

Chapter 6

Conclusion

In this thesis, we study the stability analysis problem for discrete-time neural networks with time-varying delays. The system under study involve stochastic disturbance, linear parameter dependent, norm bounded uncertainties. We used the Lyapunov stability theory, a discrete type inequality and S-procedure as our main tools. First, we obtain sufficient condition of LPD and LPD stochastic neural networks in term of LMIs which are solvable by several available algorithms. Next, we consider global stability problem of nonlinear difference with time-varying delays by using some discrete types inequalities. Finally, we study the problem of H_∞ filter for discrete-time neural networks with time-varying delays. The result have been derived in term of LMIs which are easily checked using the LMI toolbox in MATLAB. Numerical examples are also given to show the effectiveness of our theoretical results.

The results of this thesis are summarized as follows.

6.1 Robust Stability Criteria of LPD Neural Networks with time-varying delay

1. The origin of system (3.7) with polytopic type uncertainties (3.2) is robustly stable if there exist $P_i = P_i^T > 0$, $Q_i = Q_i^T > 0$, $T_i = \text{diag}\{t_{1i}, t_{2i}, \dots, t_{ni}\} \geq 0$, $S_i = \text{diag}\{s_{1i}, s_{2i}, \dots, s_{ni}\} \geq 0$ and scalars $e_{0i} > 0$, $e_i > 0$ and $e_{1i} > 0$, $i = 1, 2, \dots, N$ satisfying the following conditions

$$\left\{ \begin{array}{l} (i) \quad M_{i,i} + N_i < -I, \quad i = 1, 2, \dots, N; \\ (ii) \quad M_{i,i,j} + M_{j,i,i} + M_{i,j,i} + 2N_i + N_j < \frac{1}{(N-1)^2} I, \\ \quad i = 1, 2, \dots, N, \quad i \neq j, \quad j = 1, 2, \dots, N; \\ (iii) \quad M_{i,j,l} + M_{i,l,j} + M_{j,i,l} + M_{j,l,i} + M_{l,i,j} + M_{l,j,i} \\ \quad + 2N_i + 2N_j + 2N_l < \frac{6}{(N-1)^2} I, \\ \quad i = 1, 2, \dots, N-2, \quad j = i+1, 2, \dots, N-1, \quad l = 1, 2, \dots, N, \end{array} \right.$$

where

$$M_{i,j,l} = \begin{bmatrix} \varphi_{11} & 0 & \varphi_{13} & \varphi_{14} & \varphi_{15} & \varphi_{16} & \varphi_{17} \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \varphi_{34} & \varphi_{35} & \varphi_{36} & \varphi_{37} \\ * & * & * & * & \varphi_{45} & \varphi_{46} & \varphi_{47} \\ * & * & * & * & * & \varphi_{56} & \varphi_{57} \\ * & * & * & * & * & \varphi_{66} & \varphi_{67} \\ * & * & * & * & * & * & \varphi_{77} \end{bmatrix},$$

$$N_i = \begin{bmatrix} \Pi_{11}(i) & 0 & -2L_2T_i & 0 & 0 & 0 & 0 \\ 0 & -Q_i - 2V_1S_i & 0 & -2V_2S_i & 0 & 0 & 0 \\ -2L_2T_i & 0 & \Pi_{33}(i) & 0 & 0 & 0 & 0 \\ 0 & -2V_2S_i & 0 & \Pi_{44}(i) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{55}(i) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{66}(i) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Pi_{77}(i) \end{bmatrix},$$

and $\varphi_{11} = A_i^T P_j A_l$, $\varphi_{13} = -A_i^T P_j W_l$, $\varphi_{14} = -A_i^T P_j W_{1l}$, $\varphi_{15} = A_i^T P_j H_0$,

$\varphi_{16} = -A_i^T P_j H$, $\varphi_{17} = -A_i^T P_j H_1$, $\varphi_{34} = i^T P_j W_{1l}$, $\varphi_{35} = -W_i^T P_j H_0$,

$\varphi_{36} = W_i^T P_j H$, $\varphi_{37} = W_i^T P_j H_1$, $\varphi_{45} = -W_{1i}^T P_j H_0$, $\varphi_{46} = W_{1i}^T P_j H$,

$\varphi_{47} = W_{1i}^T P_j H_1$, $\varphi_{56} = -H_0^T P_i H$, $\varphi_{57} = -H_0^T P_i H_1$, $\varphi_{66} = H^T P_i H$,

$\varphi_{67} = H^T P_i H_1$, $\varphi_{77} = H_1^T P_i H_1$,

$\hat{\tau} = \tau_2 - \tau_1 + 1$, $\Pi_{11}(i) = e_{0i} E_0^T E_0 - P_i + \hat{\tau} Q_i - 2L_1 T_i$, $\Pi_{33}(i) = -2T_i + e_i E^T E$,

$\Pi_{44}(i) = -2S_i + e_{1i} E_1^T E_1$, $\Pi_{55}(i) = -e_{0i} I$, $\Pi_{66}(i) = -e_i I$, $\Pi_{77}(i) = -e_{1i} I$,

$L_1 = \text{diag}(l_1^+ l_1^-, l_2^+ l_2^-, \dots, l_n^+ l_n^-)$, $L_2 = \text{diag}(-\frac{l_1^+ + l_1^-}{2}, -\frac{l_2^+ + l_2^-}{2}, \dots, -\frac{l_n^+ + l_n^-}{2})$,

$V_1 = \text{diag}(v_1^+ v_1^-, v_2^+ v_2^-, \dots, v_n^+ v_n^-)$ and $V_2 = \text{diag}(-\frac{v_1^+ + v_1^-}{2}, -\frac{v_2^+ + v_2^-}{2}, \dots, -\frac{v_n^+ + v_n^-}{2})$.

6.2 Robust stability of stochastic LPD neural networks with time-varying delay

1. The DSNNs (3.22) with polytopic type uncertainties (3.23) is robustly stable in the mean square if there exist $P_i = P_i^T > 0$, $Q_i = Q_i^T > 0$, $T_i = \text{diag}\{t_{1i}, t_{2i}, \dots, t_{ni}\} \geq 0$, $S_i = \text{diag}\{s_{1i}, s_{2i}, \dots, s_{ni}\} \geq 0$ and scalars $e_{0i} > 0$, $e_i > 0$, $e_{1i} > 0$ and $\lambda_i^* > 0$, $i = 1, 2, \dots, N$ satisfying the following conditions

- (i) $M_{i,i,i} + N_i < -I$, $i = 1, 2, \dots, N$;
- (ii) $M_{i,i,j} + M_{j,i,i} + M_{i,j,i} + 2N_i + N_j < \frac{1}{(N-1)^2}I$,
 $i = 1, 2, \dots, N$, $i \neq j$, $j = 1, 2, \dots, N$;
- (iii) $M_{i,j,l} + M_{i,l,j} + M_{j,i,l} + M_{j,l,i} + M_{l,i,j} + M_{l,j,i}$
 $+ 2N_i + 2N_j + 2N_l < \frac{6}{(N-1)^2}I$,
 $i = 1, 2, \dots, N-2$, $j = i+1, 2, \dots, N-1$, $l = 1, 2, \dots, N$;
- (iv) $P_i < \lambda_i^* I$,

where

$$M_{i,j,l} = \begin{bmatrix} \varphi_{11} & 0 & \varphi_{13} & \varphi_{14} & \varphi_{15} & \varphi_{16} & \varphi_{17} \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \varphi_{34} & \varphi_{35} & \varphi_{36} & \varphi_{37} \\ * & * & * & * & \varphi_{45} & \varphi_{46} & \varphi_{47} \\ * & * & * & * & * & \varphi_{56} & \varphi_{57} \\ * & * & * & * & * & \varphi_{66} & \varphi_{67} \\ * & * & * & * & * & * & \varphi_{77} \end{bmatrix},$$

$$N_i = \begin{bmatrix} \Pi_{11}(i) & 0 & -2L_2T_i & 0 & 0 & 0 & 0 \\ 0 & \Pi_{22}(i) & 0 & -2V_2S_i & 0 & 0 & 0 \\ -2L_2T_i & 0 & \Pi_{33}(i) & 0 & 0 & 0 & 0 \\ 0 & -2V_2S_i & 0 & \Pi_{44}(i) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{55}(i) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{66}(i) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Pi_{77}(i) \end{bmatrix},$$

$$\Pi_{11}(i) = e_{0i}E_0^TE_0 - P_i + \hat{\tau}Q_i - 2L_1T_i + \lambda_i^*\rho_1I, \quad \Pi_{22}(i) = -Q_i - 2V_1S_i + \lambda_i^*\rho_2I,$$

$$\Pi_{33}(i) = -2T_i + e_iE^TE, \quad \Pi_{44}(i) = -2S_i + e_{1i}E_1^TE_1, \quad \Pi_{55}(i) = -e_{0i}I,$$

$$\Pi_{66}(i) = -e_iI, \quad \Pi_{77}(i) = -e_{1i}I, \quad \hat{\tau} = \tau_2 - \tau_1 + 1,$$

$$L_1 = \text{diag}(l_1^+l_1^-, l_2^+l_2^-, \dots, l_n^+l_n^-), \quad L_2 = \text{diag}\left(-\frac{l_1^++l_1^-}{2}, -\frac{l_2^++l_2^-}{2}, \dots, -\frac{l_n^++l_n^-}{2}\right),$$

$$V_1 = \text{diag}(v_1^+v_1^-, v_2^+v_2^-, \dots, v_n^+v_n^-), \quad \text{and } V_2 = \text{diag}\left(-\frac{v_1^++v_1^-}{2}, -\frac{v_2^++v_2^-}{2}, \dots, -\frac{v_n^++v_n^-}{2}\right).$$

6.3 New discrete type inequalities and global stability of nonlinear difference equations

1. Let $q_i \in \mathbb{R}_0^+$, $h_i \in \mathbb{Z}^+$, $i = 1, \dots, r$; $p, q_r \in \mathbb{R}^+$, where $0 = h_0 < h_1 < \dots < h_r$ and $\sum_{i=0}^r q_i < p \leq 1$, and let $\{x_j\}_{j \in \mathbb{Z}^{-h_r}}$ be a sequence of real numbers satisfying the inequality

$$\Delta x_n \leq -px_n + \sum_{i=0}^r q_i x_{n-h_i}, \quad n \in \mathbb{Z}^0. \quad (6.1)$$

Then there exists $\lambda_0 \in (0, 1)$ such that

$$x_n \leq \max\{0, x_0, x_{-1}, \dots, x_{-h_r}\} \lambda_0^n, \quad n \in \mathbb{Z}^0.$$

Moreover, λ_0 might be chosen as the smallest root of the polynomial

$$P(\lambda) = \lambda^{h_r+1} - (1 - p + q_0)\lambda^{h_r} - q_1\lambda^{h_r-h_1} \dots - q_{r-1}\lambda^{h_r-h_{r-1}} - q_r \quad (6.2)$$

which lies in the interval $(0, 1)$.

2. Let $p, \alpha_i, \beta_i \in \mathbb{R}^+$, $h_i \in \mathbb{Z}^+$, $i = 1, \dots, r$, where $0 = h_0 < h_1 < \dots < h_r$, $\sum_{i=0}^r \alpha_i = 1$ and $\prod_{i=0}^r \beta_i < p \leq 1$. Let $\{x_n\}_{n \in \mathbb{Z}^{-h_r}}$ be a sequence of real numbers such that $x_{n-h_i}^{\alpha_i}$ are defined for all $i = 1, \dots, r$; $n \in \mathbb{Z}^0$ which satisfies the inequality

$$\Delta x_n \leq -px_n + \prod_{i=0}^r \beta_i x_{n-h_i}^{\alpha_i}, \quad n \in \mathbb{Z}^0.$$

Then there exists $\lambda_0 \in (0, 1)$ such that

$$x_n \leq \max\{0, x_0, x_{-1}, \dots, x_{-h_r}\} \lambda_0^n, \quad n \in \mathbb{Z}^0.$$

Moreover, λ_0 might be chosen as the smallest root of the function

$$F(\lambda) = \lambda - \left(\prod_{i=0}^r \beta_i \right) \lambda^{-\sum_{i=1}^r h_i \alpha_i} + p - 1$$

which lies in the interval $(0, 1)$.

3. Assume that there exist $q_i \in \mathbb{R}_0^+$, $h_i \in \mathbb{Z}^+$, $i = 1, \dots, r$; $q_r \in \mathbb{R}^+$, where $\sum_{i=0}^r q_i < p \leq 1$ such that

$$|f(n, x_n, x_{n-h_1}, \dots, x_{n-h_r})| \leq \sum_{i=0}^r q_i |x_{n-h_i}|, \quad (6.3)$$

for all $(n, x_n, x_{n-h_1}, \dots, x_{n-h_r}) \in \mathbb{Z}^0 \times \mathbb{R}^{r+1}$. Then, there exists $\lambda_0 \in (0, 1)$ such that every solution $\{x_n\}$ of (4.5) satisfies

$$|x_n| \leq \left(\max_{-h_r \leq i \leq 0} \{|x_i|\} \right) \lambda_0^n, \quad n \in \mathbb{Z}^0,$$

where λ_0 is chosen as in Theorem 4.1.1.

4. Assume that there exist $p, \alpha_i, \beta_i \in \mathbb{R}^+$, $h_i \in \mathbb{Z}^+$, $i = 1, \dots, r$, where $\sum_{i=0}^r \alpha_i = 1$ and $\prod_{i=0}^r \beta_i < p \leq 1$ such that

$$|f(n, x_n, x_{n-h_1}, \dots, x_{n-h_r})| \leq \prod_{i=0}^r \beta_i |x_{n-h_i}|^{\alpha_i},$$

for all $(n, x_n, x_{n-h_1}, \dots, x_{n-h_r}) \in \mathbb{Z}^0 \times \mathbb{R}^{r+1}$. Then there exists $\lambda_0 \in (0, 1)$ such that every solution $\{x_n\}$ of (4.5) satisfies

$$|x_n| \leq \left(\max_{-h_r \leq i \leq 0} \{|x_i|\} \right) \lambda_0^n, \quad n \in \mathbb{Z}^0,$$

where λ_0 is chosen as in Theorem 4.1.2.

6.4 Global exponential stability of nonlinear discrete-time neural networks with time-varying delays

1. Let $q_i \in \mathbb{R}_0^+$, $k_i \in \mathbb{Z}^+$, $i = 1, \dots, r$; $p, q_r \in \mathbb{R}^+$, where $0 = k_0 < k_1 < \dots < k_r$ and $\sum_{i=0}^r q_i < p \leq 1$, and let $\{x_j\}_{j \in \mathbb{Z}^{-k_r}}$ be a sequence of real numbers satisfying the inequality

$$\Delta x_n \leq -p x_n + \sum_{i=0}^r q_i \sum_{j=0}^{k_i} x_{n-j}, \quad n \in \mathbb{Z}^0.$$

Then there exists $\lambda_0 \in (0, 1)$ such that

$$x_n \leq \max\{0, x_0, x_{-1}, \dots, x_{-k_r}\} \lambda_0^n, \quad n \in \mathbb{Z}^0.$$

Moreover, λ_0 might be chosen as the smallest root of the polynomial

$$P(\lambda) = \lambda^{n+1} - (1-p)\lambda^n - q_1 \sum_{j=0}^{k_1} \lambda^{n-j} - q_2 \sum_{j=0}^{k_2} \lambda^{n-j} - \dots - q_M \sum_{j=0}^{k_r} \lambda^{n-j} \quad (6.4)$$

which lies in the interval $(0, 1)$.

2. The equilibrium point of system (4.12) is globally exponentially stable if

$$c_{\max} + l\|A\| + l \sum_{j=1}^M (\tau_j + 1) \|B_j\| < 1, \quad (6.5)$$

where $l = \max_j (L_j)$ and $c_{\max} = \max_j (c_j)$.

6.5 H_∞ filtering problem for nonlinear discrete-time system with time-varying delay

1. Given the filter parameters A_F, B_F, C_F and D_F . Then, the filtering error system (5.6) with time-varying delay $\tau(k)$ satisfying (5.2) with $\omega(k) = 0$ is asymptotically stable if there exist $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $Q = Q^T > 0$, $T = \text{diag}\{t_1, t_2, \dots, t_n\} \geq 0$, $S = \text{diag}\{s_1, s_2, \dots, s_n\} \geq 0$ satisfying the following conditions

$$M = \begin{bmatrix} M_0 & M_1^T P_1 & M_2^T P_2 \\ * & -P_1 & 0 \\ * & * & -P_2 \end{bmatrix} < 0,$$

where

$$M_0 = \begin{bmatrix} -P_2 - 2L_1 T + \hat{\tau} Q & P_2 & 0 & -2L_2 T & 0 \\ * & -P_2 & 0 & 0 & 0 \\ * & * & -Q - 2V_1 S & 0 & -2V_2 S \\ * & * & * & -2T & 0 \\ * & * & * & * & -2S \end{bmatrix},$$

$$M_1 = [A_1 \ 0 \ 0 \ A_2 \ A_3], \quad M_2 = [\bar{A}_1 - A_F \ 0 \ \bar{A}_2 \ \bar{A}_3], \quad \bar{A}_1 = A_1 - B_F C_1, \quad \bar{A}_2 = A_2 - B_F C_2, \\ \bar{A}_3 = A_3 - B_F C_3, \quad \hat{\tau} = \tau_2 - \tau_1 + 1.$$

2. Given the filter parameters A_F, B_F, C_F, D_F and scalar $\gamma > 0$. Then, the filtering error system (5.6) with time-varying delay $\tau(k)$ satisfying (5.2). is asymptotically stable if there exist $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $Q = Q^T > 0$, $T = \text{diag}\{t_1, t_2, \dots, t_n\} \geq 0$, $S = \text{diag}\{s_1, s_2, \dots, s_n\} \geq 0$ satisfying the following conditions :

$$\bar{M} = \begin{bmatrix} \bar{M}_0 & \bar{M}_1^T P_1 & \bar{M}_2^T P_2 & \bar{M}_3^T \\ * & -P_1 & 0 & 0 \\ * & * & -P_2 & 0 \\ * & * & * & -I \end{bmatrix} < 0,$$

where

$$\bar{M}_0 = \begin{bmatrix} -P_2 - 2L_1 T + \hat{\tau} Q & P_2 & 0 & -2L_2 T & 0 & 0 \\ * & -P_2 & 0 & 0 & 0 & 0 \\ * & * & -Q - 2V_1 S & 0 & -2V_2 S & 0 \\ * & * & * & -2T & 0 & 0 \\ * & * & * & * & -2S & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\bar{M}_1 = [A_1 \ 0 \ 0 \ A_2 \ A_3 \ B], \quad \bar{M}_2 = [\bar{A}_1 - A_F \ 0 \ \bar{A}_2 \ \bar{A}_3 \ \bar{B}], \quad \bar{M}_3 = [\bar{K}_1 - C_F \ 0 \ \bar{K}_2 \ \bar{K}_3 \ \bar{G}],$$

$$\bar{A}_1 = A_1 - B_F C_1, \quad \bar{A}_2 = A_2 - B_F C_2, \quad \bar{A}_3 = A_3 - B_F C_3, \quad \bar{B} = B - B_F D,$$

$$\bar{K}_1 = K_1 - D_F C_1, \quad \bar{K}_2 = K_2 - D_F C_2, \quad \bar{K}_3 = K_3 - D_F C_3, \quad \bar{G} = G - D_F D,$$

$$\hat{\tau} = \tau_2 - \tau_1 + 1.$$

3. Given the scalar $\gamma > 0$. Then, the nonlinear discrete-time neural network (5.1) an H_∞ filter (5.5) can be designed such that the filtering error system (5.6) with time-varying delay $\tau(k)$ satisfying (5.2) if there exist $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $Q = Q^T > 0$, $\bar{A}_F$, $\bar{B}_F$, $\bar{C}_F$, $\bar{D}_F$, $T = \text{diag}\{t_1, t_2, \dots, t_n\} \geq 0$, $S = \text{diag}\{s_1, s_2, \dots, s_n\} \geq 0$ satisfying the following conditions :

$$\bar{M} = \begin{bmatrix} \bar{M}_0 & \bar{M}_1^T P_1 & \bar{M}_2^T P_2 & \bar{M}_3^T \\ * & -P_1 & 0 & 0 \\ * & * & -P_2 & 0 \\ * & * & * & -I \end{bmatrix} < 0$$

where $\bar{M}_0, \bar{M}_1, \bar{M}_2, \bar{M}_3$ are defined in Theorem 5.1.2. Moreover, the filter parameters of the form (5.5) is designed as follows

$$A_F = -P_2^{-1} \bar{A}_F, B_F = -P_2^{-1} \bar{B}_F, C_F = -\bar{C}_F, D_F = \bar{D}_F.$$