CHAPTER 4
DISCUSSIONS AND EXAMPLES

We now provide simulation results to discuss characteristics of our control law. Consider the planar system

$$\begin{aligned}\dot{x}_1 &= x_2^3\\ \dot{x}_2 &= u + x_2^q\\ y &= x_1\end{aligned} \tag{4.1}$$

When $1 \leq q < 4$, the system (4.1) can be globally asymptotically stabilized by output feedback under the growth conditions in [9], [10]. In the case that $0 < q \leq 1$, the methods in [9] and [10] cannot be used since they require that ϕ_i are differentiable with respect to state x_i. For illustration proposes, we choose $q = \frac{1}{3}$. the system (4.1) becomes

$$\begin{aligned}\dot{x}_1 &= x_2^3\\ \dot{x}_2 &= u + x_2^{\frac{1}{3}}\\ y &= x_1.\end{aligned} \tag{4.2}$$

Clearly, with $m_1 = 1, \tau = -\frac{2}{11}, m_2 = \frac{3}{11}$, Assumptiom 3.1 holds, that is $|x_2|^{\frac{1}{3}} \leq \hat{c}\big(|x_1|^{\frac{1}{11}} + |x_2|^{\frac{1}{3}}\big)$, for some $\hat{c} > 0$. Therefore, using Theorem 3.2, the system (4.2) can be globally asymptotically stabilized by the homogeneous dynamic output feedback control law,

$$\dot{\eta} = -Ll_1(\eta + l_1 y)^{\frac{9}{11}}, \quad u = -L^{\frac{4}{3}}\beta_2(\eta + l_1 y + \beta_1 y)^{\frac{1}{11}} \tag{4.3}$$

where β_1, β_2, l_1, and L are large enough positive constants. From Theorem 3.2, we can estimate value of $(\beta_1, \beta_2, l_1, L) = (5, 310, 200, 1)$. For simplicity, we fix $L = 1$ since it can be included with the observer gain l_1. We vary the gains, l_1, β_1 and β_2 to illustrate the effects of these gains to the closed-loop trajectories. All simulations are performed with the initial condition $\big(x_1(0), x_2(0), \eta(0)\big) = (1, -1, 0)$. From Figures 4.1, 4.2 and 4.3, increasing the gain l_1 reduces the oscillation in the states

x_1 and x_2 but increases the peak in η. State trajectories in Figures 4.4, 4.5 and 4.6 show that increasing the gain β_2 reduces the convergence times to zero of all state trajectories without promoting too much oscillations. Increasing the gain β_1, shown in Figures 4.7, 4.8 and 4.9, also reduces the convergence times of all state trajectories but promotes oscillations especially in x_2 and η.

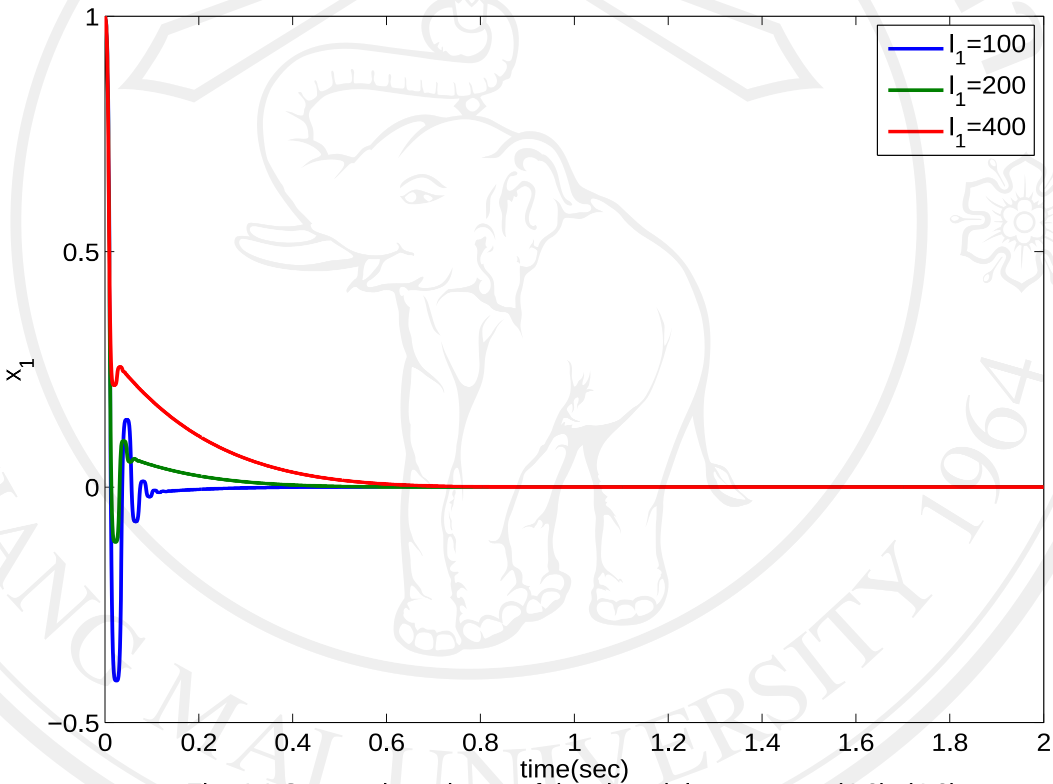

Fig. 4.1 State trajectories x_1 of the closed−loop system (4.2)−(4.3) with $(x_1(0),x_2(0),\eta(0))=(1,-1,0)$ and $(\beta_1,\beta_2)=(5,310)$

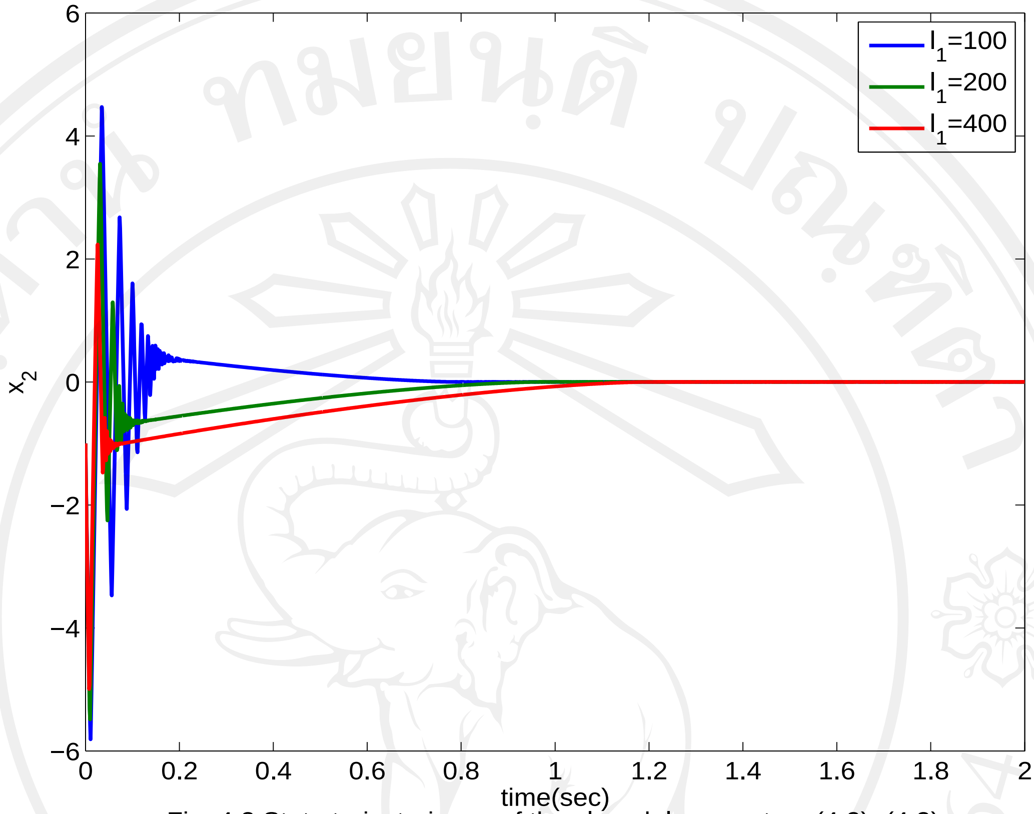

Fig. 4.2 State trajectories x_2 of the closed−loop system (4.2)−(4.3) with $(x_1(0),x_2(0),\eta(0))=(1,-1,0)$ and $(\beta_1,\beta_2)=(5,310)$

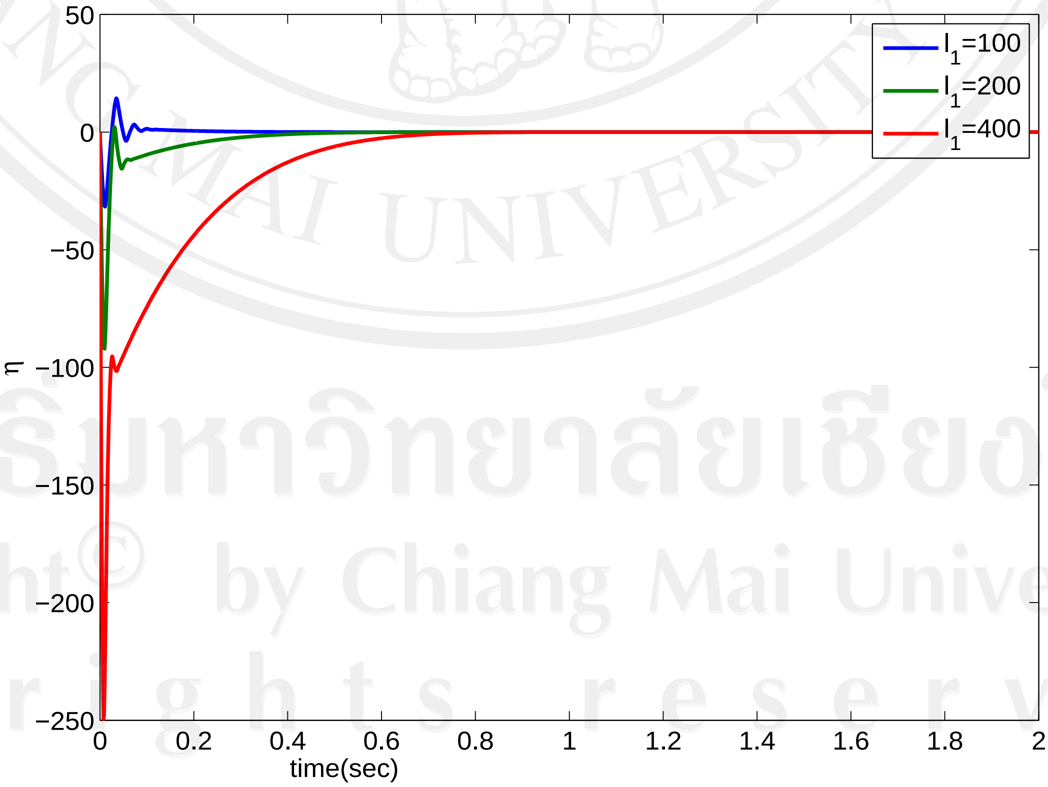

Fig. 4.3 State trajectories η of the closed−loop system (4.2)−(4.3) with $(x_1(0),x_2(0),\eta(0))=(1,-1,0)$ and $(\beta_1,\beta_2)=(5,310)$

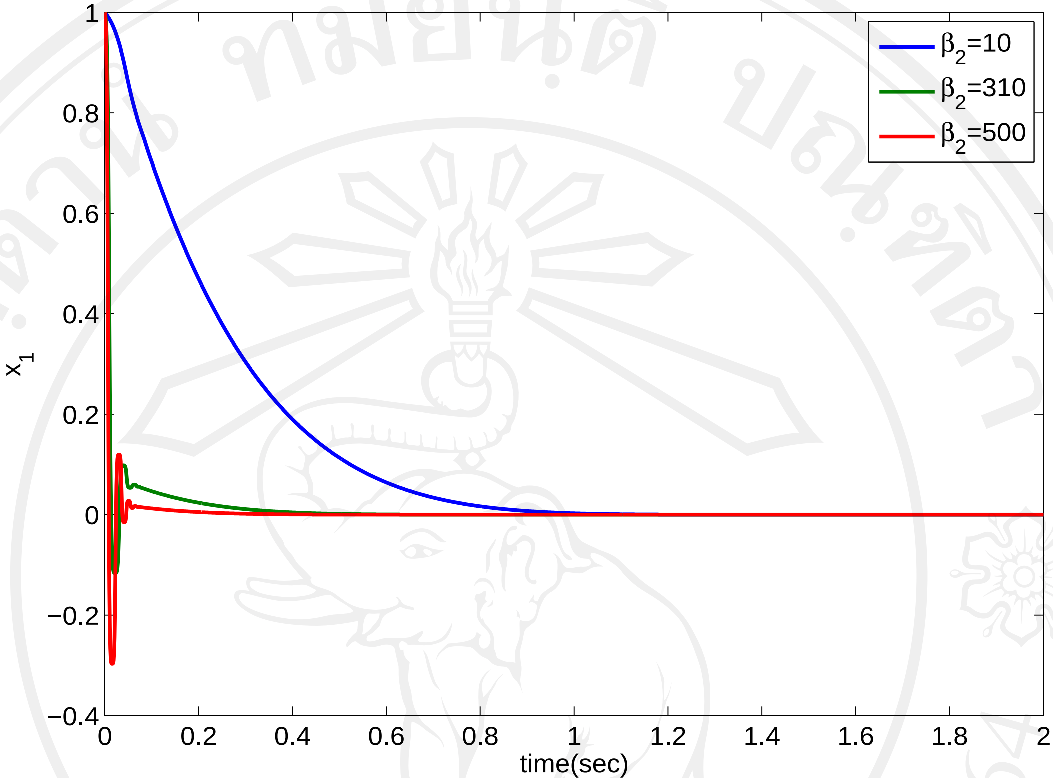

Fig. 4.4 State trajectories x_1 of the closed−loop system (4.2)−(4.3) with $(x_1(0),x_2(0),\eta(0))=(1,-1,0)$ and $(\beta_1,l_1)=(5,200)$

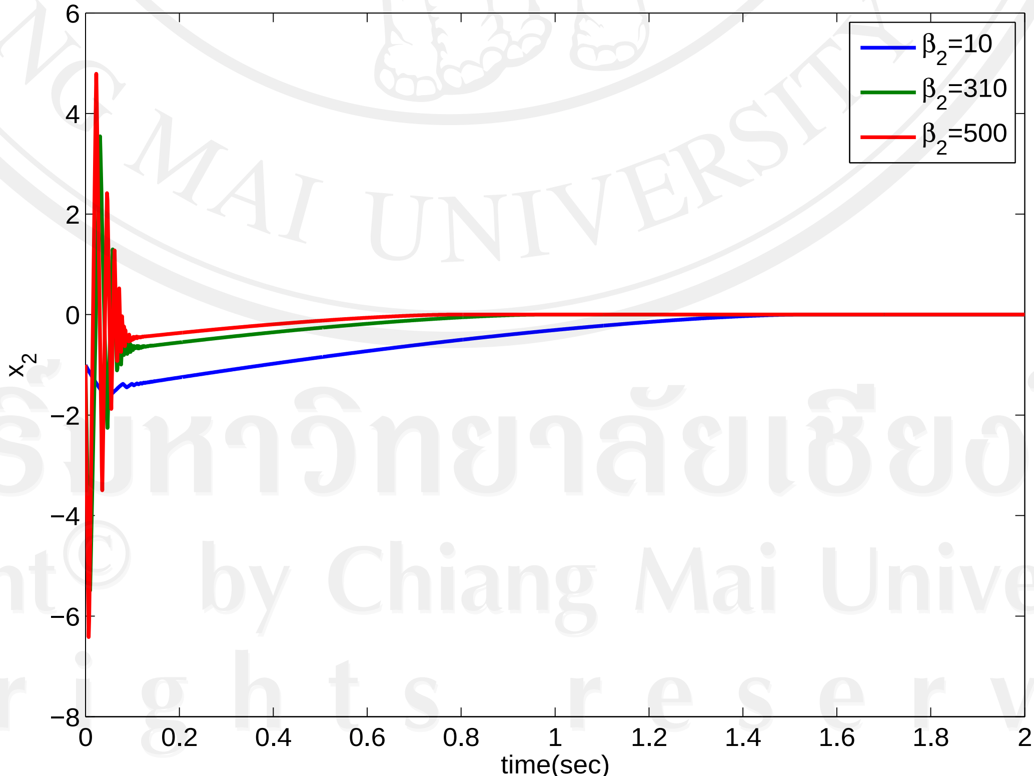

Fig. 4.5 State trajectories x_2 of the closed−loop system (4.2)−(4.3) with $(x_1(0),x_2(0),\eta(0))=(1,-1,0)$ and $(\beta_1,l_1)=(5,200)$

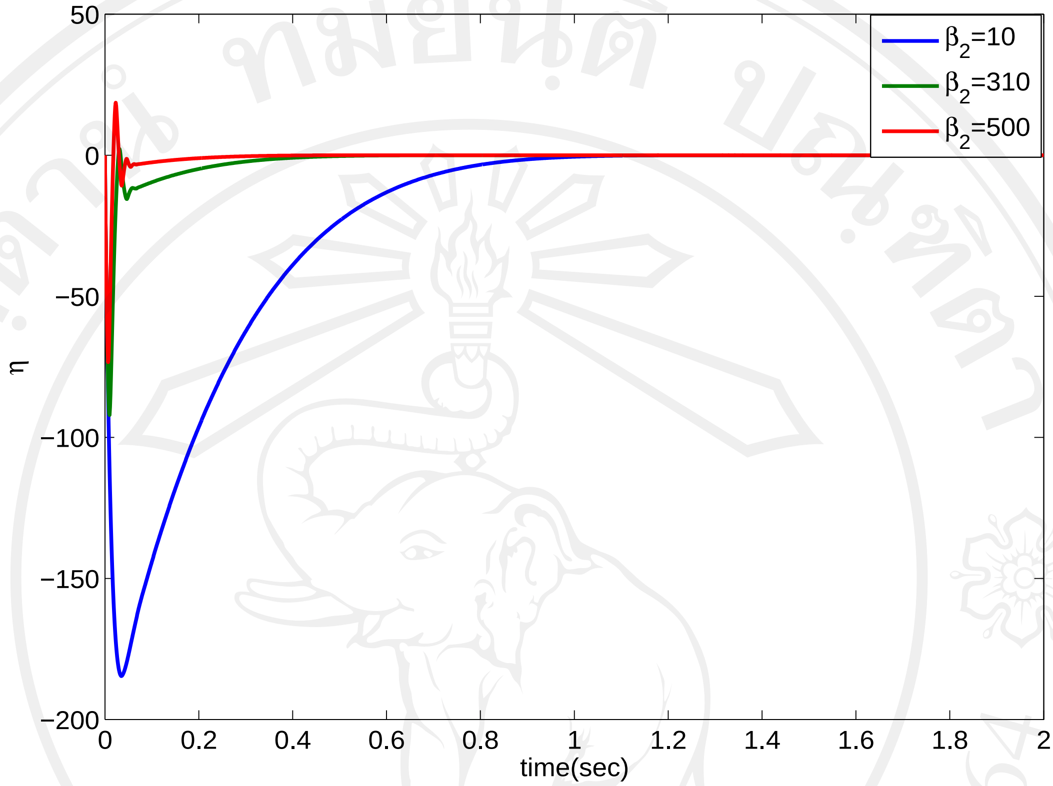

Fig. 4.6 State trajectories η of the closed−loop system (4.2)−(4.3) with $(x_1(0),x_2(0),\eta(0))=(1,-1,0)$ and $(\beta_1,l_1)=(5,200)$

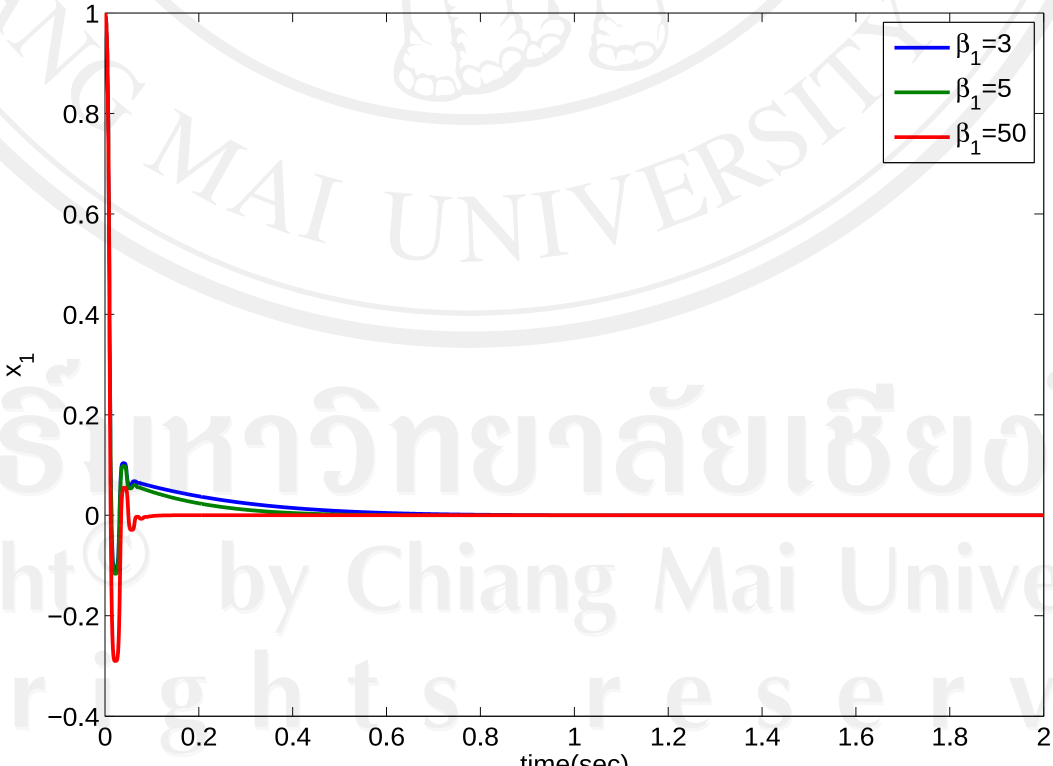

Fig. 4.7 State trajectories x_1 of the closed−loop system (4.2)−(4.3) with $(x_1(0),x_2(0),\eta(0))=(1,-1,0)$ and $(\beta_2,l_1)=(310,200)$

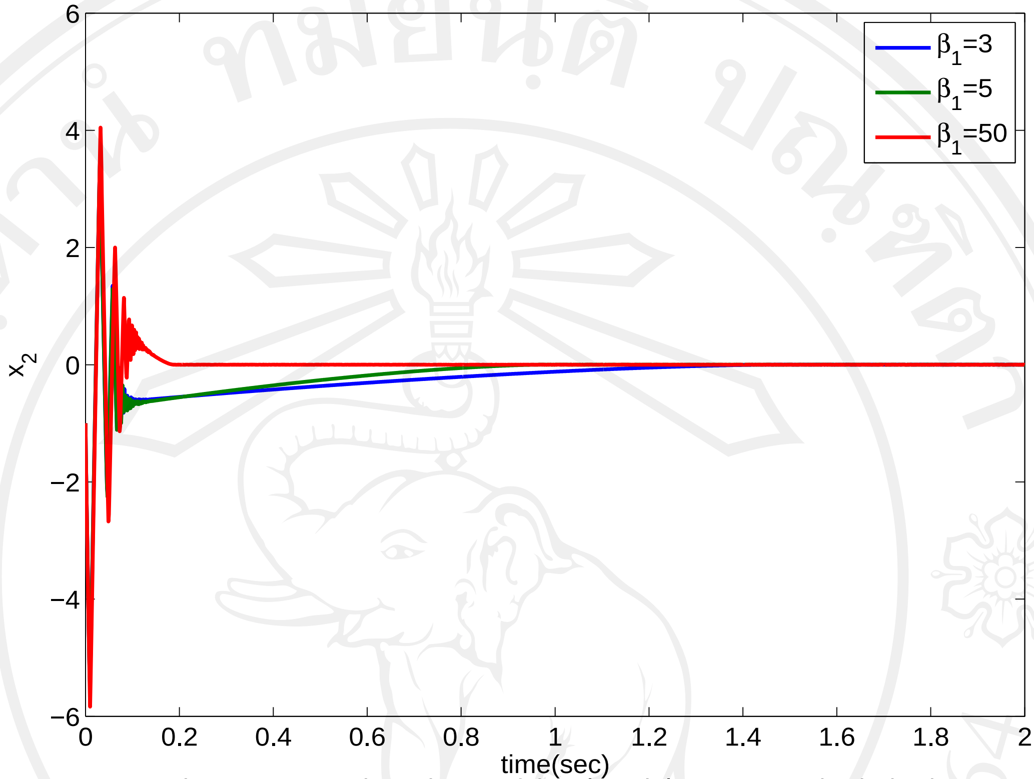

Fig. 4.8 State trajectories x_2 of the closed−loop system (4.2)−(4.3) with $(x_1(0),x_2(0),\eta(0))=(1,-1,0)$ and $(\beta_2,l_1)=(310,200)$

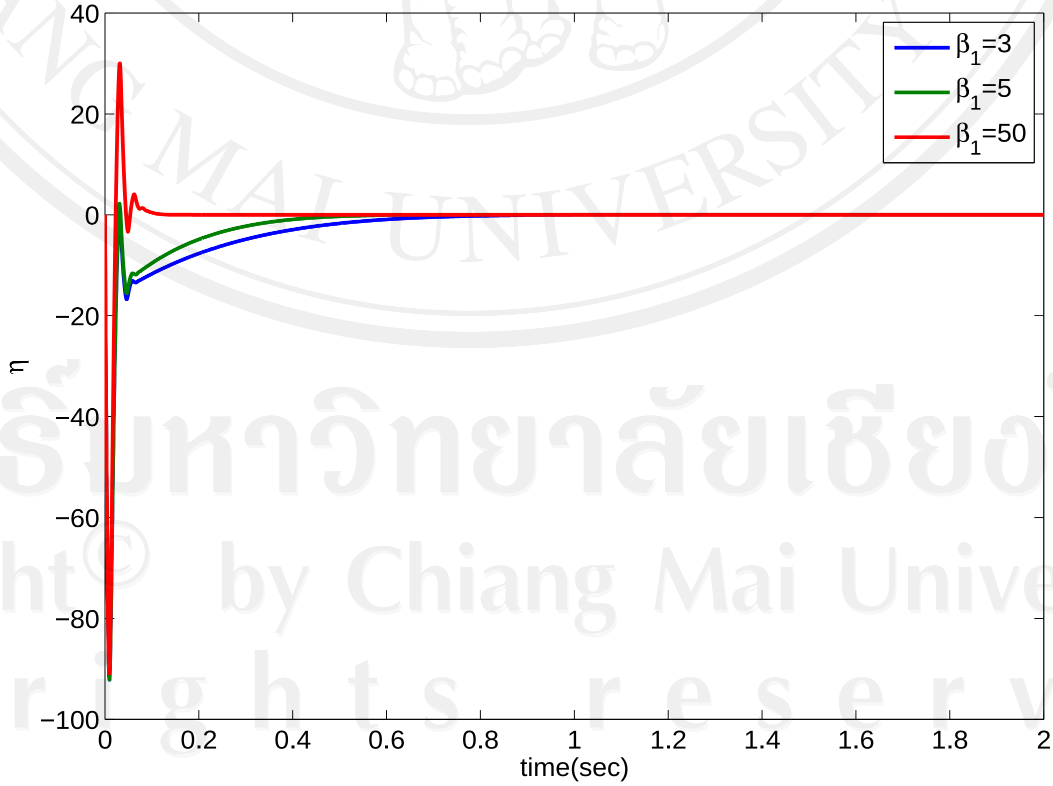

Fig. 4.9 State trajectories η of the closed−loop system (4.2)−(4.3) with $(x_1(0),x_2(0),\eta(0))=(1,-1,0)$ and $(\beta_2,l_1)=(310,200)$