z}(k)$, one obtains the following filtering system:

$$\begin{cases} \bar{x}(k+1) = \bar{A}_0 \bar{x}(k) + \bar{A}_1 E \bar{x}(k-h(k)) + \bar{B}_1 \omega(k) \\ \bar{z}(k) = \bar{L}_0 \bar{x}(k) + \bar{L}_1 E \bar{x}(k-h(k)) + \bar{B}_3 \omega(k) \\ \bar{x}(k) = [\phi(k), 0]^T \quad k = -\bar{h}, -\bar{h}+1, \dots, 0, \end{cases} \quad (1.11)$$

where $\bar{A}_0 = \begin{bmatrix} A_0 & 0 \\ B_f C_0 & A_f \end{bmatrix}$, $E = \begin{bmatrix} I & 0 \end{bmatrix}$, $\bar{A}_1 = \begin{bmatrix} A_1 \\ B_f C_1 \end{bmatrix}$, $\bar{B}_1 = \begin{bmatrix} B_1 \\ B_f B_2 \end{bmatrix}$, $\bar{L}_0 = \begin{bmatrix} L_0 - D_f C_0 & -C_f \end{bmatrix}$, $\bar{L}_1 = L_1 - D_f C_1$, $\bar{B}_3 = B_3 - D_f B_2$.

The purpose of this paper is to design a robust filter of the form (1.10) such that the system (1.11) has a prescribed H_∞ performance for all uncertainties satisfying (1.9), namely

- 1) System (1.11) with $\omega(k) = 0$ is asymptotically stable.
- 2) System (1.11) has a prescribed level γ of H_∞ noise attenuation, i.e., under the zero initial condition, $\|\bar{z}\|_2 < \gamma \|\omega\|_2$ is satisfied for any nonzero $\omega \in l_2$.

Motivated by these results, we are interested in the H_∞ filtering problem of the nonlinear discrete-time system time-varying delay. The advantages of our study are

- (i) There are nonlinear perturbations in our study.
 - (ii) The time delay is time-varying.
 - (iii) We require less free matrix variable than some existing results such as [14].
- It is noted that the former has more matrix variables than our results. Therefore, our results are less conservative than these in some existing results.

It is our future investigation to propose the robust H_∞ filtering problem for nonlinear discrete-time neural networks with polytopic types uncertainties and with time-varying delay.

This thesis is organized in six chapters that are structured as follows:

Chapter 1

The first chapter is a literature of general introduction of neural networks and the stability analysis problem is recalled in order to concepts that are necessary to the comprehension of our work and we present the structure of the thesis.

Chapter 2

In this chapter, we give some important notations mathematical definition and

we show some general concepts of stability, definitions and lemmas because it will be recalled.

Chapter 3

This chapter, we study the robust stability of discrete-time LPD and discrete-time stochastic LPD neural networks with time-varying delay. Based on Lyapunov stability theory and the S-procedure, we derive robust stability criteria in terms of linear matrix inequalities which are solvable by several available algorithms. Numerical examples are given to illustrate the effectiveness of our theoretical results.

Chapter 4

In this chapter, we introduce discrete type inequalities. Based on these inequalities, we derive new global stability conditions of nonlinear difference equations and presents a new approach to the global stability of discrete-time with time-varying delays. A numerical example is provide to demonstrate the effectiveness of the proposed designs.

Chapter 5

In chapter 5, we investigate the problem of H_∞ filter for discrete-time neural networks with time-varying delays. Based on Lyapunov stability theory and the S-procedure, we derive criteria in terms of linear matrix inequalities. Numerical examples are given to illustrate the effectiveness of our theoretical results.

Chapter 6

In the last chapter we provide a summary of our results.